Turaev in Toulouse

May 15, 2017 5:08 AM

Spin structure:

$$\begin{tikzcd} & BSpin(n) \arrow[d] \\ X^n \arrow[r] \arrow[ur, dashed] & BSO(n) \end{tikzcd}$$

X-HQFT: TQFT for mflds endowed with a maps to a fixed target space X

homotopy class of

[really, pointed mflds, maps preserve basepoints]

Example: HQFT in dim = 0+1

$$+\bullet \to V \qquad -\bullet \to V^* \qquad \begin{matrix} +\bullet \\ | \\ +\bullet \end{matrix} \xrightarrow{\alpha} (X, x)$$

$\alpha \in \pi = \pi_1(X, x)$

$\Rightarrow$ an action of π on V.

Focus on $X = K(\pi, 1)$

2-dim HQFT w/ target $X = K(\pi, 1)$

$\Updownarrow$

Crossed Frobenius bi-algebras.

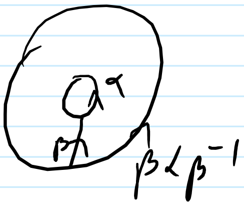
Indeed, in an HQFT τ,

$\alpha \in \pi$

$\xrightarrow{obj} L_\alpha = T(\bigcirc_\alpha)$

Pair of pants: map $L_\alpha \times L_\beta \to L_{\alpha\beta}$

$L = \bigoplus_\alpha L_\alpha$ a π-graded v.s.



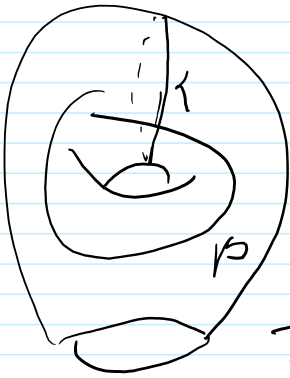
: $\varphi_\beta : L_\alpha \longrightarrow L_{\beta\alpha\beta^{-1}}$

+ unit + co-unit.

"Commutativity" $a \in L_\alpha$ $b \in L_\beta$

$$ab = b \cdot \varphi_{\beta^{-1}}(a)$$

A condition from the punctured torus:



: $L_{\alpha\beta\alpha^{-1}\beta^{-1}} \longrightarrow \mathbb{C}$

$$Tr(\hat{c}\,\varphi_\beta) = tr(\varphi_{\alpha^{-1}}\hat{c})$$

for $c \in L_{\alpha\beta\alpha^{-1}\beta^{-1}}$

π a finite group $\} \longrightarrow \Sigma^2$ a princip.

π-bundle over a closed surface Σ^2.

$$\exists \text{ section} \iff \underbrace{\sum_{\substack{\rho\in I_0 = \text{irreps of } \pi, \\ \text{stable under } \pi_1\Sigma}} (\dim\rho)^{\chi(\Sigma)} \cdot \chi^\rho([\Sigma])}_{T} \neq 0$$

in fact, $\#\{\text{sections}\} = T \cdot |\pi|^{2g(\Sigma)-1}$

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3D (w/ Alexis Virelizier)

$\rightsquigarrow$ π-graded categories.