Pensieve header: Verifying the Quesne formula.

Quesne@arXiv:math-ph/0305003: $e_q^x = \exp\left(\sum_{k\geq 1} \frac{(1-q)^k x^k}{k(1-q^k)}\right)$.

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c[k_] := (1 - q)^k / (k (1 - q^k));
Table[c[k], {k, 1, 10}]
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$\left\{1, \frac{(1-q)^2}{2(1-q^2)}, \frac{(1-q)^3}{3(1-q^3)}, \frac{(1-q)^4}{4(1-q^4)}, \frac{(1-q)^5}{5(1-q^5)}, \frac{(1-q)^6}{6(1-q^6)}, \frac{(1-q)^7}{7(1-q^7)}, \frac{(1-q)^8}{8(1-q^8)}, \frac{(1-q)^9}{9(1-q^9)}, \frac{(1-q)^{10}}{10(1-q^{10})}\right\}$

```
a[n_] := a[n] = Together@SeriesCoefficient[Exp[Sum[c[k] x^k, {k, 1, n}]], {x, 0, n}];

Table[Echo[n → a[n]], {n, 0, 10}];
```

» $0 \to 1$

» $1 \to 1$

» $2 \to \frac{1}{1+q}$

» $3 \to \frac{1}{(1+q)(1+q+q^2)}$

» $4 \to \frac{1}{(1+q)^2(1+q^2)(1+q+q^2)}$

» $5 \to \frac{1}{(1+q)^2(1+q^2)(1+q+q^2)(1+q+q^2+q^3+q^4)}$

» $6 \to \frac{1}{(1+q)^3(1+q^2)(1-q+q^2)(1+q+q^2)^2(1+q+q^2+q^3+q^4)}$

» $7 \to 1/((1+q)^3(1+q^2)(1-q+q^2)(1+q+q^2)^2(1+q+q^2+q^3+q^4)(1+q+q^2+q^3+q^4+q^5+q^6))$

» $8 \to 1/((1+q)^4(1+q^2)^2(1-q+q^2)(1+q+q^2)^2(1+q^4)(1+q+q^2+q^3+q^4)(1+q+q^2+q^3+q^4+q^5+q^6))$

» $9 \to 1/((1+q)^4(1+q^2)^2(1-q+q^2)(1+q+q^2)^3(1+q^4)(1+q+q^2+q^3+q^4)(1+q^3+q^6)(1+q+q^2+q^3+q^4+q^5+q^6))$

» $10 \to 1/((1+q)^5(1+q^2)^2(1-q+q^2)(1+q+q^2)^3(1+q^4)(1-q+q^2-q^3+q^4)(1+q+q^2+q^3+q^4)^2(1+q^3+q^6)(1+q+q^2+q^3+q^4+q^5+q^6))$

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Table[FunctionExpand@QFactorial[n, q], {n, 0, 6}]
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$\{1, 1, 1+q, (1+q)(1+q+q^2), (1+q)^2(1+q^2)(1+q+q^2), (1+q)^2(1+q^2)(1+q+q^2)(1+q+q^2+q^3+q^4), (1+q)^3(1+q^2)(1-q+q^2)(1+q+q^2)^2(1+q+q^2+q^3+q^4)\}$

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Table[a[n] FunctionExpand@QFactorial[n, q], {n, 0, 10}]
{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}
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