Old geps vs tgeps

March 11, 2017 1:24 PM

1-Smidgen sl_2 Let $\mathfrak{g}_1$ be the 4-dimensional Lie algebra $\mathfrak{g}_1 = \langle b, c, u, w\rangle$ over the ring $R = \mathbb{Q}[\epsilon]/(\epsilon^2 = 0)$, with b central and with $[w, c] = w$, $[c, u] = u$, and $[u, w] = b - 2\epsilon c$, with CYBE $r_{ij} = (b_i - \epsilon c_i)c_j + u_i w_j$ in $\mathcal{U}(\mathfrak{g}_1)^{\otimes\{i,j\}}$. Over $\mathbb{Q}$, $\mathfrak{g}_1$ is a solvable approximation of sl_2: $\mathfrak{g}_1 \supset \langle b, u, w, \epsilon b, \epsilon c, \epsilon u, \epsilon w\rangle \supset \langle b, \epsilon b, \epsilon c, \epsilon u, \epsilon w\rangle \supset 0$.
(note: $\deg(b, c, u, w, \epsilon) = (1, 0, 1, 0, 1)$)

$t\mathfrak{g}^\epsilon = \langle b, e, g, f\rangle / \left([g, e] = 2e,\ [g, f] = -2f,\ [e, f] = b + \epsilon g,\ [b, *] = 0\right)$

old	b	u	c	w	ϵ	$(b-\epsilon c)\otimes c + u\otimes w$
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
new	b	e	$\frac{1}{2}g$	f	$-\epsilon$	$(b+\frac{1}{2}\epsilon g)\otimes\frac{1}{2}g + e\otimes f$

$[c, w] = -w \longrightarrow [\frac{1}{2}g, f] = -f$ ✓

$[c, u] = u \longrightarrow [\frac{1}{2}g, e] = e$ ✓

$[u, w] = b - 2\epsilon c \longrightarrow [e, f] = b + \epsilon g$ ✓