LY 30 YYZ -> TLV

March 26, 2017 2:01 PM

Another task for these flights: sort out the degeneration story and the doubling of the Cartan.

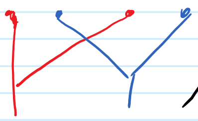
(160611) The quantum double $\mathcal{D}A := A^{*,op} \otimes A$ with $(\phi a)(\psi b) := \langle S a_1, \psi_1\rangle\langle a_3, \psi_3\rangle(\psi_2\phi)(a_2 b)$. What problem does it solve?

} Why the extra complexity rel. the below ?

(170325a) Generalized Weyl: If $f \in \mathcal{S}(V)$ and $\psi \in \mathcal{S}(V^*)$ then in $\mathcal{U}(HV)$, $f\psi = \psi_1 f_1\langle\psi_2, f_2\rangle$, where $\Delta f = \sum f_1 \otimes f_2$ and $\Delta\psi = \sum \psi_1 \otimes \psi_2$. Is there a version with $\mathcal{U}(\mathfrak{g})$ replacing $\mathcal{S}(V)$?

In other words, what conditions on $m_V, m_{V^*}, \Delta_V, \Delta_{V^*}$ make

$f\psi \longrightarrow \psi_1 f_1 \langle\psi_2, f_2\rangle$ a "swap" satisfying

(Enough for associativity of )

(1) $(fg)\psi \mapsto \psi_1 (fg)_1 \langle\psi_2, (fg)_2\rangle = \psi_1 f_1 g_1 \langle\psi_2, f_2 g_2\rangle$

(2) $(fg)\psi \mapsto f\psi_1 g_1\langle\psi_2, g_2\rangle \mapsto \psi_1 f_1 g_1 \langle\psi_2, f_2\rangle\langle\psi_3\, g_2\rangle$

Cond seems to be $\langle\psi, f\cdot g\rangle = \langle\psi_1, f\rangle\langle\psi_2, g\rangle$

presumably also $\langle\psi\phi, f\rangle = \langle\psi, f_1\rangle\langle\phi, f_2\rangle$

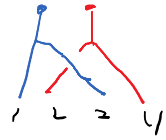
From Chari-Pressley, 4.2D: (not yet the quantum double)

Let B and C be Hopf algebras over a commutative ring k, and let $\mathcal{R}$ be an invertible element of $C \otimes B$ such that

$$(17) \qquad (\Delta^C \otimes \mathrm{id})(\mathcal{R}) = \mathcal{R}_{13}\mathcal{R}_{23}, \qquad (\mathrm{id} \otimes \Delta^B)(\mathcal{R}) = \mathcal{R}_{12}\mathcal{R}_{13},$$
$$(\mathrm{id} \otimes S^B)(\mathcal{R}) = \mathcal{R}^{-1}, \qquad (S^C \otimes \mathrm{id})(\mathcal{R}) = \mathcal{R}^{-1}.$$

PROPOSITION 4.2.10 *If $\mathcal{R} \in C \otimes B$ satisfies (17), then $B \otimes C$, with the usual algebra structure and with comultiplication*

$$\Delta(b \otimes c) = \mathcal{R}_{23}\Delta^B_{13}(b)\Delta^C_{24}(c)\mathcal{R}_{23}^{-1},$$



antipode

$$S(b \otimes c) = \mathcal{R}_{21}^{-1}(S^B(b) \otimes S^C(c))\mathcal{R}_{21},$$

and counit

$$\epsilon(b \otimes c) = \epsilon^B(b)\epsilon^C(c),$$

is a Hopf algebra, which we denote by $B \underset{\mathcal{R}}{\otimes} C$.

Makes sense in AW!
C = tails w/ the KV coproduct.
B = heads w/ no mods.

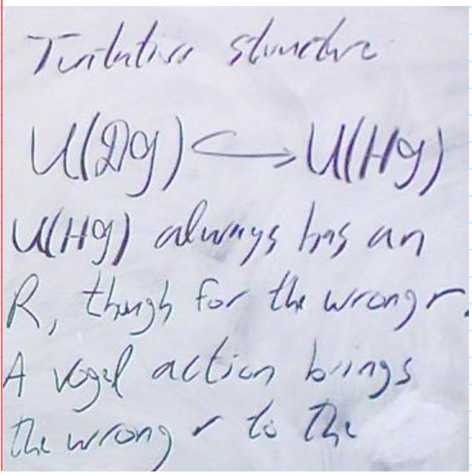
$B \otimes C$ is the "split presentation", or $\pi_2 \times \pi_1$ (not $\pi_2 \ltimes \pi_1$), as algebra.
Weird coproduct! What does it do in w-land?

Cp later proceed to dualize:

Thus, we may define the *quantum double* of A to be

$$\mathcal{D}(A) = (A^* \underset{\mathcal{R}}{\otimes} A_{\mathrm{op}})^*.$$

wherefore?



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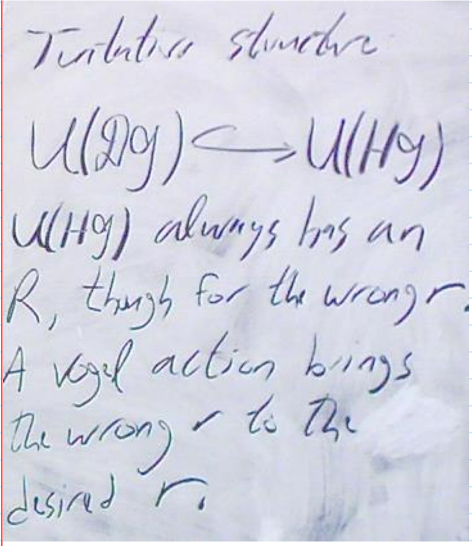


: Dror - 170323 @ BBS

From Drinfeld's ICM paper:

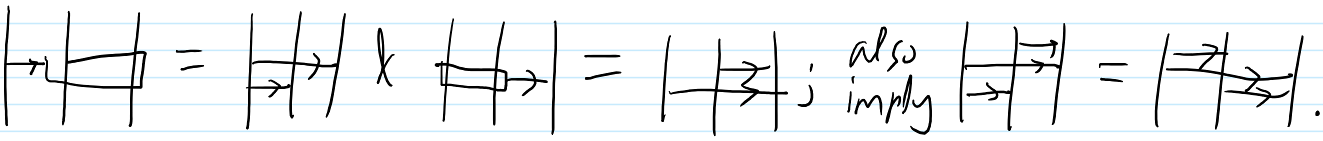
Now let us define the *quantum double*. Let A be a Hopf algebra. Denote by A° the algebra A^* with the opposite comultiplication. It can be shown that there is a unique quasitriangular Hopf algebra $(D(A), R)$ such that (1) $D(A)$ contains A and A° as Hopf subalgebras, (2) R is the image of the canonical element of $A \otimes A^\circ$ under the embedding $A \otimes A^\circ \hookrightarrow D(A) \otimes D(A)$, and (3) the linear mapping $A \otimes A^\circ \to D(A)$ given by $a \otimes b \mapsto ab$ is bijective. As a vector space, $D(A)$ can be identified with $A \otimes A^\circ$, and the Hopf algebra structure on $A \otimes A^\circ$ can be found from the commutation relations

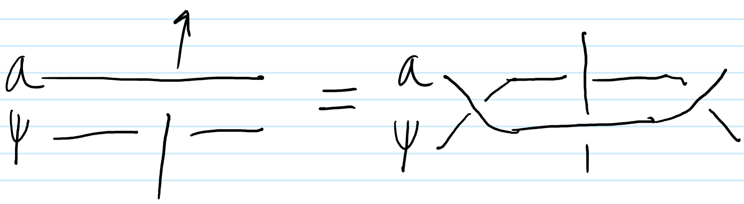
$$e_s e^t = \mu_s^{kjn} m_{plk}^t \sigma_n^p e^l e_j \quad \text{and} \quad e^t e_s = \mu_s^{njk} m_{klp}^t \sigma_n^p e_j e^l,$$

which follow from (1)–(3). Here $\{e_s\}$ and $\{e^t\}$ are dual bases of A and A°, while $\sigma, m,$ and μ are the matrices of the skew antipode $A \to A$, the multiplication map $A \otimes A \otimes A \to A$, and the comultiplication map $A \to A \otimes A \otimes A$. If A is a QUE-algebra we will understand A° in the QUE-sense. Then $D(A)$ is a QUE-algebra, too. If $\mathfrak{g}$ is the classical limit of A then $(D(\mathfrak{g}), r)$ is the classical limit of $(D(A), R)$

$\Delta^{op} = R\Delta R^{-1}$, $(\Delta 1)R = RR$, $(1\Delta)R = RR$

But in qsl_2 R is not the image of the canonical element?





The key DD identity