$R(5,5)\leq 48$

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For the purpose of this discussion, any pair of people in the world either love each other or hate each other, and the feelings are always mutual.

Within a group of people, can you always find a triple such that they all love each other or all hate each other?

If the group is size 4, the answer is no. Arrange the people in a square, and it may be that each person loves her neighbors but hates the people diagonally away from her. Within such an arrangement you cannot find a love triangle, nor a hate triangle.

The exact same thing works for pentagons (see e.e. the first image at https://en.wikipedia.org/wiki/Ramsey%27s_theorem), but fails to work for hexagons, because in a hexagon you can find a triangle made entirely of diagonals, so there would be a hate triangle.

Indeed it is not too hard to show that within a group of six people you will always find either a love triangle or a hate triangle. In Ramsey Theory Talk, R(3,3)=6.

Suppose now you had a party with n people and you wanted to find a love square or a hate square - a group of four people who all love each other or all hate each other. It turns out that in a party of 18 people this is always possible, yet in a party of 17 it may not be possible. In Ramsey Theory Talk, R(5,5)=18.

An old theorem of Ramsey says that for any k, R(k,k) exists. In other words, given k, in a large enough party you will always be able to find a group of k people who all love each other or who all hate each other. But Ramsey does not tell us what the number R(k,k) really is. As per the discussion above, R(3,3)=6 and R(5,5)=18, but other than that and the even smaller cases (which are silly), no other R(k,k) number is known.

The title of Brendan's paper, "R(5,5)<=48", sufficiently explains the main part of what he did (with Angeltveit). See https://arxiv.org/abs/1703.08768.

The problem is hard because the number of possible relationships to inspect within a given party grows very rapidly as the size of the party grows. Hence one has to be *very* clever about how to study parties with around 50 participants *without* having to inspect all possible relationships between them. Even then, when cleverness runs its course, it remains a huge computational challenge. I think the real interest in Brendan's result is not so much in it in itself, but more in the cleverness that was employed.

Gil Kalai: https://gilkalai.wordpress.com/2017/03/29/r55-%E2%89%A4-48/