

Pensieve header: Implementing  $\mathcal{U}(sl^+_2)$ . Modifies pensieve://2017-03/nb/sl2.pdf.

```
B[0, _] = 0; B[_, 0] = 0;
B[c_*x : (c | e | f | h), y_] := Expand[c B[x, y]];
B[y_, c_*x : (c | e | f | h)] := Expand[c B[y, x]];
B[x_Plus, y_] := B[#, y] & /@ x;
B[x_, y_Plus] := B[x, #] & /@ y;
B[x_, x_] = 0;
B[y_, x_] := Expand[-B[x, y]];
```

```
B[h, e] = 2 e; B[h, f] = -2 f; B[e, f] = h + c; B[c, _] = 0;
Basis[] := {c, e, f, h};
```

Basis[]

{c, e, f, h}

Table[{x, y} → B[x, y], {x, Basis[]}, {y, Basis[]} // MatrixForm

$$\begin{pmatrix} \{c, c\} \rightarrow 0 & \{c, e\} \rightarrow 0 & \{c, f\} \rightarrow 0 & \{c, h\} \rightarrow 0 \\ \{e, c\} \rightarrow 0 & \{e, e\} \rightarrow 0 & \{e, f\} \rightarrow c + h & \{e, h\} \rightarrow -2 e \\ \{f, c\} \rightarrow 0 & \{f, e\} \rightarrow -c - h & \{f, f\} \rightarrow 0 & \{f, h\} \rightarrow 2 f \\ \{h, c\} \rightarrow 0 & \{h, e\} \rightarrow 2 e & \{h, f\} \rightarrow -2 f & \{h, h\} \rightarrow 0 \end{pmatrix}$$

Union@Table[{x, y} = t; B[x, y] + B[y, x], {t, Tuples[Basis[], 2]}]

{0}

```
DeleteCases[Table[
  ({x, y, z} = t) → B[x, B[y, z]] + B[y, B[z, x]] + B[z, B[x, y]],
  {t, Tuples[Basis[], 3]}
], _ → 0]
{}
```

```
PBWRule = {};
x_ ≤ y_ := OrderedQ[{x, y} /. PBWRule]; x_ < y_ := ! OrderedQ[{y, x} /. PBWRule];
UU_i_[ε_] := ε /. (x : c | e | f | h) ⇒ U_i[x];
Simp[ε_] := Collect[ε, Times[U[___] ..], Expand];
Simp[ε_] := Expand[ε];
```

```
m_s_[0] = 0; m_s_[x_Plus] := m_s_ /@ x;
m_i→j_[ε_] := ε /. U_i → U_j;
```

```
m_i,j→k_[c_. U_i[x___] U_j[]] := c U_k[x];
m_i,j→k_[c_. U_i[] U_j[y___]] := c U_k[y];
m_i,j→k_[c_. U_i[xx___, x_] U_j[y_, yy___]] := If[x ≤ y,
  c U_k[xx, x, y, yy],
  ((U_i[xx] (U_j[y, x] + UU_j[B[x, y]])) // Expand // m_i,j→i) U_j[yy] // Expand // m_i,j→k)
  c // Simp
];
```

```

c U1[e, f] U2[e, e, f] + (a + b) U1[h] U2[f, f] // m1,2→3
-4 a U3[f, f] - 4 b U3[f, f] + 2 c U3[e, e, f] + a U3[f, f, h] +
b U3[f, f, h] - 2 c U3[c, e, e, f] - 2 c U3[e, e, f, h] + c U3[e, e, e, f, f]

```

```

UProducts[{}, 0] = {1}; UProducts[{}, d_Integer] /; d > 0 = {};
UProducts[{i_, is___}, d_Integer] := Sort@Flatten@
Table[{Ui@@Subsets[Basis[], {j}]} u, {j, 0, d}, {u, UProducts[{is}, d-j]}}];

```

```
UProducts[{1}, 1]
```

```
{U1[c], U1[e], U1[f], U1[h]}
```

```
Union[{u ↦ m1,3→1[m1,2→1[u]] - m1,2→1[m2,3→2[u]]} /@ UProducts[{1, 2, 3}, 3]]
```

```
{0}
```

```
S[ $\mathcal{E}$ ] := Union@Cases[{ $\mathcal{E}$ }, Ui[____] ⇔ i, ∞];
```

```

Unprotect[NonCommutativeMultiply];
NonCommutativeMultiply[x_] := x;
x_ ** y_ := Module[{is = S[x] ∩ S[y], σ, z},
z = x; Do[z = mi→σ@i[z], {i, is}];
z = Expand[y z]; Do[z = mσ@i, i→i[z], {i, is}]; z];
UB[x_, y_] := Simp[x ** y - y ** x];

```

```
{x = RandomChoice[UProducts[{1, 2}, 3]], y = RandomChoice[UProducts[{1, 2, 3}, 3]], x ** y}
```

```
{U1[ ] U2[c, f, h], U1[h] U2[h] U3[f], U1[h] U2[c, f, h, h] U3[f]}
```

$$r_{i,j} := U_i[e] U_j[f] + \frac{1}{4} U_i[h] U_j[h] + \frac{1}{2} \beta U_i[c] U_j[h] + \frac{1}{2} (1 - \beta) U_i[h] U_j[c];$$

```
UB[r1,2, r1,3] + UB[r1,2, r2,3] + UB[r1,3, r2,3]
```

```
0
```