

Pensieve header: Simplifying the logos formula.

```
DPx→Dα,y→Dβ[P-][f-] :=
  Total[CoefficientRules[P, {x, y}] /. ({m-, n-} → c-) ⇒ c D[f, {α, m}, {β, n}]]
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OΛk[e-, L-, f-, α-, β-, δ-] := OΛk[e, L, f, α, β, δ] = Module[
  {ρh, ρe, ρl, ρf, eqn, a, b, c, sol, q, λ, v},
  ρh =  $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ ; ρe =  $\begin{pmatrix} 0 & 0 \\ -\epsilon & 0 \end{pmatrix}$ ; ρl =  $\begin{pmatrix} -(1+1/\epsilon)/2 & 0 \\ 0 & (1-1/\epsilon)/2 \end{pmatrix}$ ; ρf =  $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ ;
  eqn = MatrixExp[α ρf].MatrixExp[β ρe] ==
    MatrixExp[a ρe].MatrixExp[c (ρh - 2 ϵ ρl)].MatrixExp[b ρf];
  Echo[sol = Solve[Thread[Flatten/@eqn], {a, b, c}][[1]] /. C[1] → 0];
  λ = Simplify[e-f α - e β + h α β Normal@Series[ec h + a e - 2 ϵ c l + b f /. sol, {ϵ, 0, k}]];
  q = ev (f α + e β - h α β + e f δ);
  Collect[q-1 DPα→Df,β→De[λ][q] /. v → (1 + h δ)-1, ϵ, Simplify]];
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$\text{OA}_2[\mathbf{e}, \mathbf{l}, \mathbf{f}, \alpha, \beta, \delta]$

$\{ \mathbf{a} \rightarrow -\frac{\beta}{-1+\alpha\beta\epsilon}, \mathbf{b} \rightarrow -\frac{\alpha}{-1+\alpha\beta\epsilon}, \mathbf{c} \rightarrow \frac{\text{Log}[1-\alpha\beta\epsilon]}{\epsilon} \}$

1 +

$$\frac{1}{2(1+h\delta)^4} \left(2e\alpha\beta^2 - h\alpha^2\beta^2 + 4e\beta\delta - 4h\alpha\beta\delta + 2e^2\beta^2\delta - 2h\delta^2 + 4eh\beta\delta^2 - 4h^2\alpha\beta\delta^2 + e^2h\beta^2\delta^2 - 4h^2\delta^3 - 2h^3\delta^4 + 4l(1+h\delta)^2(\alpha(\beta+f\delta) + \delta(1+e\beta+ef\delta+h\delta)) + f^2\delta(\alpha+e\delta)(\alpha(2+h\delta) + e\delta(4+3h\delta)) + 2f(\alpha^2\beta + 2\alpha\delta(1+h\delta) + e\beta(2+h\delta)) + e\delta^2(4+6h\delta + 2h^2\delta^2 + e\beta(3+2h\delta)) \right) \epsilon + \frac{1}{24(1+h\delta)^8}$$

$$\begin{aligned} & (24l(1+h\delta)^4((\alpha+e\delta)^2(\beta+f\delta)^2 + 4\delta(\alpha+e\delta)(\beta+f\delta)(1+h\delta) + 2\delta^2(1+h\delta)^2) + \\ & 48l^2(1+h\delta)^4((\alpha+e\delta)^2(\beta+f\delta)^2 + 4\delta(\alpha+e\delta)(\beta+f\delta)(1+h\delta) + 2\delta^2(1+h\delta)^2) + \\ & 24f(\alpha+e\delta)(1+h\delta)^3 \\ & \left((\alpha+e\delta)^2(\beta+f\delta)^2 + 6\delta(\alpha+e\delta)(\beta+f\delta)(1+h\delta) + 6\delta^2(1+h\delta)^2 \right) + 48fl(\alpha+e\delta) \\ & (1+h\delta)^3 \left((\alpha+e\delta)^2(\beta+f\delta)^2 + 6\delta(\alpha+e\delta)(\beta+f\delta)(1+h\delta) + 6\delta^2(1+h\delta)^2 \right) + 24e \\ & (\beta+f\delta)(1+h\delta)^3 \left((\alpha+e\delta)^2(\beta+f\delta)^2 + 6\delta(\alpha+e\delta)(\beta+f\delta)(1+h\delta) + 6\delta^2(1+h\delta)^2 \right) + \\ & 48el(\beta+f\delta)(1+h\delta)^3 \left((\alpha+e\delta)^2(\beta+f\delta)^2 + 6\delta(\alpha+e\delta)(\beta+f\delta)(1+h\delta) + \right. \\ & \left. 6\delta^2(1+h\delta)^2 \right) + 12(\beta+f\delta)^2(e+eh\delta)^2 \\ & \left((\alpha+e\delta)^2(\beta+f\delta)^2 + 8\delta(\alpha+e\delta)(\beta+f\delta)(1+h\delta) + 12\delta^2(1+h\delta)^2 \right) + 12(\alpha+e\delta)^2 \\ & (f+fh\delta)^2 \left((\alpha+e\delta)^2(\beta+f\delta)^2 + 8\delta(\alpha+e\delta)(\beta+f\delta)(1+h\delta) + 12\delta^2(1+h\delta)^2 \right) + \\ & 24ef(1+h\delta)^2 \left((\alpha+e\delta)^3(\beta+f\delta)^3 + 9\delta(\alpha+e\delta)^2(\beta+f\delta)^2(1+h\delta) + \right. \\ & \left. 18\delta^2(\alpha+e\delta)(\beta+f\delta)(1+h\delta)^2 + 6\delta^3(1+h\delta)^3 \right) - 8h(1+h\delta)^2 \left((\alpha+e\delta)^3(\beta+f\delta)^3 + \right. \\ & \left. 9\delta(\alpha+e\delta)^2(\beta+f\delta)^2(1+h\delta) + 18\delta^2(\alpha+e\delta)(\beta+f\delta)(1+h\delta)^2 + 6\delta^3(1+h\delta)^3 \right) - \\ & 24hl(1+h\delta)^2 \left((\alpha+e\delta)^3(\beta+f\delta)^3 + 9\delta(\alpha+e\delta)^2(\beta+f\delta)^2(1+h\delta) + \right. \\ & \left. 18\delta^2(\alpha+e\delta)(\beta+f\delta)(1+h\delta)^2 + 6\delta^3(1+h\delta)^3 \right) - \\ & 12fh(\alpha+e\delta)(1+h\delta) \left((\alpha+e\delta)^3(\beta+f\delta)^3 + 12\delta(\alpha+e\delta)^2(\beta+f\delta)^2(1+h\delta) + \right. \\ & \left. 36\delta^2(\alpha+e\delta)(\beta+f\delta)(1+h\delta)^2 + 24\delta^3(1+h\delta)^3 \right) - \\ & 12eh(\beta+f\delta)(1+h\delta) \left((\alpha+e\delta)^3(\beta+f\delta)^3 + 12\delta(\alpha+e\delta)^2(\beta+f\delta)^2(1+h\delta) + \right. \\ & \left. 36\delta^2(\alpha+e\delta)(\beta+f\delta)(1+h\delta)^2 + 24\delta^3(1+h\delta)^3 \right) + \\ & 3h^2 \left((\alpha+e\delta)^4(\beta+f\delta)^4 + 16\delta(\alpha+e\delta)^3(\beta+f\delta)^3(1+h\delta) + 72\delta^2(\alpha+e\delta)^2 \right. \\ & \left. (\beta+f\delta)^2(1+h\delta)^2 + 96\delta^3(\alpha+e\delta)(\beta+f\delta)(1+h\delta)^3 + 24\delta^4(1+h\delta)^4 \right) \epsilon^2 \end{aligned}$$

$\text{FullSimplify}\left[e^{ch+ae-2\epsilon cl+bf} /. \left\{ \mathbf{a} \rightarrow -\frac{\beta}{-1+\alpha\beta\epsilon}, \mathbf{b} \rightarrow -\frac{\alpha}{-1+\alpha\beta\epsilon}, \mathbf{c} \rightarrow \frac{\text{Log}[1-\alpha\beta\epsilon]}{\epsilon} \right\} \right]$

$e^{\frac{f\alpha+e\beta}{1-\alpha\beta\epsilon}} (1-\alpha\beta\epsilon)^{-2l+\frac{h}{\epsilon}}$

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Lambda[e_, l_, f_, alpha_, beta_, delta_] := Module[{lambda},
  lambda = Echo[Expand@Normal@Series[e^{\frac{f\alpha+e\beta}{1-\alpha\beta\epsilon}} (1-\alpha\beta\epsilon)^{-2l+\frac{h}{\epsilon}}, {\epsilon, 0, k}] /. e -> 1];
  Collect[DP_{alpha>Df, beta>Dc}[lambda][e^{(f\alpha+e\beta+ef\delta)/(1+h\delta)}] /. e -> 1, \epsilon, Simplify];

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Simplify[$0\Lambda_2[e, l, f, \alpha, \beta, \delta] == \Lambda_2[e, l, f, \alpha, \beta, \delta]$]

True

$\Lambda_1[e, l, f, \alpha, \beta, \delta]$

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$$1 + 2 l \alpha \beta \epsilon + f \alpha^2 \beta \epsilon + e \alpha \beta^2 \epsilon - \frac{1}{2} h \alpha^2 \beta^2 \epsilon$$

$$1 + \frac{1}{2 (1+h\delta)^4} \left(4 l (1+h\delta)^2 \left((\alpha+e\delta) (\beta+f\delta) + \delta (1+h\delta) \right) + 2 f (\alpha+e\delta) (1+h\delta) \right. \\ \left. \left((\alpha+e\delta) (\beta+f\delta) + 2 \delta (1+h\delta) \right) + 2 e (\beta+f\delta) (1+h\delta) \left((\alpha+e\delta) (\beta+f\delta) + 2 \delta (1+h\delta) \right) - \right. \\ \left. h \left((\alpha+e\delta)^2 (\beta+f\delta)^2 + 4 \delta (\alpha+e\delta) (\beta+f\delta) (1+h\delta) + 2 \delta^2 (1+h\delta)^2 \right) \right) \epsilon$$

$\Lambda_2[e, l, f, \alpha, \beta, \delta]$

$$\begin{aligned}
 & 1 + 2l\alpha\beta\epsilon + f\alpha^2\beta\epsilon + e\alpha\beta^2\epsilon - \frac{1}{2}h\alpha^2\beta^2\epsilon + l\alpha^2\beta^2\epsilon^2 + 2l^2\alpha^2\beta^2\epsilon^2 + \\
 & f\alpha^3\beta^2\epsilon^2 + 2fl\alpha^3\beta^2\epsilon^2 + \frac{1}{2}f^2\alpha^4\beta^2\epsilon^2 + e\alpha^2\beta^3\epsilon^2 + 2el\alpha^2\beta^3\epsilon^2 + ef\alpha^3\beta^3\epsilon^2 - \\
 & \frac{1}{3}h\alpha^3\beta^3\epsilon^2 - hl\alpha^3\beta^3\epsilon^2 - \frac{1}{2}fh\alpha^4\beta^3\epsilon^2 + \frac{1}{2}e^2\alpha^2\beta^4\epsilon^2 - \frac{1}{2}eh\alpha^3\beta^4\epsilon^2 + \frac{1}{8}h^2\alpha^4\beta^4\epsilon^2 \\
 & 1 + \frac{1}{2(1+h\delta)^4} \left(4l(1+h\delta)^2((\alpha+e\delta)(\beta+f\delta) + \delta(1+h\delta)) + 2f(\alpha+e\delta)(1+h\delta) \right. \\
 & \quad \left. ((\alpha+e\delta)(\beta+f\delta) + 2\delta(1+h\delta)) + 2e(\beta+f\delta)(1+h\delta)((\alpha+e\delta)(\beta+f\delta) + 2\delta(1+h\delta)) - \right. \\
 & \quad \left. h((\alpha+e\delta)^2(\beta+f\delta)^2 + 4\delta(\alpha+e\delta)(\beta+f\delta)(1+h\delta) + 2\delta^2(1+h\delta)^2) \right) \epsilon + \frac{1}{24(1+h\delta)^8} \\
 & \left(24l(1+h\delta)^4((\alpha+e\delta)^2(\beta+f\delta)^2 + 4\delta(\alpha+e\delta)(\beta+f\delta)(1+h\delta) + 2\delta^2(1+h\delta)^2) + \right. \\
 & \quad 48l^2(1+h\delta)^4((\alpha+e\delta)^2(\beta+f\delta)^2 + 4\delta(\alpha+e\delta)(\beta+f\delta)(1+h\delta) + 2\delta^2(1+h\delta)^2) + \\
 & \quad 24f(\alpha+e\delta)(1+h\delta)^3 \\
 & \quad \left. \left((\alpha+e\delta)^2(\beta+f\delta)^2 + 6\delta(\alpha+e\delta)(\beta+f\delta)(1+h\delta) + 6\delta^2(1+h\delta)^2 \right) + 48fl(\alpha+e\delta) \right. \\
 & \quad \left. (1+h\delta)^3 \left((\alpha+e\delta)^2(\beta+f\delta)^2 + 6\delta(\alpha+e\delta)(\beta+f\delta)(1+h\delta) + 6\delta^2(1+h\delta)^2 \right) + 24e \right. \\
 & \quad \left. (\beta+f\delta)(1+h\delta)^3 \left((\alpha+e\delta)^2(\beta+f\delta)^2 + 6\delta(\alpha+e\delta)(\beta+f\delta)(1+h\delta) + 6\delta^2(1+h\delta)^2 \right) + \right. \\
 & \quad 48el(\beta+f\delta)(1+h\delta)^3 \left((\alpha+e\delta)^2(\beta+f\delta)^2 + 6\delta(\alpha+e\delta)(\beta+f\delta)(1+h\delta) + \right. \\
 & \quad \left. 6\delta^2(1+h\delta)^2 \right) + 12(\beta+f\delta)^2(e+eh\delta)^2 \\
 & \quad \left. \left((\alpha+e\delta)^2(\beta+f\delta)^2 + 8\delta(\alpha+e\delta)(\beta+f\delta)(1+h\delta) + 12\delta^2(1+h\delta)^2 \right) + 12(\alpha+e\delta)^2 \right. \\
 & \quad \left. (f+fh\delta)^2 \left((\alpha+e\delta)^2(\beta+f\delta)^2 + 8\delta(\alpha+e\delta)(\beta+f\delta)(1+h\delta) + 12\delta^2(1+h\delta)^2 \right) + \right. \\
 & \quad 24ef(1+h\delta)^2 \left((\alpha+e\delta)^3(\beta+f\delta)^3 + 9\delta(\alpha+e\delta)^2(\beta+f\delta)^2(1+h\delta) + \right. \\
 & \quad \left. 18\delta^2(\alpha+e\delta)(\beta+f\delta)(1+h\delta)^2 + 6\delta^3(1+h\delta)^3 \right) - 8h(1+h\delta)^2 \left((\alpha+e\delta)^3(\beta+f\delta)^3 + \right. \\
 & \quad \left. 9\delta(\alpha+e\delta)^2(\beta+f\delta)^2(1+h\delta) + 18\delta^2(\alpha+e\delta)(\beta+f\delta)(1+h\delta)^2 + 6\delta^3(1+h\delta)^3 \right) - \\
 & \quad 24hl(1+h\delta)^2 \left((\alpha+e\delta)^3(\beta+f\delta)^3 + 9\delta(\alpha+e\delta)^2(\beta+f\delta)^2(1+h\delta) + \right. \\
 & \quad \left. 18\delta^2(\alpha+e\delta)(\beta+f\delta)(1+h\delta)^2 + 6\delta^3(1+h\delta)^3 \right) - \\
 & \quad 12fh(\alpha+e\delta)(1+h\delta) \left((\alpha+e\delta)^3(\beta+f\delta)^3 + 12\delta(\alpha+e\delta)^2(\beta+f\delta)^2(1+h\delta) + \right. \\
 & \quad \left. 36\delta^2(\alpha+e\delta)(\beta+f\delta)(1+h\delta)^2 + 24\delta^3(1+h\delta)^3 \right) - \\
 & \quad 12eh(\beta+f\delta)(1+h\delta) \left((\alpha+e\delta)^3(\beta+f\delta)^3 + 12\delta(\alpha+e\delta)^2(\beta+f\delta)^2(1+h\delta) + \right. \\
 & \quad \left. 36\delta^2(\alpha+e\delta)(\beta+f\delta)(1+h\delta)^2 + 24\delta^3(1+h\delta)^3 \right) + \\
 & \quad \left. 3h^2 \left((\alpha+e\delta)^4(\beta+f\delta)^4 + 16\delta(\alpha+e\delta)^3(\beta+f\delta)^3(1+h\delta) + 72\delta^2(\alpha+e\delta)^2 \right. \right. \\
 & \quad \left. \left. (\beta+f\delta)^2(1+h\delta)^2 + 96\delta^3(\alpha+e\delta)(\beta+f\delta)(1+h\delta)^3 + 24\delta^4(1+h\delta)^4 \right) \right) \epsilon^2
 \end{aligned}$$

$\Lambda_3[e, l, f, \alpha, \beta, \delta]$

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$$\begin{aligned}
 & 1 + 2 l \alpha \beta \epsilon + f \alpha^2 \beta \epsilon + e \alpha \beta^2 \epsilon - \frac{1}{2} h \alpha^2 \beta^2 \epsilon + l \alpha^2 \beta^2 \epsilon^2 + 2 l^2 \alpha^2 \beta^2 \epsilon^2 + f \alpha^3 \beta^2 \epsilon^2 + 2 f l \alpha^3 \beta^2 \epsilon^2 + \\
 & \frac{1}{2} f^2 \alpha^4 \beta^2 \epsilon^2 + e \alpha^2 \beta^3 \epsilon^2 + 2 e l \alpha^2 \beta^3 \epsilon^2 + e f \alpha^3 \beta^3 \epsilon^2 - \frac{1}{3} h \alpha^3 \beta^3 \epsilon^2 - h l \alpha^3 \beta^3 \epsilon^2 - \frac{1}{2} f h \alpha^4 \beta^3 \epsilon^2 + \\
 & \frac{1}{2} e^2 \alpha^2 \beta^4 \epsilon^2 - \frac{1}{2} e h \alpha^3 \beta^4 \epsilon^2 + \frac{1}{8} h^2 \alpha^4 \beta^4 \epsilon^2 + \frac{2}{3} l \alpha^3 \beta^3 \epsilon^3 + 2 l^2 \alpha^3 \beta^3 \epsilon^3 + \frac{4}{3} l^3 \alpha^3 \beta^3 \epsilon^3 + \\
 & f \alpha^4 \beta^3 \epsilon^3 + 3 f l \alpha^4 \beta^3 \epsilon^3 + 2 f l^2 \alpha^4 \beta^3 \epsilon^3 + f^2 \alpha^5 \beta^3 \epsilon^3 + f^2 l \alpha^5 \beta^3 \epsilon^3 + \frac{1}{6} f^3 \alpha^6 \beta^3 \epsilon^3 + e \alpha^3 \beta^4 \epsilon^3 + \\
 & 3 e l \alpha^3 \beta^4 \epsilon^3 + 2 e l^2 \alpha^3 \beta^4 \epsilon^3 + 2 e f \alpha^4 \beta^4 \epsilon^3 - \frac{1}{4} h \alpha^4 \beta^4 \epsilon^3 + 2 e f l \alpha^4 \beta^4 \epsilon^3 - \frac{7}{6} h l \alpha^4 \beta^4 \epsilon^3 - \\
 & h l^2 \alpha^4 \beta^4 \epsilon^3 + \frac{1}{2} e f^2 \alpha^5 \beta^4 \epsilon^3 - \frac{5}{6} f h \alpha^5 \beta^4 \epsilon^3 - f h l \alpha^5 \beta^4 \epsilon^3 - \frac{1}{4} f^2 h \alpha^6 \beta^4 \epsilon^3 + e^2 \alpha^3 \beta^5 \epsilon^3 + \\
 & e^2 l \alpha^3 \beta^5 \epsilon^3 + \frac{1}{2} e^2 f \alpha^4 \beta^5 \epsilon^3 - \frac{5}{6} e h \alpha^4 \beta^5 \epsilon^3 - e h l \alpha^4 \beta^5 \epsilon^3 - \frac{1}{2} e f h \alpha^5 \beta^5 \epsilon^3 + \frac{1}{6} h^2 \alpha^5 \beta^5 \epsilon^3 + \\
 & \frac{1}{4} h^2 l \alpha^5 \beta^5 \epsilon^3 + \frac{1}{8} f h^2 \alpha^6 \beta^5 \epsilon^3 + \frac{1}{6} e^3 \alpha^3 \beta^6 \epsilon^3 - \frac{1}{4} e^2 h \alpha^4 \beta^6 \epsilon^3 + \frac{1}{8} e h^2 \alpha^5 \beta^6 \epsilon^3 - \frac{1}{48} h^3 \alpha^6 \beta^6 \epsilon^3 \\
 & 1 + \frac{1}{2 (1+h\delta)^4} \left(4 l (1+h\delta)^2 ((\alpha+e\delta)(\beta+f\delta)+\delta(1+h\delta)) + 2 f (\alpha+e\delta)(1+h\delta) \right. \\
 & \quad \left. ((\alpha+e\delta)(\beta+f\delta)+2\delta(1+h\delta)) + 2 e (\beta+f\delta)(1+h\delta) ((\alpha+e\delta)(\beta+f\delta)+2\delta(1+h\delta)) - \right. \\
 & \quad \left. h ((\alpha+e\delta)^2(\beta+f\delta)^2+4\delta(\alpha+e\delta)(\beta+f\delta)(1+h\delta)+2\delta^2(1+h\delta)^2) \right) \epsilon + \frac{1}{24 (1+h\delta)^8} \\
 & \left(24 l (1+h\delta)^4 ((\alpha+e\delta)^2(\beta+f\delta)^2+4\delta(\alpha+e\delta)(\beta+f\delta)(1+h\delta)+2\delta^2(1+h\delta)^2) + \right. \\
 & \quad 48 l^2 (1+h\delta)^4 ((\alpha+e\delta)^2(\beta+f\delta)^2+4\delta(\alpha+e\delta)(\beta+f\delta)(1+h\delta)+2\delta^2(1+h\delta)^2) + \\
 & \quad 24 f (\alpha+e\delta)(1+h\delta)^3 \\
 & \quad \left. \left((\alpha+e\delta)^2(\beta+f\delta)^2+6\delta(\alpha+e\delta)(\beta+f\delta)(1+h\delta)+6\delta^2(1+h\delta)^2 \right) + 48 f l (\alpha+e\delta) \right. \\
 & \quad \left. (1+h\delta)^3 \left((\alpha+e\delta)^2(\beta+f\delta)^2+6\delta(\alpha+e\delta)(\beta+f\delta)(1+h\delta)+6\delta^2(1+h\delta)^2 \right) + 24 e \right. \\
 & \quad \left. (\beta+f\delta)(1+h\delta)^3 \left((\alpha+e\delta)^2(\beta+f\delta)^2+6\delta(\alpha+e\delta)(\beta+f\delta)(1+h\delta)+6\delta^2(1+h\delta)^2 \right) \right) + \\
 & \quad 48 e l (\beta+f\delta)(1+h\delta)^3 \left((\alpha+e\delta)^2(\beta+f\delta)^2+6\delta(\alpha+e\delta)(\beta+f\delta)(1+h\delta)+ \right. \\
 & \quad \left. 6\delta^2(1+h\delta)^2 \right) + 12 (\beta+f\delta)^2 (e+eh\delta)^2 \\
 & \quad \left((\alpha+e\delta)^2(\beta+f\delta)^2+8\delta(\alpha+e\delta)(\beta+f\delta)(1+h\delta)+12\delta^2(1+h\delta)^2 \right) + 12 (\alpha+e\delta)^2 \\
 & \quad \left. (f+f h\delta)^2 \left((\alpha+e\delta)^2(\beta+f\delta)^2+8\delta(\alpha+e\delta)(\beta+f\delta)(1+h\delta)+12\delta^2(1+h\delta)^2 \right) + \right. \\
 & \quad 24 e f (1+h\delta)^2 \left((\alpha+e\delta)^3(\beta+f\delta)^3+9\delta(\alpha+e\delta)^2(\beta+f\delta)^2(1+h\delta)+ \right. \\
 & \quad \left. 18\delta^2(\alpha+e\delta)(\beta+f\delta)(1+h\delta)^2+6\delta^3(1+h\delta)^3 \right) - 8 h (1+h\delta)^2 \left((\alpha+e\delta)^3(\beta+f\delta)^3+ \right. \\
 & \quad \left. 9\delta(\alpha+e\delta)^2(\beta+f\delta)^2(1+h\delta)+18\delta^2(\alpha+e\delta)(\beta+f\delta)(1+h\delta)^2+6\delta^3(1+h\delta)^3 \right) - \\
 & \quad 24 h l (1+h\delta)^2 \left((\alpha+e\delta)^3(\beta+f\delta)^3+9\delta(\alpha+e\delta)^2(\beta+f\delta)^2(1+h\delta)+ \right. \\
 & \quad \left. 18\delta^2(\alpha+e\delta)(\beta+f\delta)(1+h\delta)^2+6\delta^3(1+h\delta)^3 \right) - \\
 & \quad 12 f h (\alpha+e\delta)(1+h\delta) \left((\alpha+e\delta)^3(\beta+f\delta)^3+12\delta(\alpha+e\delta)^2(\beta+f\delta)^2(1+h\delta)+ \right. \\
 & \quad \left. 36\delta^2(\alpha+e\delta)(\beta+f\delta)(1+h\delta)^2+24\delta^3(1+h\delta)^3 \right) - \\
 & \quad 12 e h (\beta+f\delta)(1+h\delta) \left((\alpha+e\delta)^3(\beta+f\delta)^3+12\delta(\alpha+e\delta)^2(\beta+f\delta)^2(1+h\delta)+ \right. \\
 & \quad \left. 36\delta^2(\alpha+e\delta)(\beta+f\delta)(1+h\delta)^2+24\delta^3(1+h\delta)^3 \right) + \\
 & \quad 3 h^2 \left((\alpha+e\delta)^4(\beta+f\delta)^4+16\delta(\alpha+e\delta)^3(\beta+f\delta)^3(1+h\delta)+72\delta^2(\alpha+e\delta)^2 \right. \\
 & \quad \left. (\beta+f\delta)^2(1+h\delta)^2+96\delta^3(\alpha+e\delta)(\beta+f\delta)(1+h\delta)^3+24\delta^4(1+h\delta)^4 \right) \epsilon^2 + \\
 & \quad \frac{1}{48 (1+h\delta)^{12}} \left(32 l (1+h\delta)^6 \left((\alpha+e\delta)^3(\beta+f\delta)^3+9\delta(\alpha+e\delta)^2(\beta+f\delta)^2(1+h\delta)+ \right. \right. \\
 & \quad \left. \left. 18\delta^2(\alpha+e\delta)(\beta+f\delta)(1+h\delta)^2+6\delta^3(1+h\delta)^3 \right) + \right.
 \end{aligned}$$

$$\begin{aligned}
 & 72 \delta^2 (\alpha + e \delta)^2 (\beta + f \delta)^2 (1 + h \delta)^2 + 96 \delta^3 (\alpha + e \delta) (\beta + f \delta) (1 + h \delta)^3 + 24 \delta^4 (1 + h \delta)^4 + \\
 & 24 e f^2 (\alpha + e \delta) (1 + h \delta)^3 \left((\alpha + e \delta)^4 (\beta + f \delta)^4 + 20 \delta (\alpha + e \delta)^3 (\beta + f \delta)^3 (1 + h \delta) + 120 \right. \\
 & \quad \left. \delta^2 (\alpha + e \delta)^2 (\beta + f \delta)^2 (1 + h \delta)^2 + 240 \delta^3 (\alpha + e \delta) (\beta + f \delta) (1 + h \delta)^3 + 120 \delta^4 (1 + h \delta)^4 \right) - \\
 & 40 f h (\alpha + e \delta) (1 + h \delta)^3 \left((\alpha + e \delta)^4 (\beta + f \delta)^4 + 20 \delta (\alpha + e \delta)^3 (\beta + f \delta)^3 (1 + h \delta) + 120 \right. \\
 & \quad \left. \delta^2 (\alpha + e \delta)^2 (\beta + f \delta)^2 (1 + h \delta)^2 + 240 \delta^3 (\alpha + e \delta) (\beta + f \delta) (1 + h \delta)^3 + 120 \delta^4 (1 + h \delta)^4 \right) - \\
 & 48 f h l (\alpha + e \delta) (1 + h \delta)^3 \left((\alpha + e \delta)^4 (\beta + f \delta)^4 + 20 \delta (\alpha + e \delta)^3 (\beta + f \delta)^3 (1 + h \delta) + 120 \right. \\
 & \quad \left. \delta^2 (\alpha + e \delta)^2 (\beta + f \delta)^2 (1 + h \delta)^2 + 240 \delta^3 (\alpha + e \delta) (\beta + f \delta) (1 + h \delta)^3 + 120 \delta^4 (1 + h \delta)^4 \right) + \\
 & 24 e^2 f (\beta + f \delta) (1 + h \delta)^3 \left((\alpha + e \delta)^4 (\beta + f \delta)^4 + 20 \delta (\alpha + e \delta)^3 (\beta + f \delta)^3 (1 + h \delta) + 120 \right. \\
 & \quad \left. \delta^2 (\alpha + e \delta)^2 (\beta + f \delta)^2 (1 + h \delta)^2 + 240 \delta^3 (\alpha + e \delta) (\beta + f \delta) (1 + h \delta)^3 + 120 \delta^4 (1 + h \delta)^4 \right) - \\
 & 40 e h (\beta + f \delta) (1 + h \delta)^3 \left((\alpha + e \delta)^4 (\beta + f \delta)^4 + 20 \delta (\alpha + e \delta)^3 (\beta + f \delta)^3 (1 + h \delta) + 120 \right. \\
 & \quad \left. \delta^2 (\alpha + e \delta)^2 (\beta + f \delta)^2 (1 + h \delta)^2 + 240 \delta^3 (\alpha + e \delta) (\beta + f \delta) (1 + h \delta)^3 + 120 \delta^4 (1 + h \delta)^4 \right) - \\
 & 48 e h l (\beta + f \delta) (1 + h \delta)^3 \left((\alpha + e \delta)^4 (\beta + f \delta)^4 + 20 \delta (\alpha + e \delta)^3 (\beta + f \delta)^3 (1 + h \delta) + 120 \right. \\
 & \quad \left. \delta^2 (\alpha + e \delta)^2 (\beta + f \delta)^2 (1 + h \delta)^2 + 240 \delta^3 (\alpha + e \delta) (\beta + f \delta) (1 + h \delta)^3 + 120 \delta^4 (1 + h \delta)^4 \right) - \\
 & 12 h (\beta + f \delta)^2 (e + e h \delta)^2 \left((\alpha + e \delta)^4 (\beta + f \delta)^4 + 24 \delta (\alpha + e \delta)^3 (\beta + f \delta)^3 (1 + h \delta) + 180 \right. \\
 & \quad \left. \delta^2 (\alpha + e \delta)^2 (\beta + f \delta)^2 (1 + h \delta)^2 + 480 \delta^3 (\alpha + e \delta) (\beta + f \delta) (1 + h \delta)^3 + 360 \delta^4 (1 + h \delta)^4 \right) - \\
 & 12 h (\alpha + e \delta)^2 (f + f h \delta)^2 \left((\alpha + e \delta)^4 (\beta + f \delta)^4 + 24 \delta (\alpha + e \delta)^3 (\beta + f \delta)^3 (1 + h \delta) + 180 \right. \\
 & \quad \left. \delta^2 (\alpha + e \delta)^2 (\beta + f \delta)^2 (1 + h \delta)^2 + 480 \delta^3 (\alpha + e \delta) (\beta + f \delta) (1 + h \delta)^3 + 360 \delta^4 (1 + h \delta)^4 \right) - \\
 & 24 e f h (1 + h \delta)^2 \left((\alpha + e \delta)^5 (\beta + f \delta)^5 + 25 \delta (\alpha + e \delta)^4 (\beta + f \delta)^4 (1 + h \delta) + \right. \\
 & \quad \left. 200 \delta^2 (\alpha + e \delta)^3 (\beta + f \delta)^3 (1 + h \delta)^2 + 600 \delta^3 (\alpha + e \delta)^2 (\beta + f \delta)^2 (1 + h \delta)^3 + \right. \\
 & \quad \left. 600 \delta^4 (\alpha + e \delta) (\beta + f \delta) (1 + h \delta)^4 + 120 \delta^5 (1 + h \delta)^5 \right) + \\
 & 8 h^2 (1 + h \delta)^2 \left((\alpha + e \delta)^5 (\beta + f \delta)^5 + 25 \delta (\alpha + e \delta)^4 (\beta + f \delta)^4 (1 + h \delta) + \right. \\
 & \quad \left. 200 \delta^2 (\alpha + e \delta)^3 (\beta + f \delta)^3 (1 + h \delta)^2 + 600 \delta^3 (\alpha + e \delta)^2 (\beta + f \delta)^2 (1 + h \delta)^3 + \right. \\
 & \quad \left. 600 \delta^4 (\alpha + e \delta) (\beta + f \delta) (1 + h \delta)^4 + 120 \delta^5 (1 + h \delta)^5 \right) + \\
 & 12 h^2 l (1 + h \delta)^2 \left((\alpha + e \delta)^5 (\beta + f \delta)^5 + 25 \delta (\alpha + e \delta)^4 (\beta + f \delta)^4 (1 + h \delta) + \right. \\
 & \quad \left. 200 \delta^2 (\alpha + e \delta)^3 (\beta + f \delta)^3 (1 + h \delta)^2 + 600 \delta^3 (\alpha + e \delta)^2 (\beta + f \delta)^2 (1 + h \delta)^3 + \right. \\
 & \quad \left. 600 \delta^4 (\alpha + e \delta) (\beta + f \delta) (1 + h \delta)^4 + 120 \delta^5 (1 + h \delta)^5 \right) + \\
 & 6 f h^2 (\alpha + e \delta) (1 + h \delta) \left((\alpha + e \delta)^5 (\beta + f \delta)^5 + 30 \delta (\alpha + e \delta)^4 (\beta + f \delta)^4 (1 + h \delta) + \right. \\
 & \quad \left. 300 \delta^2 (\alpha + e \delta)^3 (\beta + f \delta)^3 (1 + h \delta)^2 + 1200 \delta^3 (\alpha + e \delta)^2 (\beta + f \delta)^2 (1 + h \delta)^3 + \right. \\
 & \quad \left. 1800 \delta^4 (\alpha + e \delta) (\beta + f \delta) (1 + h \delta)^4 + 720 \delta^5 (1 + h \delta)^5 \right) + \\
 & 6 e h^2 (\beta + f \delta) (1 + h \delta) \left((\alpha + e \delta)^5 (\beta + f \delta)^5 + 30 \delta (\alpha + e \delta)^4 (\beta + f \delta)^4 (1 + h \delta) + \right. \\
 & \quad \left. 300 \delta^2 (\alpha + e \delta)^3 (\beta + f \delta)^3 (1 + h \delta)^2 + 1200 \delta^3 (\alpha + e \delta)^2 (\beta + f \delta)^2 (1 + h \delta)^3 + \right. \\
 & \quad \left. 1800 \delta^4 (\alpha + e \delta) (\beta + f \delta) (1 + h \delta)^4 + 720 \delta^5 (1 + h \delta)^5 \right) - \\
 & h^3 \left((\alpha + e \delta)^6 (\beta + f \delta)^6 + 36 \delta (\alpha + e \delta)^5 (\beta + f \delta)^5 (1 + h \delta) + 450 \delta^2 (\alpha + e \delta)^4 (\beta + f \delta)^4 \right. \\
 & \quad \left. (1 + h \delta)^2 + 2400 \delta^3 (\alpha + e \delta)^3 (\beta + f \delta)^3 (1 + h \delta)^3 + 5400 \delta^4 (\alpha + e \delta)^2 (\beta + f \delta)^2 (1 + h \delta)^4 + \right. \\
 & \quad \left. 4320 \delta^5 (\alpha + e \delta) (\beta + f \delta) (1 + h \delta)^5 + 720 \delta^6 (1 + h \delta)^6 \right) \epsilon^3
 \end{aligned}$$

```
lA[k_] := Collect[Normal@Series[Log[Ak[e, l, f, α, β, δ]], {ε, 0, k}], ε, Simplify]
```

Exponent [1Λ[2] /. {x: (α | β | e | f) => ħ x, 1 → ħ² 1}, ħ]

$$1 + 2 l \alpha \beta \epsilon + f \alpha^2 \beta \epsilon + e \alpha \beta^2 \epsilon - \frac{1}{2} h \alpha^2 \beta^2 \epsilon + l \alpha^2 \beta^2 \epsilon^2 + 2 l^2 \alpha^2 \beta^2 \epsilon^2 + f \alpha^3 \beta^2 \epsilon^2 + 2 f l \alpha^3 \beta^2 \epsilon^2 + \frac{1}{2} f^2 \alpha^4 \beta^2 \epsilon^2 + e \alpha^2 \beta^3 \epsilon^2 + 2 e l \alpha^2 \beta^3 \epsilon^2 + e f \alpha^3 \beta^3 \epsilon^2 - \frac{1}{3} h \alpha^3 \beta^3 \epsilon^2 - h l \alpha^3 \beta^3 \epsilon^2 - \frac{1}{2} f h \alpha^4 \beta^3 \epsilon^2 + \frac{1}{2} e^2 \alpha^2 \beta^4 \epsilon^2 - \frac{1}{2} e h \alpha^3 \beta^4 \epsilon^2 + \frac{1}{8} h^2 \alpha^4 \beta^4 \epsilon^2$$

6

Exponent [1Λ[3] /. {x: (α | β | e | f) => ħ x, 1 → ħ² 1}, ħ]

$$1 + 2 l \alpha \beta \epsilon + f \alpha^2 \beta \epsilon + e \alpha \beta^2 \epsilon - \frac{1}{2} h \alpha^2 \beta^2 \epsilon + l \alpha^2 \beta^2 \epsilon^2 + 2 l^2 \alpha^2 \beta^2 \epsilon^2 + f \alpha^3 \beta^2 \epsilon^2 + 2 f l \alpha^3 \beta^2 \epsilon^2 + \frac{1}{2} f^2 \alpha^4 \beta^2 \epsilon^2 + e \alpha^2 \beta^3 \epsilon^2 + 2 e l \alpha^2 \beta^3 \epsilon^2 + e f \alpha^3 \beta^3 \epsilon^2 - \frac{1}{3} h \alpha^3 \beta^3 \epsilon^2 - h l \alpha^3 \beta^3 \epsilon^2 - \frac{1}{2} f h \alpha^4 \beta^3 \epsilon^2 + \frac{1}{2} e^2 \alpha^2 \beta^4 \epsilon^2 - \frac{1}{2} e h \alpha^3 \beta^4 \epsilon^2 + \frac{1}{8} h^2 \alpha^4 \beta^4 \epsilon^2 + \frac{2}{3} l \alpha^3 \beta^3 \epsilon^3 + 2 l^2 \alpha^3 \beta^3 \epsilon^3 + \frac{4}{3} l^3 \alpha^3 \beta^3 \epsilon^3 + f \alpha^4 \beta^3 \epsilon^3 + 3 f l \alpha^4 \beta^3 \epsilon^3 + 2 f l^2 \alpha^4 \beta^3 \epsilon^3 + f^2 \alpha^5 \beta^3 \epsilon^3 + f^2 l \alpha^5 \beta^3 \epsilon^3 + \frac{1}{6} f^3 \alpha^6 \beta^3 \epsilon^3 + e \alpha^3 \beta^4 \epsilon^3 + 3 e l \alpha^3 \beta^4 \epsilon^3 + 2 e l^2 \alpha^3 \beta^4 \epsilon^3 + 2 e f \alpha^4 \beta^4 \epsilon^3 - \frac{1}{4} h \alpha^4 \beta^4 \epsilon^3 + 2 e f l \alpha^4 \beta^4 \epsilon^3 - \frac{7}{6} h l \alpha^4 \beta^4 \epsilon^3 - h l^2 \alpha^4 \beta^4 \epsilon^3 + \frac{1}{2} e f^2 \alpha^5 \beta^4 \epsilon^3 - \frac{5}{6} f h \alpha^5 \beta^4 \epsilon^3 - f h l \alpha^5 \beta^4 \epsilon^3 - \frac{1}{4} f^2 h \alpha^6 \beta^4 \epsilon^3 + e^2 \alpha^3 \beta^5 \epsilon^3 + e^2 l \alpha^3 \beta^5 \epsilon^3 + \frac{1}{2} e^2 f \alpha^4 \beta^5 \epsilon^3 - \frac{5}{6} e h \alpha^4 \beta^5 \epsilon^3 - e h l \alpha^4 \beta^5 \epsilon^3 - \frac{1}{2} e f h \alpha^5 \beta^5 \epsilon^3 + \frac{1}{6} h^2 \alpha^5 \beta^5 \epsilon^3 + \frac{1}{4} h^2 l \alpha^5 \beta^5 \epsilon^3 + \frac{1}{8} f h^2 \alpha^6 \beta^5 \epsilon^3 + \frac{1}{6} e^3 \alpha^3 \beta^6 \epsilon^3 - \frac{1}{4} e^2 h \alpha^4 \beta^6 \epsilon^3 + \frac{1}{8} e h^2 \alpha^5 \beta^6 \epsilon^3 - \frac{1}{48} h^3 \alpha^6 \beta^6 \epsilon^3$$

8

Exponent [1Δ[4] /. {x: (α | β | e | f) :-> ħ x, 1 -> ħ^2 1}, ħ]

$$\begin{aligned}
 & 1 + 2 l \alpha \beta \epsilon + f \alpha^2 \beta \epsilon + e \alpha \beta^2 \epsilon - \frac{1}{2} h \alpha^2 \beta^2 \epsilon + l \alpha^2 \beta^2 \epsilon^2 + 2 l^2 \alpha^2 \beta^2 \epsilon^2 + f \alpha^3 \beta^2 \epsilon^2 + 2 f l \alpha^3 \beta^2 \epsilon^2 + \\
 & \frac{1}{2} f^2 \alpha^4 \beta^2 \epsilon^2 + e \alpha^2 \beta^3 \epsilon^2 + 2 e l \alpha^2 \beta^3 \epsilon^2 + e f \alpha^3 \beta^3 \epsilon^2 - \frac{1}{3} h \alpha^3 \beta^3 \epsilon^2 - h l \alpha^3 \beta^3 \epsilon^2 - \frac{1}{2} f h \alpha^4 \beta^3 \epsilon^2 + \\
 & \frac{1}{2} e^2 \alpha^2 \beta^4 \epsilon^2 - \frac{1}{2} e h \alpha^3 \beta^4 \epsilon^2 + \frac{1}{8} h^2 \alpha^4 \beta^4 \epsilon^2 + \frac{2}{3} l \alpha^3 \beta^3 \epsilon^3 + 2 l^2 \alpha^3 \beta^3 \epsilon^3 + \frac{4}{3} l^3 \alpha^3 \beta^3 \epsilon^3 + f \alpha^4 \beta^3 \epsilon^3 + \\
 & 3 f l \alpha^4 \beta^3 \epsilon^3 + 2 f l^2 \alpha^4 \beta^3 \epsilon^3 + f^2 \alpha^5 \beta^3 \epsilon^3 + f^2 l \alpha^5 \beta^3 \epsilon^3 + \frac{1}{6} f^3 \alpha^6 \beta^3 \epsilon^3 + e \alpha^3 \beta^4 \epsilon^3 + 3 e l \alpha^3 \beta^4 \epsilon^3 + \\
 & 2 e l^2 \alpha^3 \beta^4 \epsilon^3 + 2 e f \alpha^4 \beta^4 \epsilon^3 - \frac{1}{4} h \alpha^4 \beta^4 \epsilon^3 + 2 e f l \alpha^4 \beta^4 \epsilon^3 - \frac{7}{6} h l \alpha^4 \beta^4 \epsilon^3 - h l^2 \alpha^4 \beta^4 \epsilon^3 + \\
 & \frac{1}{2} e f^2 \alpha^5 \beta^4 \epsilon^3 - \frac{5}{6} f h \alpha^5 \beta^4 \epsilon^3 - f h l \alpha^5 \beta^4 \epsilon^3 - \frac{1}{4} f^2 h \alpha^6 \beta^4 \epsilon^3 + e^2 \alpha^3 \beta^5 \epsilon^3 + e^2 l \alpha^3 \beta^5 \epsilon^3 + \\
 & \frac{1}{2} e^2 f \alpha^4 \beta^5 \epsilon^3 - \frac{5}{6} e h \alpha^4 \beta^5 \epsilon^3 - e h l \alpha^4 \beta^5 \epsilon^3 - \frac{1}{2} e f h \alpha^5 \beta^5 \epsilon^3 + \frac{1}{6} h^2 \alpha^5 \beta^5 \epsilon^3 + \frac{1}{4} h^2 l \alpha^5 \beta^5 \epsilon^3 + \\
 & \frac{1}{8} f h^2 \alpha^6 \beta^5 \epsilon^3 + \frac{1}{6} e^3 \alpha^3 \beta^6 \epsilon^3 - \frac{1}{4} e^2 h \alpha^4 \beta^6 \epsilon^3 + \frac{1}{8} e h^2 \alpha^5 \beta^6 \epsilon^3 - \frac{1}{48} h^3 \alpha^6 \beta^6 \epsilon^3 + \frac{1}{2} l \alpha^4 \beta^4 \epsilon^4 + \\
 & \frac{11}{6} l^2 \alpha^4 \beta^4 \epsilon^4 + 2 l^3 \alpha^4 \beta^4 \epsilon^4 + \frac{2}{3} l^4 \alpha^4 \beta^4 \epsilon^4 + f \alpha^5 \beta^4 \epsilon^4 + \frac{11}{3} f l \alpha^5 \beta^4 \epsilon^4 + 4 f l^2 \alpha^5 \beta^4 \epsilon^4 + \frac{4}{3} f l^3 \alpha^5 \beta^4 \epsilon^4 + \\
 & \frac{3}{2} f^2 \alpha^6 \beta^4 \epsilon^4 + \frac{5}{2} f^2 l \alpha^6 \beta^4 \epsilon^4 + f^2 l^2 \alpha^6 \beta^4 \epsilon^4 + \frac{1}{2} f^3 \alpha^7 \beta^4 \epsilon^4 + \frac{1}{3} f^3 l \alpha^7 \beta^4 \epsilon^4 + \frac{1}{24} f^4 \alpha^8 \beta^4 \epsilon^4 + e \alpha^4 \beta^5 \epsilon^4 + \\
 & \frac{11}{3} e l \alpha^4 \beta^5 \epsilon^4 + 4 e l^2 \alpha^4 \beta^5 \epsilon^4 + \frac{4}{3} e l^3 \alpha^4 \beta^5 \epsilon^4 + 3 e f \alpha^5 \beta^5 \epsilon^4 - \frac{1}{5} h \alpha^5 \beta^5 \epsilon^4 + 5 e f l \alpha^5 \beta^5 \epsilon^4 - \\
 & \frac{7}{6} h l \alpha^5 \beta^5 \epsilon^4 + 2 e f l^2 \alpha^5 \beta^5 \epsilon^4 - \frac{5}{3} h l^2 \alpha^5 \beta^5 \epsilon^4 - \frac{2}{3} h l^3 \alpha^5 \beta^5 \epsilon^4 + \frac{3}{2} e f^2 \alpha^6 \beta^5 \epsilon^4 - \frac{13}{12} f h \alpha^6 \beta^5 \epsilon^4 + \\
 & e f^2 l \alpha^6 \beta^5 \epsilon^4 - \frac{13}{6} f h l \alpha^6 \beta^5 \epsilon^4 - f h l^2 \alpha^6 \beta^5 \epsilon^4 + \frac{1}{6} e f^3 \alpha^7 \beta^5 \epsilon^4 - \frac{2}{3} f^2 h \alpha^7 \beta^5 \epsilon^4 - \frac{1}{2} f^2 h l \alpha^7 \beta^5 \epsilon^4 - \\
 & \frac{1}{12} f^3 h \alpha^8 \beta^5 \epsilon^4 + \frac{3}{2} e^2 \alpha^4 \beta^6 \epsilon^4 + \frac{5}{2} e^2 l \alpha^4 \beta^6 \epsilon^4 + e^2 l^2 \alpha^4 \beta^6 \epsilon^4 + \frac{3}{2} e^2 f \alpha^5 \beta^6 \epsilon^4 - \frac{13}{12} e h \alpha^5 \beta^6 \epsilon^4 + \\
 & e^2 f l \alpha^5 \beta^6 \epsilon^4 - \frac{13}{6} e h l \alpha^5 \beta^6 \epsilon^4 - e h l^2 \alpha^5 \beta^6 \epsilon^4 + \frac{1}{4} e^2 f^2 \alpha^6 \beta^6 \epsilon^4 - \frac{4}{3} e f h \alpha^6 \beta^6 \epsilon^4 + \frac{13}{72} h^2 \alpha^6 \beta^6 \epsilon^4 - \\
 & e f h l \alpha^6 \beta^6 \epsilon^4 + \frac{11}{24} h^2 l \alpha^6 \beta^6 \epsilon^4 + \frac{1}{4} h^2 l^2 \alpha^6 \beta^6 \epsilon^4 - \frac{1}{4} e f^2 h \alpha^7 \beta^6 \epsilon^4 + \frac{7}{24} f h^2 \alpha^7 \beta^6 \epsilon^4 + \frac{1}{4} f h^2 l \alpha^7 \beta^6 \epsilon^4 + \\
 & \frac{1}{16} f^2 h^2 \alpha^8 \beta^6 \epsilon^4 + \frac{1}{2} e^3 \alpha^4 \beta^7 \epsilon^4 + \frac{1}{3} e^3 l \alpha^4 \beta^7 \epsilon^4 + \frac{1}{6} e^3 f \alpha^5 \beta^7 \epsilon^4 - \frac{2}{3} e^2 h \alpha^5 \beta^7 \epsilon^4 - \frac{1}{2} e^2 h l \alpha^5 \beta^7 \epsilon^4 - \\
 & \frac{1}{4} e^2 f h \alpha^6 \beta^7 \epsilon^4 + \frac{7}{24} e h^2 \alpha^6 \beta^7 \epsilon^4 + \frac{1}{4} e h^2 l \alpha^6 \beta^7 \epsilon^4 + \frac{1}{8} e f h^2 \alpha^7 \beta^7 \epsilon^4 - \frac{1}{24} h^3 \alpha^7 \beta^7 \epsilon^4 - \frac{1}{24} h^3 l \alpha^7 \beta^7 \epsilon^4 - \\
 & \frac{1}{48} f h^3 \alpha^8 \beta^7 \epsilon^4 + \frac{1}{24} e^4 \alpha^4 \beta^8 \epsilon^4 - \frac{1}{12} e^3 h \alpha^5 \beta^8 \epsilon^4 + \frac{1}{16} e^2 h^2 \alpha^6 \beta^8 \epsilon^4 - \frac{1}{48} e h^3 \alpha^7 \beta^8 \epsilon^4 + \frac{1}{384} h^4 \alpha^8 \beta^8 \epsilon^4
 \end{aligned}$$