Pensieve header: The Logos at $k=2$, adapted from pensieve://Classes/17-1350-AKT/nb/170307-geps.pdf.

Representing $\mathfrak{g}^\epsilon = \langle h, e, l, f\rangle / ([e, l] = -e,\ [f, l] = f,\ [e, f] = h - 2\epsilon l,\ [h, *] = 0)$

```
ρh = {{-1, 0}, {0, -1}}; ρe = {{0, 0}, {-ϵ, 0}}; ρl = {{-(1 + 1/ϵ)/2, 0}, {0, (1 - 1/ϵ)/2}}; ρf = {{0, 1}, {0, 0}};
MB[x_?MatrixQ, y_?MatrixQ] := x.y - y.x;
Simplify@{MB[ρe, ρl] == -ρe, MB[ρf, ρl] == ρf, MB[ρe, ρf] == ρh - 2 ϵ ρl}
```

```
{True, True, True}
```

```
MatrixExp[α ρf].MatrixExp[β ρe] // Simplify // MatrixForm
```

$$\begin{pmatrix} 1-\alpha\beta\epsilon & \alpha \\ -\beta\epsilon & 1 \end{pmatrix}$$

```
eqn = MatrixExp[α ρf].MatrixExp[β ρe] == MatrixExp[a ρe].MatrixExp[c (ρh - 2 ϵ ρl)].MatrixExp[b ρf]
```

$$\{\{1-\alpha\beta\epsilon,\ \alpha\},\ \{-\beta\epsilon,\ 1\}\} == \left\{\left\{e^{c\epsilon},\ b\,e^{c\epsilon}\right\},\ \left\{-a\,e^{c\epsilon}\epsilon,\ e^{-c\epsilon} - a\,b\,e^{c\epsilon}\epsilon\right\}\right\}$$

```
sol = Solve[Thread[Flatten /@ eqn], {a, b, c}][[1]]
```

$$\left\{a \to -\frac{\beta}{-1+\alpha\beta\epsilon},\ b \to -\frac{\alpha}{-1+\alpha\beta\epsilon},\ c \to \text{ConditionalExpression}\left[\frac{2\,i\,\pi\,C[1] + \text{Log}[1-\alpha\beta\epsilon]}{\epsilon},\ C[1] \in \text{Integers}\right]\right\}$$

```
sol = sol /. C[1] → 0
```

$$\left\{a \to -\frac{\beta}{-1+\alpha\beta\epsilon},\ b \to -\frac{\alpha}{-1+\alpha\beta\epsilon},\ c \to \frac{\text{Log}[1-\alpha\beta\epsilon]}{\epsilon}\right\}$$

And so,

3. $\mathbb{O}\left(e^{\alpha f+\beta e} \,\middle|\, f\,e\right) = \mathbb{O}\left(e^{c\,h+a\,e-2\epsilon c\,l+b\,f} \,\middle|\, e\,l\,f\right)$, with $\left\{a \to -\frac{\beta}{-1+\alpha\beta\epsilon},\ b \to -\frac{\alpha}{-1+\alpha\beta\epsilon},\ c \to \frac{\text{Log}[1-\alpha\beta\epsilon]}{\epsilon}\right\}$.

At $\epsilon = 0$ get

```
Limit[{a, b, c} /. sol, ϵ → 0]
```

$$\{\beta,\ \alpha,\ -\alpha\beta\}$$

The $k = 1$ logos

```
Series[{a, b, c} /. sol, {ϵ, 0, 1}] // Normal
```

$$\left\{\beta + \alpha\beta^2\epsilon,\ \alpha + \alpha^2\beta\epsilon,\ -\alpha\beta - \frac{1}{2}\alpha^2\beta^2\epsilon\right\}$$

```
λ1 = Simplify[e^(-f α - e β + h α β) Normal@Series[e^(c h + a e - 2 ϵ c l + b f) /. sol, {ϵ, 0, 1}]]
```

$$1 + 2\,l\,\alpha\beta\epsilon + f\,\alpha^2\beta\epsilon + e\,\alpha\beta^2\epsilon - \frac{1}{2}h\,\alpha^2\beta^2\epsilon$$

Which means $\mathbb{O}\left(e^{\alpha f+\beta e} \,\middle|\, f\,e\right) = \mathbb{O}\left(e^{-\alpha\beta h+\beta e+\alpha f}\left(1+\epsilon\left(2\,l\,\alpha\beta + f\,\alpha^2\beta + e\,\alpha\beta^2 - \frac{1}{2}h\,\alpha^2\beta^2\right)\right) \,\middle|\, e\,l\,f\right)$.

And so,

3. $\mathbb{O}\left(e^{\alpha f+\beta e+\delta ef} \,\middle|\, f\,e\right) = \mathbb{O}\left(v(1+\epsilon\Lambda)\,e^{v(-h\alpha\beta+\alpha f+\beta e+\delta ef)} \,\middle|\, e\,l\,f\right)$, with $v = (1+h\,\delta)^{-1}$ and Λ, the "logos", as below.

```
DP_{x_→D_α, y_→D_β}[P_][f_] := Total[CoefficientRules[P, {x, y}] /. ({m_, n_} → c_) :> c D[f, {α, m}, {β, n}]]
```

```
Λ1 = Collect[
  With[{q = e^(ν (f α+e β-h α β+e f δ))}, q^-1 DP_{α→D_f,β→D_e}[λ1][q] /. ν → (1+h δ)^-1],
  ϵ, Simplify]
```

$$1+\frac{1}{2\,(1+h\,\delta)^4}\left(2\,e\,\alpha\,\beta^2-h\,\alpha^2\,\beta^2+4\,e\,\beta\,\delta-4\,h\,\alpha\,\beta\,\delta+2\,e^2\,\beta^2\,\delta-2\,h\,\delta^2+4\,e\,h\,\beta\,\delta^2-4\,h^2\,\alpha\,\beta\,\delta^2+e^2\,h\,\beta^2\,\delta^2-4\,h^2\,\delta^3-2\,h^3\,\delta^4+4\,l\,(1+h\,\delta)^2\,(\alpha\,(\beta+f\,\delta)+\delta\,(1+e\,\beta+e\,f\,\delta+h\,\delta))+f^2\,\delta\,(\alpha+e\,\delta)\,(\alpha\,(2+h\,\delta)+e\,\delta\,(4+3\,h\,\delta))+2\,f\,(\alpha^2\,\beta+2\,\alpha\,\delta\,(1+h\,\delta+e\,\beta\,(2+h\,\delta))+e\,\delta^2\,(4+6\,h\,\delta+2\,h^2\,\delta^2+e\,\beta\,(3+2\,h\,\delta)))\right)\epsilon$$

```
oldΛ = Simplify[
  1/(2 μ^4) (-b α^2 β^2 + u^2 β^2 δ (2 + b δ) + w^2 δ (α + u δ) (α (2 + b δ) + u δ (4 + 3 b δ)) - 4 b α β δ μ + 4 c α β μ^2 - 2 b δ^2 μ^2 +
    4 c δ μ^3 + 2 u β (α β + 2 δ μ (1 + c μ)) + 2 w (α^2 β + 2 α δ (u β (2 + b δ) + μ (1 + c μ))) +
      u δ^2 (u β (3 + 2 b δ) + 2 μ (2 + b δ + c μ)))) /. {μ → ν^-1, b → h, c → l, u → e, w → f}]
```

$$\frac{1}{2}\,\nu\left(4\,l\left(\delta+\alpha\,\beta\,\nu+f\,\alpha\,\delta\,\nu+e\,\beta\,\delta\,\nu+e\,f\,\delta^2\,\nu\right)+\nu\left(2\left(e\,\beta+f\left(\alpha+2\,e\,\delta\right)\right)\nu\left(\alpha\,\beta\,\nu+e\,f\,\delta^2\,\nu+\delta\left(2+f\,\alpha\,\nu+e\,\beta\,\nu\right)\right)+h\left(-4\,\alpha\,\beta\,\delta\,\nu-\alpha^2\,\beta^2\,\nu^2+3\,e^2\,f^2\,\delta^4\,\nu^2+4\,e\,f\,\delta^3\,\nu\left(1+f\,\alpha\,\nu+e\,\beta\,\nu\right)+\delta^2\left(-2+f^2\,\alpha^2\,\nu^2+4\,e\,f\,\alpha\,\beta\,\nu^2+e^2\,\beta^2\,\nu^2\right)\right)\right)\right)$$

```
Simplify[Λ1 == 1 + ϵ oldΛ /. ν → (1+h δ)^-1]
```

```
True
```

The $k=2$ logos

```
Series[{a, b, c} /. sol, {ϵ, 0, 2}] // Normal
```

$$\left\{\beta+\alpha\,\beta^2\,\epsilon+\alpha^2\,\beta^3\,\epsilon^2,\ \alpha+\alpha^2\,\beta\,\epsilon+\alpha^3\,\beta^2\,\epsilon^2,\ -\alpha\,\beta-\frac{1}{2}\,\alpha^2\,\beta^2\,\epsilon-\frac{1}{3}\,\alpha^3\,\beta^3\,\epsilon^2\right\}$$

```
λ2 = Simplify[e^(-f α-e β+h α β) Normal@Series[e^(c h + a e - 2 ϵ c l + b f) /. sol, {ϵ, 0, 2}]]
```

$$\frac{1}{2}\left(2-\alpha\,\beta\left(-4\,l-2\,f\,\alpha-2\,e\,\beta+h\,\alpha\,\beta\right)\epsilon+\frac{1}{12}\,\alpha^2\,\beta^2\left(3\left(4\,l+2\,f\,\alpha+2\,e\,\beta-h\,\alpha\,\beta\right)^2+8\left(3\,l+3\,f\,\alpha+3\,e\,\beta-h\,\alpha\,\beta\right)\right)\epsilon^2\right)$$

```
Λ2 = Collect[
  With[{q = ℯ^(ν (f α+e β-h α β+e f δ))}, q^-1 DP_{α→D_f, β→D_e}[λ2][q] /. ν → (1+h δ)^-1],
  ϵ, Simplify]
```

$$
\begin{aligned}
&1+\\
&\frac{1}{2(1+h\delta)^4}\Big(2e\alpha\beta^2-h\alpha^2\beta^2+4e\beta\delta-4h\alpha\beta\delta+2e^2\beta^2\delta-2h\delta^2+4eh\beta\delta^2-4h^2\alpha\beta\delta^2+e^2h\beta^2\delta^2-4h^2\delta^3-2h^3\delta^4+\\
&\quad 4l(1+h\delta)^2\big(\alpha(\beta+f\delta)+\delta(1+e\beta+ef\delta+h\delta)\big)+f^2\delta(\alpha+e\delta)\big(\alpha(2+h\delta)+e\delta(4+3h\delta)\big)+\\
&\quad 2f\big(\alpha^2\beta+2\alpha\delta(1+h\delta+e\beta(2+h\delta))+e\delta^2(4+6h\delta+2h^2\delta^2+e\beta(3+2h\delta))\big)\Big)\epsilon+\\
&\frac{1}{24(1+h\delta)^8}\Big(24l(1+h\delta)^4\big((\alpha+e\delta)^2(\beta+f\delta)^2+4\delta(\alpha+e\delta)(\beta+f\delta)(1+h\delta)+2\delta^2(1+h\delta)^2\big)+\\
&\quad 48l^2(1+h\delta)^4\big((\alpha+e\delta)^2(\beta+f\delta)^2+4\delta(\alpha+e\delta)(\beta+f\delta)(1+h\delta)+2\delta^2(1+h\delta)^2\big)+\\
&\quad 24f(\alpha+e\delta)(1+h\delta)^3\big((\alpha+e\delta)^2(\beta+f\delta)^2+6\delta(\alpha+e\delta)(\beta+f\delta)(1+h\delta)+6\delta^2(1+h\delta)^2\big)+\\
&\quad 48fl(\alpha+e\delta)(1+h\delta)^3\big((\alpha+e\delta)^2(\beta+f\delta)^2+6\delta(\alpha+e\delta)(\beta+f\delta)(1+h\delta)+6\delta^2(1+h\delta)^2\big)+\\
&\quad 24e(\beta+f\delta)(1+h\delta)^3\big((\alpha+e\delta)^2(\beta+f\delta)^2+6\delta(\alpha+e\delta)(\beta+f\delta)(1+h\delta)+6\delta^2(1+h\delta)^2\big)+\\
&\quad 48el(\beta+f\delta)(1+h\delta)^3\big((\alpha+e\delta)^2(\beta+f\delta)^2+6\delta(\alpha+e\delta)(\beta+f\delta)(1+h\delta)+6\delta^2(1+h\delta)^2\big)+\\
&\quad 12(\beta+f\delta)^2(e+eh\delta)^2\big((\alpha+e\delta)^2(\beta+f\delta)^2+8\delta(\alpha+e\delta)(\beta+f\delta)(1+h\delta)+12\delta^2(1+h\delta)^2\big)+\\
&\quad 12(\alpha+e\delta)^2(f+fh\delta)^2\big((\alpha+e\delta)^2(\beta+f\delta)^2+8\delta(\alpha+e\delta)(\beta+f\delta)(1+h\delta)+12\delta^2(1+h\delta)^2\big)+24ef(1+h\delta)^2\\
&\qquad\big((\alpha+e\delta)^3(\beta+f\delta)^3+9\delta(\alpha+e\delta)^2(\beta+f\delta)^2(1+h\delta)+18\delta^2(\alpha+e\delta)(\beta+f\delta)(1+h\delta)^2+6\delta^3(1+h\delta)^3\big)-\\
&\quad 8h(1+h\delta)^2\big((\alpha+e\delta)^3(\beta+f\delta)^3+9\delta(\alpha+e\delta)^2(\beta+f\delta)^2(1+h\delta)+\\
&\qquad 18\delta^2(\alpha+e\delta)(\beta+f\delta)(1+h\delta)^2+6\delta^3(1+h\delta)^3\big)-24hl(1+h\delta)^2\\
&\qquad\big((\alpha+e\delta)^3(\beta+f\delta)^3+9\delta(\alpha+e\delta)^2(\beta+f\delta)^2(1+h\delta)+18\delta^2(\alpha+e\delta)(\beta+f\delta)(1+h\delta)^2+6\delta^3(1+h\delta)^3\big)-\\
&\quad 12fh(\alpha+e\delta)(1+h\delta)\big((\alpha+e\delta)^3(\beta+f\delta)^3+12\delta(\alpha+e\delta)^2(\beta+f\delta)^2(1+h\delta)+\\
&\qquad 36\delta^2(\alpha+e\delta)(\beta+f\delta)(1+h\delta)^2+24\delta^3(1+h\delta)^3\big)-12eh(\beta+f\delta)(1+h\delta)\\
&\qquad\big((\alpha+e\delta)^3(\beta+f\delta)^3+12\delta(\alpha+e\delta)^2(\beta+f\delta)^2(1+h\delta)+36\delta^2(\alpha+e\delta)(\beta+f\delta)(1+h\delta)^2+24\delta^3(1+h\delta)^3\big)+\\
&\quad 3h^2\big((\alpha+e\delta)^4(\beta+f\delta)^4+16\delta(\alpha+e\delta)^3(\beta+f\delta)^3(1+h\delta)+72\delta^2(\alpha+e\delta)^2(\beta+f\delta)^2(1+h\delta)^2+\\
&\qquad 96\delta^3(\alpha+e\delta)(\beta+f\delta)(1+h\delta)^3+24\delta^4(1+h\delta)^4\big)\big)\epsilon^2
\end{aligned}
$$

```
(Λ2 /. ϵ^2 → 0) == Λ1
```

True

Implementing g_1 at $k=2$

```
PBWRule = {e → 1, l → 2, f → 3};
B[U@e, U@l] = -U@e; B[U@f, U@l] = U@f; B[U@e, U@f] = h U[] -2 ϵ U[l];
ϵ /: ϵ^3|4 = 0;
```

```
$TD = 3; ħ /: ħ^d_. /; d > $TD := 0;
```

```
x_ ≼ y_ := OrderedQ[{x, y} /. PBWRule]; x_ ≺ y_ := ! OrderedQ[{y, x} /. PBWRule];
Simp[ℰ_] := Collect[ℰ, _U, Expand];
```

```
Uᵢ_[ℰ_] := ℰ /. {h → hᵢ, t → tᵢ, u_U :> Replace[u, x_ :> xᵢ, 1]};
B[U[(x_)ᵢ_], U[(y_)ᵢ_]] := B[U[xᵢ], U[yᵢ]] = Uᵢ[B[U@x, U@y]];
B[U[(x_)ᵢ_], U[(y_)ⱼ_]] /; i =!= j := 0;
B[x_, x_] = 0;
B[U[y_], U[x_]] := B[U[y], U[x]] = Simp[-B[U[x], U[y]]];
B[x_, y_] := x ** y - y ** x;
```

```
Unprotect[NonCommutativeMultiply];
NonCommutativeMultiply[x_] := x;
0 ** _ = _ ** 0 = 0;
x_ ** U[] := x; U[] ** x_ := x;
(a_ * x_U) ** (b_ * y_U) := If[a b === 0, 0, Simp[a b (x ** y)]];
(a_ * x_U) ** y_ := Simp[a (x ** y)]; x_ ** (a_ * y_U) := Simp[a (x ** y)];
(x_Plus) ** y_ := (# ** y) & /@ x; x_ ** (y_Plus) := (x ** #) & /@ y;
```

```
U[xx___, x_] ** U[y_, yy___] := If[x ≤ y, U[xx, x, y, yy], U@xx ** (U@y ** U@x + B[U@x, U@y]) ** U@yy];
```

```
UU[l___, x_^n_, r___] := UU[l, Sequence @@ Table[x, {n}], r];
UU[l___, 1, r___] := UU[l, r];
UU[] = U[];
UU[l_, r___] := U[l] ** UU[r];
```

```
UProducts[{}, 0] = {UU[]};
UProducts[{}, n_Integer] /; n > 0 = {};
UProducts[{x_, xs___}, n_Integer] :=
  Sort@Flatten@Table[UU[x^k] ** u, {k, 0, n}, {u, UProducts[{xs}, n - k]}];
UProducts[xs_List, k_Integer, n_Integer] := UProducts[Flatten@Table[xⱼ, {x, xs}, {j, k}], n];
UProducts[any__, {n_}] := Flatten@Table[UProducts[any, k], {k, 0, n}];
```

```
rᵢ_,ⱼ_ := Simp[ħ (hᵢ UU[lⱼ] + UU[eᵢ, fⱼ])]
```

```
UExp[u_] := Module[{s, t, k},
   s = t = U[]; k = 0;
   While[k < 20 ∧ 0 =!= (t = t ** u), s += t / (++k)!];
   Simp[s]];
Rᵢ_,ⱼ_ := UExp[rᵢ,ⱼ];
```

```
m[i_, j_, k_][ℰ_] := Simp[ℰ /. {
    u_U :> UU @@ Join[DeleteCases[u, x_(i|j)], U @@ Cases[u, x_i :> xₖ], U @@ Cases[u, x_j :> xₖ]],
    h_(i|j) → hₖ}]
```

```
𝕆[poly_, specs___] := Module[{vs, us, z},
  vs = Join @@ (First /@ {specs});
  us = Join @@ ({specs} /. (l_ → s_) :> (l /. x_i_ :> xₛ));
  Simp@Total[CoefficientRules[Normal@Series[poly, {ħ, 0, $TD}], vs] /. (p_ → c_) :> c UU @@ (us^p)]
 ]
```

Verifying the full $\mathfrak{g}_1$ lemma at $k = 2$

3. $\mathbb{O}\left(e^{\alpha f + \beta e + \delta e f} \mid f e\right) = \mathbb{O}\left(v \Lambda_2 e^{v(-h\alpha\beta + \alpha f + \beta e + \delta e f)} \mid e f\right)$, with $v = (1 + h\delta)^{-1}$ and Λ_2, the "2-logos", as above.

```
degrule = {α → ħ α, h → ħ h1, e → ħ e1, l → l1, f → f1, ϵ → ħ ϵ};

$TD = 2; 𝕆[e^(α f+β e+δ e f) /. degrule, {f1, e1} → 1]
```

$$\left(1-\delta\,\hbar\,h_1-\alpha\,\beta\,\hbar^2\,h_1-\delta^2\,\epsilon\,\hbar^2\,h_1+\delta^2\,\hbar^2\,h_1^2\right)U[\,]+\left(\beta\,\hbar+2\,\beta\,\delta\,\epsilon\,\hbar^2-2\,\beta\,\delta\,\hbar^2\,h_1\right)U[e_1]+$$
$$\left(\alpha\,\hbar+2\,\alpha\,\delta\,\epsilon\,\hbar^2-2\,\alpha\,\delta\,\hbar^2\,h_1\right)U[f_1]+\left(2\,\delta\,\epsilon\,\hbar+2\,\alpha\,\beta\,\epsilon\,\hbar^2+2\,\delta^2\,\epsilon^2\,\hbar^2-4\,\delta^2\,\epsilon\,\hbar^2\,h_1\right)U[l_1]+\frac{1}{2}\beta^2\,\hbar^2\,U[e_1,e_1]+$$
$$\left(\delta\,\hbar+\alpha\,\beta\,\hbar^2+4\,\delta^2\,\epsilon\,\hbar^2-2\,\delta^2\,\hbar^2\,h_1\right)U[e_1,f_1]+4\,\beta\,\delta\,\epsilon\,\hbar^2\,U[e_1,l_1]+\frac{1}{2}\alpha^2\,\hbar^2\,U[f_1,f_1]+4\,\alpha\,\delta\,\epsilon\,\hbar^2\,U[l_1,f_1]+$$
$$4\,\delta^2\,\epsilon^2\,\hbar^2\,U[l_1,l_1]+\beta\,\delta\,\hbar^2\,U[e_1,e_1,f_1]+\alpha\,\delta\,\hbar^2\,U[e_1,f_1,f_1]+4\,\delta^2\,\epsilon\,\hbar^2\,U[e_1,l_1,f_1]+\frac{1}{2}\delta^2\,\hbar^2\,U[e_1,e_1,f_1,f_1]$$

```
$TD = 10; Simp[
  𝕆[e^(α f+β e+δ e f) /. degrule, {f1, e1} → 1] - 𝕆[ν Λ2 e^(ν (-α β h+α f+β e+δ e f)) /. ν → (1 + h δ)^-1 /. degrule, {e1, l1, f1} → 1]
]
```

```
0
```

The *k* = 3 logos

```
Clear[ϵ]

Series[{a, b, c} /. sol, {ϵ, 0, 3}] // Normal
```

$$\left\{\beta+\alpha\,\beta^2\,\epsilon+\alpha^2\,\beta^3\,\epsilon^2+\alpha^3\,\beta^4\,\epsilon^3,\ \alpha+\alpha^2\,\beta\,\epsilon+\alpha^3\,\beta^2\,\epsilon^2+\alpha^4\,\beta^3\,\epsilon^3,\ -\alpha\,\beta-\frac{1}{2}\alpha^2\,\beta^2\,\epsilon-\frac{1}{3}\alpha^3\,\beta^3\,\epsilon^2-\frac{1}{4}\alpha^4\,\beta^4\,\epsilon^3\right\}$$

```
λ3 = Simplify[e^(-f α-e β+h α β) Normal@Series[e^(c h+a e-2 ϵ c l+b f) /. sol, {ϵ, 0, 3}]]
```

$$\frac{1}{6}\left(6-3\,\alpha\,\beta\left(-4\,l-2\,f\,\alpha-2\,e\,\beta+h\,\alpha\,\beta\right)\epsilon+\frac{1}{4}\alpha^2\,\beta^2\left(3\left(4\,l+2\,f\,\alpha+2\,e\,\beta-h\,\alpha\,\beta\right)^2+8\left(3\,l+3\,f\,\alpha+3\,e\,\beta-h\,\alpha\,\beta\right)\right)\epsilon^2+\right.$$
$$\frac{1}{24}\alpha^3\,\beta^3\left(12\left(8\,l+12\,f\,\alpha+12\,e\,\beta-3\,h\,\alpha\,\beta\right)+16\left(4\,l+2\,f\,\alpha+2\,e\,\beta-h\,\alpha\,\beta\right)\left(3\,l+3\,f\,\alpha+3\,e\,\beta-h\,\alpha\,\beta\right)+\right.$$
$$\left.\left.\left(4\,l+2\,f\,\alpha+2\,e\,\beta-h\,\alpha\,\beta\right)\left(3\left(4\,l+2\,f\,\alpha+2\,e\,\beta-h\,\alpha\,\beta\right)^2+8\left(3\,l+3\,f\,\alpha+3\,e\,\beta-h\,\alpha\,\beta\right)\right)\right)\epsilon^3\right)$$

```
Λ3 = Collect[
  With[{q = e^(ν (f α+e β-h α β+e f δ))}, q^-1 DP_(α→D_f, β→D_e)[λ3][q] /. ν → (1 + h δ)^-1],
  ϵ, Simplify]
```

$$1+\frac{1}{2\left(1+h\,\delta\right)^4}\left(2\,e\,\alpha\,\beta^2-h\,\alpha^2\,\beta^2+4\,e\,\beta\,\delta-4\,h\,\alpha\,\beta\,\delta+2\,e^2\,\beta^2\,\delta-2\,h\,\delta^2+4\,e\,h\,\beta\,\delta^2-4\,h^2\,\alpha\,\beta\,\delta^2+e^2\,h\,\beta^2\,\delta^2-4\,h^2\,\delta^3-2\,h^3\,\delta^4+\right.$$
$$4\,l\left(1+h\,\delta\right)^2\left(\alpha\left(\beta+f\,\delta\right)+\delta\left(1+e\,\beta+e\,f\,\delta+h\,\delta\right)\right)+f^2\,\delta\left(\alpha+e\,\delta\right)\left(\alpha\left(2+h\,\delta\right)+e\,\delta\left(4+3\,h\,\delta\right)\right)+$$
$$\left.2\,f\left(\alpha^2\,\beta+2\,\alpha\,\delta\left(1+h\,\delta+e\,\beta\left(2+h\,\delta\right)\right)+e\,\delta^2\left(4+6\,h\,\delta+2\,h^2\,\delta^2+e\,\beta\left(3+2\,h\,\delta\right)\right)\right)\right)\epsilon+$$
$$\frac{1}{24\left(1+h\,\delta\right)^8}\left(24\,l\left(1+h\,\delta\right)^4\left(\left(\alpha+e\,\delta\right)^2\left(\beta+f\,\delta\right)^2+4\,\delta\left(\alpha+e\,\delta\right)\left(\beta+f\,\delta\right)\left(1+h\,\delta\right)+2\,\delta^2\left(1+h\,\delta\right)^2\right)+\right.$$
$$48\,l^2\left(1+h\,\delta\right)^4\left(\left(\alpha+e\,\delta\right)^2\left(\beta+f\,\delta\right)^2+4\,\delta\left(\alpha+e\,\delta\right)\left(\beta+f\,\delta\right)\left(1+h\,\delta\right)+2\,\delta^2\left(1+h\,\delta\right)^2\right)+$$
$$24\,f\left(\alpha+e\,\delta\right)\left(1+h\,\delta\right)^3\left(\left(\alpha+e\,\delta\right)^2\left(\beta+f\,\delta\right)^2+6\,\delta\left(\alpha+e\,\delta\right)\left(\beta+f\,\delta\right)\left(1+h\,\delta\right)+6\,\delta^2\left(1+h\,\delta\right)^2\right)+$$
$$48\,f\,l\left(\alpha+e\,\delta\right)\left(1+h\,\delta\right)^3\left(\left(\alpha+e\,\delta\right)^2\left(\beta+f\,\delta\right)^2+6\,\delta\left(\alpha+e\,\delta\right)\left(\beta+f\,\delta\right)\left(1+h\,\delta\right)+6\,\delta^2\left(1+h\,\delta\right)^2\right)+$$
$$24\,e\left(\beta+f\,\delta\right)\left(1+h\,\delta\right)^3\left(\left(\alpha+e\,\delta\right)^2\left(\beta+f\,\delta\right)^2+6\,\delta\left(\alpha+e\,\delta\right)\left(\beta+f\,\delta\right)\left(1+h\,\delta\right)+6\,\delta^2\left(1+h\,\delta\right)^2\right)+$$
$$48\,e\,l\left(\beta+f\,\delta\right)\left(1+h\,\delta\right)^3\left(\left(\alpha+e\,\delta\right)^2\left(\beta+f\,\delta\right)^2+6\,\delta\left(\alpha+e\,\delta\right)\left(\beta+f\,\delta\right)\left(1+h\,\delta\right)+6\,\delta^2\left(1+h\,\delta\right)^2\right)+$$
$$12\left(\beta+f\,\delta\right)^2\left(e+e\,h\,\delta\right)^2\left(\left(\alpha+e\,\delta\right)^2\left(\beta+f\,\delta\right)^2+8\,\delta\left(\alpha+e\,\delta\right)\left(\beta+f\,\delta\right)\left(1+h\,\delta\right)+12\,\delta^2\left(1+h\,\delta\right)^2\right)+$$
$$12\left(\alpha+e\,\delta\right)^2\left(f+f\,h\,\delta\right)^2\left(\left(\alpha+e\,\delta\right)^2\left(\beta+f\,\delta\right)^2+8\,\delta\left(\alpha+e\,\delta\right)\left(\beta+f\,\delta\right)\left(1+h\,\delta\right)+12\,\delta^2\left(1+h\,\delta\right)^2\right)+24\,e\,f\left(1+h\,\delta\right)^2$$
$$\left(\left(\alpha+e\,\delta\right)^3\left(\beta+f\,\delta\right)^3+9\,\delta\left(\alpha+e\,\delta\right)^2\left(\beta+f\,\delta\right)^2\left(1+h\,\delta\right)+18\,\delta^2\left(\alpha+e\,\delta\right)\left(\beta+f\,\delta\right)\left(1+h\,\delta\right)^2+6\,\delta^3\left(1+h\,\delta\right)^3\right)-$$
$$8\,h\left(1+h\,\delta\right)^2\left(\left(\alpha+e\,\delta\right)^3\left(\beta+f\,\delta\right)^3+9\,\delta\left(\alpha+e\,\delta\right)^2\left(\beta+f\,\delta\right)^2\left(1+h\,\delta\right)+\right.$$
$$\left.18\,\delta^2\left(\alpha+e\,\delta\right)\left(\beta+f\,\delta\right)\left(1+h\,\delta\right)^2+6\,\delta^3\left(1+h\,\delta\right)^3\right)-24\,h\,l\left(1+h\,\delta\right)^2$$
$$\left(\left(\alpha+e\,\delta\right)^3\left(\beta+f\,\delta\right)^3+9\,\delta\left(\alpha+e\,\delta\right)^2\left(\beta+f\,\delta\right)^2\left(1+h\,\delta\right)+18\,\delta^2\left(\alpha+e\,\delta\right)\left(\beta+f\,\delta\right)\left(1+h\,\delta\right)^2+6\,\delta^3\left(1+h\,\delta\right)^3\right)-$$
$$12\,f\,h\left(\alpha+e\,\delta\right)\left(1+h\,\delta\right)\left(\left(\alpha+e\,\delta\right)^3\left(\beta+f\,\delta\right)^3+12\,\delta\left(\alpha+e\,\delta\right)^2\left(\beta+f\,\delta\right)^2\left(1+h\,\delta\right)+\right.$$

$$36\,\delta^2\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^2+24\,\delta^3\,(1+h\,\delta)^3\Big)-12\,e\,h\,(\beta+f\,\delta)\,(1+h\,\delta)$$
$$\Big((\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3+12\,\delta\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)+36\,\delta^2\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^2+24\,\delta^3\,(1+h\,\delta)^3\Big)+$$
$$3\,h^2\Big((\alpha+e\,\delta)^4\,(\beta+f\,\delta)^4+16\,\delta\,(\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3\,(1+h\,\delta)+72\,\delta^2\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)^2+$$
$$96\,\delta^3\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^3+24\,\delta^4\,(1+h\,\delta)^4\Big)\Big)\,\epsilon^2+$$
$$\frac{1}{48\,(1+h\,\delta)^{12}}\Big(32\,l\,(1+h\,\delta)^6\Big((\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3+9\,\delta\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)+$$
$$18\,\delta^2\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^2+6\,\delta^3\,(1+h\,\delta)^3\Big)+96\,l^2\,(1+h\,\delta)^6\Big((\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3+$$
$$9\,\delta\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)+18\,\delta^2\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^2+6\,\delta^3\,(1+h\,\delta)^3\Big)+64\,l^3\,(1+h\,\delta)^6$$
$$\Big((\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3+9\,\delta\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)+18\,\delta^2\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^2+6\,\delta^3\,(1+h\,\delta)^3\Big)+$$
$$48\,f\,(\alpha+e\,\delta)\,(1+h\,\delta)^5\Big((\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3+12\,\delta\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)+$$
$$36\,\delta^2\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^2+24\,\delta^3\,(1+h\,\delta)^3\Big)+144\,f\,l\,(\alpha+e\,\delta)\,(1+h\,\delta)^5$$
$$\Big((\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3+12\,\delta\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)+36\,\delta^2\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^2+24\,\delta^3\,(1+h\,\delta)^3\Big)+$$
$$96\,f\,l^2\,(\alpha+e\,\delta)\,(1+h\,\delta)^5\Big((\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3+12\,\delta\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)+$$
$$36\,\delta^2\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^2+24\,\delta^3\,(1+h\,\delta)^3\Big)+48\,e\,(\beta+f\,\delta)\,(1+h\,\delta)^5$$
$$\Big((\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3+12\,\delta\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)+36\,\delta^2\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^2+24\,\delta^3\,(1+h\,\delta)^3\Big)+$$
$$144\,e\,l\,(\beta+f\,\delta)\,(1+h\,\delta)^5\Big((\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3+12\,\delta\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)+$$
$$36\,\delta^2\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^2+24\,\delta^3\,(1+h\,\delta)^3\Big)+96\,e\,l^2\,(\beta+f\,\delta)\,(1+h\,\delta)^5$$
$$\Big((\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3+12\,\delta\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)+36\,\delta^2\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^2+24\,\delta^3\,(1+h\,\delta)^3\Big)+$$
$$48\,f^2\,(\alpha+e\,\delta)^2\,(1+h\,\delta)^4\Big((\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3+15\,\delta\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)+$$
$$60\,\delta^2\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^2+60\,\delta^3\,(1+h\,\delta)^3\Big)+48\,f^2\,l\,(\alpha+e\,\delta)^2\,(1+h\,\delta)^4$$
$$\Big((\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3+15\,\delta\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)+60\,\delta^2\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^2+60\,\delta^3\,(1+h\,\delta)^3\Big)+$$
$$48\,e^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)^4\Big((\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3+15\,\delta\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)+$$
$$60\,\delta^2\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^2+60\,\delta^3\,(1+h\,\delta)^3\Big)+48\,e^2\,l\,(\beta+f\,\delta)^2\,(1+h\,\delta)^4$$
$$\Big((\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3+15\,\delta\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)+60\,\delta^2\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^2+60\,\delta^3\,(1+h\,\delta)^3\Big)+$$
$$8\,(\beta+f\,\delta)^3\,(e+e\,h\,\delta)^3\Big((\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3+18\,\delta\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)+$$
$$90\,\delta^2\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^2+120\,\delta^3\,(1+h\,\delta)^3\Big)+8\,(\alpha+e\,\delta)^3\,(f+f\,h\,\delta)^3$$
$$\Big((\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3+18\,\delta\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)+90\,\delta^2\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^2+120\,\delta^3\,(1+h\,\delta)^3\Big)+$$
$$96\,e\,f\,(1+h\,\delta)^4\Big((\alpha+e\,\delta)^4\,(\beta+f\,\delta)^4+16\,\delta\,(\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3\,(1+h\,\delta)+$$
$$72\,\delta^2\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)^2+96\,\delta^3\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^3+24\,\delta^4\,(1+h\,\delta)^4\Big)-$$
$$12\,h\,(1+h\,\delta)^4\Big((\alpha+e\,\delta)^4\,(\beta+f\,\delta)^4+16\,\delta\,(\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3\,(1+h\,\delta)+$$
$$72\,\delta^2\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)^2+96\,\delta^3\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^3+24\,\delta^4\,(1+h\,\delta)^4\Big)+$$
$$96\,e\,f\,l\,(1+h\,\delta)^4\Big((\alpha+e\,\delta)^4\,(\beta+f\,\delta)^4+16\,\delta\,(\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3\,(1+h\,\delta)+$$
$$72\,\delta^2\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)^2+96\,\delta^3\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^3+24\,\delta^4\,(1+h\,\delta)^4\Big)-$$
$$56\,h\,l\,(1+h\,\delta)^4\Big((\alpha+e\,\delta)^4\,(\beta+f\,\delta)^4+16\,\delta\,(\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3\,(1+h\,\delta)+$$
$$72\,\delta^2\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)^2+96\,\delta^3\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^3+24\,\delta^4\,(1+h\,\delta)^4\Big)-$$
$$48\,h\,l^2\,(1+h\,\delta)^4\Big((\alpha+e\,\delta)^4\,(\beta+f\,\delta)^4+16\,\delta\,(\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3\,(1+h\,\delta)+$$
$$72\,\delta^2\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)^2+96\,\delta^3\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^3+24\,\delta^4\,(1+h\,\delta)^4\Big)+$$
$$24\,e\,f^2\,(\alpha+e\,\delta)\,(1+h\,\delta)^3\Big((\alpha+e\,\delta)^4\,(\beta+f\,\delta)^4+20\,\delta\,(\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3\,(1+h\,\delta)+$$
$$120\,\delta^2\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)^2+240\,\delta^3\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^3+120\,\delta^4\,(1+h\,\delta)^4\Big)-$$
$$40\,f\,h\,(\alpha+e\,\delta)\,(1+h\,\delta)^3\Big((\alpha+e\,\delta)^4\,(\beta+f\,\delta)^4+20\,\delta\,(\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3\,(1+h\,\delta)+$$
$$120\,\delta^2\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)^2+240\,\delta^3\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^3+120\,\delta^4\,(1+h\,\delta)^4\Big)-$$
$$48\,f\,h\,l\,(\alpha+e\,\delta)\,(1+h\,\delta)^3\Big((\alpha+e\,\delta)^4\,(\beta+f\,\delta)^4+20\,\delta\,(\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3\,(1+h\,\delta)+$$
$$120\,\delta^2\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)^2+240\,\delta^3\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^3+120\,\delta^4\,(1+h\,\delta)^4\Big)+$$

$$24\,e^2\,f\,(\beta+f\,\delta)\,(1+h\,\delta)^3\Big((\alpha+e\,\delta)^4\,(\beta+f\,\delta)^4+20\,\delta\,(\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3\,(1+h\,\delta)+120\,\delta^2\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)^2+240\,\delta^3\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^3+120\,\delta^4\,(1+h\,\delta)^4\Big)-$$
$$40\,e\,h\,(\beta+f\,\delta)\,(1+h\,\delta)^3\Big((\alpha+e\,\delta)^4\,(\beta+f\,\delta)^4+20\,\delta\,(\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3\,(1+h\,\delta)+120\,\delta^2\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)^2+240\,\delta^3\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^3+120\,\delta^4\,(1+h\,\delta)^4\Big)-$$
$$48\,e\,h\,l\,(\beta+f\,\delta)\,(1+h\,\delta)^3\Big((\alpha+e\,\delta)^4\,(\beta+f\,\delta)^4+20\,\delta\,(\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3\,(1+h\,\delta)+120\,\delta^2\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)^2+240\,\delta^3\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^3+120\,\delta^4\,(1+h\,\delta)^4\Big)-$$
$$12\,h\,(\beta+f\,\delta)^2\,(e+e\,h\,\delta)^2\Big((\alpha+e\,\delta)^4\,(\beta+f\,\delta)^4+24\,\delta\,(\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3\,(1+h\,\delta)+180\,\delta^2\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)^2+480\,\delta^3\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^3+360\,\delta^4\,(1+h\,\delta)^4\Big)-$$
$$12\,h\,(\alpha+e\,\delta)^2\,(f+f\,h\,\delta)^2\Big((\alpha+e\,\delta)^4\,(\beta+f\,\delta)^4+24\,\delta\,(\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3\,(1+h\,\delta)+180\,\delta^2\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)^2+480\,\delta^3\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^3+360\,\delta^4\,(1+h\,\delta)^4\Big)-$$
$$24\,e\,f\,h\,(1+h\,\delta)^2\Big((\alpha+e\,\delta)^5\,(\beta+f\,\delta)^5+25\,\delta\,(\alpha+e\,\delta)^4\,(\beta+f\,\delta)^4\,(1+h\,\delta)+200\,\delta^2\,(\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3\,(1+h\,\delta)^2+600\,\delta^3\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)^3+600\,\delta^4\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^4+120\,\delta^5\,(1+h\,\delta)^5\Big)+$$
$$8\,h^2\,(1+h\,\delta)^2\Big((\alpha+e\,\delta)^5\,(\beta+f\,\delta)^5+25\,\delta\,(\alpha+e\,\delta)^4\,(\beta+f\,\delta)^4\,(1+h\,\delta)+200\,\delta^2\,(\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3\,(1+h\,\delta)^2+600\,\delta^3\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)^3+600\,\delta^4\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^4+120\,\delta^5\,(1+h\,\delta)^5\Big)+$$
$$12\,h^2\,l\,(1+h\,\delta)^2\Big((\alpha+e\,\delta)^5\,(\beta+f\,\delta)^5+25\,\delta\,(\alpha+e\,\delta)^4\,(\beta+f\,\delta)^4\,(1+h\,\delta)+200\,\delta^2\,(\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3\,(1+h\,\delta)^2+600\,\delta^3\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)^3+600\,\delta^4\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^4+120\,\delta^5\,(1+h\,\delta)^5\Big)+6\,f\,h^2\,(\alpha+e\,\delta)\,(1+h\,\delta)\Big((\alpha+e\,\delta)^5\,(\beta+f\,\delta)^5+30\,\delta\,(\alpha+e\,\delta)^4\,(\beta+f\,\delta)^4\,(1+h\,\delta)+300\,\delta^2\,(\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3\,(1+h\,\delta)^2+1200\,\delta^3\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)^3+1800\,\delta^4\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^4+720\,\delta^5\,(1+h\,\delta)^5\Big)+$$
$$6\,e\,h^2\,(\beta+f\,\delta)\,(1+h\,\delta)\Big((\alpha+e\,\delta)^5\,(\beta+f\,\delta)^5+30\,\delta\,(\alpha+e\,\delta)^4\,(\beta+f\,\delta)^4\,(1+h\,\delta)+300\,\delta^2\,(\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3\,(1+h\,\delta)^2+1200\,\delta^3\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)^3+1800\,\delta^4\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^4+720\,\delta^5\,(1+h\,\delta)^5\Big)-$$
$$h^3\Big((\alpha+e\,\delta)^6\,(\beta+f\,\delta)^6+36\,\delta\,(\alpha+e\,\delta)^5\,(\beta+f\,\delta)^5\,(1+h\,\delta)+450\,\delta^2\,(\alpha+e\,\delta)^4\,(\beta+f\,\delta)^4\,(1+h\,\delta)^2+2400\,\delta^3\,(\alpha+e\,\delta)^3\,(\beta+f\,\delta)^3\,(1+h\,\delta)^3+5400\,\delta^4\,(\alpha+e\,\delta)^2\,(\beta+f\,\delta)^2\,(1+h\,\delta)^4+4320\,\delta^5\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,(1+h\,\delta)^5+720\,\delta^6\,(1+h\,\delta)^6\Big)\,\epsilon^3$$

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Collect[Λ1 /. x : (α | β | e | l | f) :> ħ x, ħ, Simplify]
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$$\frac{1+h^2\,\delta^2+h\,\delta\,(2-\delta\,\epsilon)}{(1+h\,\delta)^2}+\frac{2\,l\,\delta\,\epsilon\,\hbar}{1+h\,\delta}+\frac{2\,\delta\,\big((e-h\,\alpha)\,\beta+f\,(\alpha+e\,\delta\,(2+h\,\delta))\big)\,\epsilon\,\hbar^2}{(1+h\,\delta)^3}+\frac{2\,l\,(\alpha+e\,\delta)\,(\beta+f\,\delta)\,\epsilon\,\hbar^3}{(1+h\,\delta)^2}+$$
$$\frac{1}{2\,(1+h\,\delta)^4}(\alpha+e\,\delta)\,(\beta+f\,\delta)\,\big(\beta\,(-h\,\alpha+e\,(2+h\,\delta))+f\,(\alpha\,(2+h\,\delta)+e\,\delta\,(4+3\,h\,\delta))\big)\,\epsilon\,\hbar^4$$

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Collect[Λ2 /. x : (α | β | e | l | f) :> ħ x, ħ, Simplify]
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$$\frac{1}{(1+h\delta)^4}\left(1+h^4\delta^4+h^3\delta^3(4-\delta\epsilon)+h^2\delta^2\left(6-2\delta\epsilon+\delta^2\epsilon^2\right)-h\delta\left(-4+\delta\epsilon+2\delta^2\epsilon^2\right)\right)+$$
$$\frac{2l\delta\epsilon\left(1+h^2\delta^2+\delta\epsilon-2h\delta\left(-1+\delta\epsilon\right)\right)\hbar}{(1+h\delta)^3}+$$
$$\frac{1}{(1+h\delta)^5}2\delta\epsilon\left(-h\alpha\beta-2h^2\alpha\beta\delta-h^3\alpha\beta\delta^2+2l^2\delta\epsilon-3h\alpha\beta\delta\epsilon+6hl^2\delta^2\epsilon+\right.$$
$$3h^2\alpha\beta\delta^2\epsilon+6h^2l^2\delta^3\epsilon+2h^3l^2\delta^4\epsilon+e\beta\left(1+h^2\delta^2+3\delta\epsilon+h\delta\left(2-3\delta\epsilon\right)\right)+$$
$$\left.f\left(\alpha\left(1+h^2\delta^2+3\delta\epsilon+h\delta\left(2-3\delta\epsilon\right)\right)+e\delta\left(2+4h^2\delta^2+h^3\delta^3+9\delta\epsilon+h\delta\left(5+3\delta\epsilon\right)\right)\right)\right)\hbar^2+$$
$$\frac{1}{(1+h\delta)^4}2l\epsilon\left(\alpha\left(\beta\left(1+h^2\delta^2+2\delta\epsilon+h\delta\left(2-7\delta\epsilon\right)\right)+f\delta\left(1+h^2\delta^2+8\delta\epsilon+h\delta\left(2-\delta\epsilon\right)\right)\right)+\right.$$
$$\left.e\delta\left(\beta\left(1+h^2\delta^2+8\delta\epsilon+h\delta\left(2-\delta\epsilon\right)\right)+f\delta\left(1+h^2\delta^2+14\delta\epsilon+h\delta\left(2+5\delta\epsilon\right)\right)\right)\right)\hbar^3+$$
$$\frac{1}{2(1+h\delta)^6}\epsilon\left(f^2\delta\left(\alpha^2\left(2+4h^2\delta^2+h^3\delta^3+24\delta\epsilon+h\delta(5+6\delta\epsilon)\right)+\right.\right.$$
$$2e\alpha\delta\left(3+2h^3\delta^3+48\delta\epsilon+h^2\delta^2(7+6\delta\epsilon)+4h\delta\left(2+9\delta\epsilon\right)\right)+e^2\delta^2$$
$$\left(4+3h^3\delta^3+84\delta\epsilon+2h^2\delta^2\left(5+12\delta\epsilon\right)+h\delta\left(11+90\delta\epsilon\right)\right)\Big)+2f\Big(\alpha^2\beta\left(1+h^2\delta^2+6\delta\epsilon+2h\delta\left(1-6\delta\epsilon\right)\right)+$$
$$2\alpha\delta\left(4l^2\delta(1+h\delta)^3\epsilon+e\beta\left(2+5h\delta+h^3\delta^3+21\delta\epsilon+h^2\delta^2\left(4-3\delta\epsilon\right)\right)\right)+$$
$$e\delta^2\left(8l^2\delta(1+h\delta)^3\epsilon+e\beta\left(3+2h^3\delta^3+48\delta\epsilon+h^2\delta^2(7+6\delta\epsilon)+4h\delta\left(2+9\delta\epsilon\right)\right)\right)\Big)+$$
$$\beta\left(e^2\beta\delta\left(2+4h^2\delta^2+h^3\delta^3+24\delta\epsilon+h\delta(5+6\delta\epsilon)\right)+\right.$$
$$2e\left(8l^2\delta^2\left(1+h\delta\right)^3\epsilon+\alpha\beta\left(1+h^2\delta^2+6\delta\epsilon+2h\delta\left(1-6\delta\epsilon\right)\right)\right)+$$
$$\left.\left.\alpha\left(16l^2\delta\epsilon+h^3\left(-\alpha\beta\delta^2+16l^2\delta^4\epsilon\right)+2h^2\delta\left(24l^2\delta^2\epsilon+\alpha\beta\left(-1+6\delta\epsilon\right)\right)+h\left(48l^2\delta^2\epsilon-\alpha\left(\beta+6\beta\delta\epsilon\right)\right)\right)\right)\right)$$
$$\hbar^4+\frac{1}{(1+h\delta)^5}l(\alpha+e\delta)\left(\beta+f\delta\right)\left(\alpha\left(\beta-8h\beta\delta+f\delta\left(13+4h\delta\right)\right)+e\delta\left(\beta\left(13+4h\delta\right)+f\delta\left(25+16h\delta\right)\right)\right)$$
$$\epsilon^2\hbar^5+\frac{1}{3(1+h\delta)^7}$$
$$(\alpha+e\delta)\left(\beta+f\delta\right)$$
$$\left(6l^2(\alpha+e\delta)\left(\beta+f\delta\right)(1+h\delta)^3+\beta^2\left(h\alpha^2\left(-1+5h\delta\right)+e\alpha\left(3-14h\delta-5h^2\delta^2\right)+e^2\delta\left(15+11h\delta+2h^2\delta^2\right)\right)+\right.$$
$$f\beta\left(e\alpha\delta\left(39+2h\delta-13h^2\delta^2\right)+\alpha^2\left(3-14h\delta-5h^2\delta^2\right)+4e^2\delta^2\left(15+16h\delta+4h^2\delta^2\right)\right)+$$
$$\left.f^2\delta\left(\alpha^2\left(15+11h\delta+2h^2\delta^2\right)+4e\alpha\delta\left(15+16h\delta+4h^2\delta^2\right)+e^2\delta^2\left(57+77h\delta+26h^2\delta^2\right)\right)\right)\epsilon^2\hbar^6+$$
$$\frac{1}{(1+h\delta)^6}l(\alpha+e\delta)^2\left(\beta+f\delta\right)^2\left(\beta\left(-h\alpha+e\left(2+h\delta\right)\right)+f\left(\alpha\left(2+h\delta\right)+e\delta\left(4+3h\delta\right)\right)\right)$$
$$\epsilon^2$$
$$\hbar^7+$$
$$\frac{1}{8(1+h\delta)^8}(\alpha+e\delta)^2$$
$$\left(\beta+f\delta\right)^2$$
$$\left(\beta\left(-h\alpha+e\left(2+h\delta\right)\right)+f\left(\alpha\left(2+h\delta\right)+e\delta\left(4+3h\delta\right)\right)\right)^2$$
$$\epsilon^2$$
$$\hbar^8$$

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Collect[Λ3 /. x : (α | β | e | l | f) :> ħ x, ħ, Simplify]
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$$\frac{1}{(1+h\delta)^6}\left(1+h^6\delta^6+h^5\delta^5(6-\delta\epsilon)+h^4\delta^4\left(15-4\delta\epsilon+\delta^2\epsilon^2\right)-h^3\delta^3\left(-20+6\delta\epsilon+\delta^3\epsilon^3\right)-\right.$$
$$\left.h\delta\left(-6+\delta\epsilon+2\delta^2\epsilon^2+6\delta^3\epsilon^3\right)+h^2\delta^2\left(15-4\delta\epsilon-3\delta^2\epsilon^2+8\delta^3\epsilon^3\right)\right)+\frac{1}{(1+h\delta)^5}$$
$$2l\delta\epsilon\left(1+h^4\delta^4+\delta\epsilon+2\delta^2\epsilon^2-2h^3\delta^3\left(-2+\delta\epsilon\right)+3h^2\delta^2\left(2-\delta\epsilon+\delta^2\epsilon^2\right)+h\left(4\delta-10\delta^3\epsilon^2\right)\right)\hbar+$$
$$\frac{1}{(1+h\delta)^7}2\delta\epsilon\left(-h\alpha\beta-4h^2\alpha\beta\delta-6h^3\alpha\beta\delta^2-4h^4\alpha\beta\delta^3-h^5\alpha\beta\delta^4+2l^2\delta\epsilon-3h\alpha\beta\delta\epsilon+10hl^2\delta^2\epsilon-\right.$$
$$3h^2\alpha\beta\delta^2\epsilon+20h^2l^2\delta^3\epsilon+3h^3\alpha\beta\delta^3\epsilon+20h^3l^2\delta^4\epsilon+3h^4\alpha\beta\delta^4\epsilon+10h^4l^2\delta^5\epsilon+2h^5l^2\delta^6\epsilon+$$
$$6l^2\delta^2\epsilon^2-12h\alpha\beta\delta^2\epsilon^2+12hl^2\delta^3\epsilon^2+26h^2\alpha\beta\delta^3\epsilon^2-7h^3\alpha\beta\delta^4\epsilon^2-12h^3l^2\delta^5\epsilon^2-6h^4l^2\delta^6\epsilon^2+$$
$$e\beta\left(1+h^4\delta^4+3\delta\epsilon+12\delta^2\epsilon^2+h^3\delta^3\left(4-3\delta\epsilon\right)+h\delta\left(4+3\delta\epsilon-26\delta^2\epsilon^2\right)+h^2\delta^2\left(6-3\delta\epsilon+7\delta^2\epsilon^2\right)\right)+$$
$$f\left(e\delta\left(2+6h^4\delta^4+h^5\delta^5+9\delta\epsilon+48\delta^2\epsilon^2+h^2\delta^2\left(16+15\delta\epsilon\right)+h^3\delta^3\left(14+3\delta\epsilon+\delta^2\epsilon^2\right)+h\delta\left(9+21\delta\epsilon+2\delta^2\epsilon^2\right)\right)+\right.$$

$$\alpha\left(1+h^4\delta^4+3\delta\epsilon+12\delta^2\epsilon^2+h^3\delta^3\left(4-3\delta\epsilon\right)+h\delta\left(4+3\delta\epsilon-26\delta^2\epsilon^2\right)+h^2\delta^2\left(6-3\delta\epsilon+7\delta^2\epsilon^2\right)\right)\Big)\Big)\hbar^2+$$

$$\frac{1}{\left(1+h\delta\right)^6}2l\epsilon\Big(\alpha\left(f\delta\left(1+h^4\delta^4+8\delta\epsilon+42\delta^2\epsilon^2+h^3\delta^3\left(4-\delta\epsilon\right)+h\delta\left(4+15\delta\epsilon-32\delta^2\epsilon^2\right)+h^2\delta^2\left(6+6\delta\epsilon+\delta^2\epsilon^2\right)\right)+$$

$$\beta\left(1+h^4\delta^4+2\delta\epsilon+6\delta^2\epsilon^2+h^3\delta^3\left(4-7\delta\epsilon\right)+h\delta\left(4-3\delta\epsilon-44\delta^2\epsilon^2\right)+h^2\delta^2\left(6-12\delta\epsilon+25\delta^2\epsilon^2\right)\right)\Big)+$$

$$\delta\Big(4l^2\delta^2\left(1+h\delta\right)^3\epsilon^2+e\left(\beta\left(1+h^4\delta^4+8\delta\epsilon+42\delta^2\epsilon^2+h^3\delta^3\left(4-\delta\epsilon\right)+h\delta\left(4+15\delta\epsilon-32\delta^2\epsilon^2\right)+$$

$$h^2\delta^2\left(6+6\delta\epsilon+\delta^2\epsilon^2\right)\right)+f\delta\left(1+h^4\delta^4+14\delta\epsilon+102\delta^2\epsilon^2+h^3\delta^3\left(4+5\delta\epsilon\right)+$$

$$h^2\delta^2\left(6+24\delta\epsilon+\delta^2\epsilon^2\right)+h\delta\left(4+33\delta\epsilon+28\delta^2\epsilon^2\right)\right)\Big)\Big)\hbar^3+$$

$$\frac{1}{2\left(1+h\delta\right)^8}\epsilon\Big(f^2\delta\Big(\alpha^2\left(2+6h^4\delta^4+h^5\delta^5+24\delta\epsilon+192\delta^2\epsilon^2+h\delta\left(9+54\delta\epsilon-40\delta^2\epsilon^2\right)+2h^2\delta^2\left(8+18\delta\epsilon-3\delta^2\epsilon^2\right)+$$

$$h^3\delta^3\left(14+6\delta\epsilon+\delta^2\epsilon^2\right)\right)+2e\alpha\delta\left(3+2h^5\delta^5+48\delta\epsilon+480\delta^2\epsilon^2+h^4\delta^4\left(11+6\delta\epsilon\right)+$$

$$2h^3\delta^3\left(12+24\delta\epsilon+7\delta^2\epsilon^2\right)+h^2\delta^2\left(26+126\delta\epsilon+83\delta^2\epsilon^2\right)+2h\delta\left(7+66\delta\epsilon+162\delta^2\epsilon^2\right)\right)+$$

$$e^2\delta^2\left(4+3h^5\delta^5+84\delta\epsilon+1008\delta^2\epsilon^2+8h^4\delta^4\left(2+3\delta\epsilon\right)+h^3\delta^3\left(34+138\delta\epsilon+87\delta^2\epsilon^2\right)+$$

$$4h^2\delta^2\left(9+72\delta\epsilon+133\delta^2\epsilon^2\right)+h\delta\left(19+258\delta\epsilon+1228\delta^2\epsilon^2\right)\right)+$$

$$2f\Big(\alpha^2\beta\left(1+h^4\delta^4+6\delta\epsilon+36\delta^2\epsilon^2+4h^3\delta^3\left(1-3\delta\epsilon\right)+h^2\delta^2\left(6-18\delta\epsilon+61\delta^2\epsilon^2\right)+h\left(4\delta-128\delta^3\epsilon^2\right)\right)+$$

$$2\alpha\delta\Big(2l^2\delta\left(1+h\delta\right)^3\epsilon\left(2+2h^2\delta^2+21\delta\epsilon+h\delta\left(4-3\delta\epsilon\right)\right)+e\beta\Big(2+h^5\delta^5+21\delta\epsilon+168\delta^2\epsilon^2-3h^4\delta^4$$

$$\left(-2+\delta\epsilon\right)+h\delta\left(9+42\delta\epsilon-82\delta^2\epsilon^2\right)+h^3\delta^3\left(14-6\delta\epsilon+7\delta^2\epsilon^2\right)-2h^2\delta^2\left(-8-9\delta\epsilon+9\delta^2\epsilon^2\right)\Big)\Big)+$$

$$e\delta^2\Big(4l^2\delta\left(1+h\delta\right)^3\epsilon\left(2+2h^2\delta^2+33\delta\epsilon+h\delta\left(4+9\delta\epsilon\right)\right)+e\beta\Big(3+2h^5\delta^5+48\delta\epsilon+480\delta^2\epsilon^2+h^4\delta^4\left(11+6\delta\epsilon\right)+2h^3\delta^3\left(12+24\delta\epsilon+7\delta^2\epsilon^2\right)+h^2\delta^2\left(26+126\delta\epsilon+83\delta^2\epsilon^2\right)+2h\delta\left(7+66\delta\epsilon+162\delta^2\epsilon^2\right)\Big)\Big)+$$

$$\beta\Big(e^2\beta\delta\left(2+6h^4\delta^4+h^5\delta^5+24\delta\epsilon+192\delta^2\epsilon^2+h\delta\left(9+54\delta\epsilon-40\delta^2\epsilon^2\right)+2h^2\delta^2\left(8+18\delta\epsilon-3\delta^2\epsilon^2\right)+$$

$$h^3\delta^3\left(14+6\delta\epsilon+\delta^2\epsilon^2\right)\right)-$$

$$\alpha\left(-8l^2\delta\epsilon\left(2+9\delta\epsilon\right)+h^5\left(\alpha\beta\delta^4-16l^2\delta^6\epsilon\right)+4h^4\delta^3\left(10l^2\delta^2\epsilon\left(-2+3\delta\epsilon\right)+\alpha\left(\beta-3\beta\delta\epsilon\right)\right)-$$

$$4h^2\left(4l^2\delta^3\epsilon\left(10-9\delta\epsilon\right)+\alpha\beta\delta\left(-1+32\delta^2\epsilon^2\right)\right)+h\left(-16l^2\delta^2\epsilon\left(5+6\delta\epsilon\right)+\alpha\beta\left(1+6\delta\epsilon+36\delta^2\epsilon^2\right)\right)+$$

$$h^3\delta^2\left(32l^2\delta^2\epsilon\left(-5+9\delta\epsilon\right)+\alpha\beta\left(6-18\delta\epsilon+61\delta^2\epsilon^2\right)\right)\Big)+$$

$$2e\Big(4l^2\delta^2\left(1+h\delta\right)^3\epsilon\left(2+2h^2\delta^2+21\delta\epsilon+h\delta\left(4-3\delta\epsilon\right)\right)+$$

$$\alpha\beta\left(1+h^4\delta^4+6\delta\epsilon+36\delta^2\epsilon^2+4h^3\delta^3\left(1-3\delta\epsilon\right)+h^2\delta^2\left(6-18\delta\epsilon+61\delta^2\epsilon^2\right)+h\left(4\delta-128\delta^3\epsilon^2\right)\right)\Big)\Big)\Big)\hbar^4+$$

$$\frac{1}{\left(1+h\delta\right)^7}l\epsilon^2\Big(\alpha^2\Big(2f\beta\delta\left(7-2h^3\delta^3+60\delta\epsilon+12h\delta\left(1-7\delta\epsilon\right)+3h^2\delta^2\left(1+2\delta\epsilon\right)\right)+$$

$$f^2\delta^2\left(13+21h^2\delta^2+4h^3\delta^3+174\delta\epsilon+6h\delta\left(5+4\delta\epsilon\right)\right)+$$

$$\beta^2\left(1-8h^3\delta^3+6\delta\epsilon-6h\delta\left(1+12\delta\epsilon\right)+3h^2\delta^2\left(-5+24\delta\epsilon\right)\right)\Big)+$$

$$2\alpha\delta\Big(12l^2\delta\left(\beta+f\delta\right)\left(1+h\delta\right)^3\epsilon+e\Big(\beta^2\left(7-2h^3\delta^3+60\delta\epsilon+12h\delta\left(1-7\delta\epsilon\right)+3h^2\delta^2\left(1+2\delta\epsilon\right)\right)+$$

$$2f\beta\delta\left(13+30h\delta+4h^3\delta^3+162\delta\epsilon-3h^2\delta^2\left(-7+4\delta\epsilon\right)\right)+$$

$$f^2\delta^2\left(19+10h^3\delta^3+324\delta\epsilon+3h^2\delta^2\left(13+10\delta\epsilon\right)+12h\delta\left(4+17\delta\epsilon\right)\right)\Big)\Big)+$$

$$e\delta^2\Big(24l^2\delta\left(\beta+f\delta\right)\left(1+h\delta\right)^3\epsilon+e\Big(\beta^2\left(13+21h^2\delta^2+4h^3\delta^3+174\delta\epsilon+6h\delta\left(5+4\delta\epsilon\right)\right)+$$

$$2f\beta\delta\left(19+10h^3\delta^3+324\delta\epsilon+3h^2\delta^2\left(13+10\delta\epsilon\right)+12h\delta\left(4+17\delta\epsilon\right)\right)+$$

$$f^2\delta^2\left(25+16h^3\delta^3+534\delta\epsilon+3h^2\delta^2\left(19+40\delta\epsilon\right)+6h\delta\left(11+84\delta\epsilon\right)\right)\Big)\Big)\Big)\hbar^5+$$

$$\frac{1}{3\left(1+h\delta\right)^9}\epsilon^2\Big(\beta^3\Big(-3e\alpha^2\left(-1+5h^3\delta^3-12\delta\epsilon+h^2\delta^2\left(9-62\delta\epsilon\right)+h\delta\left(3+76\delta\epsilon\right)\right)+$$

$$h\alpha^3\left(-1+5h^3\delta^3-12\delta\epsilon+h^2\delta^2\left(9-62\delta\epsilon\right)+h\delta\left(3+76\delta\epsilon\right)\right)-$$

$$3e^2\alpha\delta\left(-6+h^4\delta^4-84\delta\epsilon+h^3\delta^3\left(3-2\delta\epsilon\right)+3h^2\delta^2\left(-1+4\delta\epsilon\right)+h\delta\left(-11+80\delta\epsilon\right)\right)+$$

$$e^3\delta^2\left(2h^4\delta^4+h^3\delta^3\left(15+4\delta\epsilon\right)+h^2\delta^2\left(39+34\delta\epsilon\right)+3\left(5+92\delta\epsilon\right)+h\delta\left(41+156\delta\epsilon\right)\right)\Big)-$$

$$3f\beta^2\Big(-3e^2\alpha\delta^2\left(13+7h^3\delta^3+232\delta\epsilon-9h^2\delta^2\left(-3+2\delta\epsilon\right)+h\delta\left(33+64\delta\epsilon\right)\right)+$$

$$\alpha^3\left(-1+5h^3\delta^3-12\delta\epsilon+h^2\delta^2\left(9-62\delta\epsilon\right)+h\delta\left(3+76\delta\epsilon\right)\right)+$$

$$3e\alpha^2\delta\left(-5+2h^4\delta^4-72\delta\epsilon+h^3\delta^3\left(7-10\delta\epsilon\right)+h^2\delta^2\left(3+8\delta\epsilon\right)+h\delta\left(-7+96\delta\epsilon\right)\right)-$$

$$e^3\delta^3\left(25+6h^4\delta^4+552\delta\epsilon+h^3\delta^3\left(37+30\delta\epsilon\right)+h^2\delta^2\left(81+212\delta\epsilon\right)+h\delta\left(75+584\delta\epsilon\right)\right)\Big)+$$

$$3f^2\beta\delta\Big(\alpha^3\left(6-h^4\delta^4+84\delta\epsilon+h\delta\left(11-80\delta\epsilon\right)+3h^2\delta^2\left(1-4\delta\epsilon\right)+h^3\delta^3\left(-3+2\delta\epsilon\right)\right)+$$

$$3e\alpha^2\delta\left(13+7h^3\delta^3+232\delta\epsilon-9h^2\delta^2\left(-3+2\delta\epsilon\right)+h\delta\left(33+64\delta\epsilon\right)\right)+$$

$$3e^2\alpha\delta^2\left(5h^4\delta^4+3h^3\delta^3\left(11+6\delta\epsilon\right)+3h^2\delta^2\left(25+52\delta\epsilon\right)+8\left(3+65\delta\epsilon\right)+h\delta\left(71+508\delta\epsilon\right)\right)+$$

$$e^3\delta^3\left(39+14h^4\delta^4+1008\delta\epsilon+h^3\delta^3\left(75+116\delta\epsilon\right)+3h^2\delta^2\left(49+230\delta\epsilon\right)+h\delta\left(125+1432\delta\epsilon\right)\right)\Big)+$$

$$f^3\delta^2\Big(\alpha^3\left(2h^4\delta^4+h^3\delta^3\left(15+4\delta\epsilon\right)+h^2\delta^2\left(39+34\delta\epsilon\right)+3\left(5+92\delta\epsilon\right)+h\delta\left(41+156\delta\epsilon\right)\right)+$$

$$3e\alpha^2\delta\left(25+6h^4\delta^4+552\delta\epsilon+h^3\delta^3\left(37+30\delta\epsilon\right)+h^2\delta^2\left(81+212\delta\epsilon\right)+h\delta\left(75+584\delta\epsilon\right)\right)+$$

$$3\,e^2\,\alpha\,\delta^2\,\left(39+14\,h^4\,\delta^4+1008\,\delta\,\epsilon+h^3\,\delta^3\,\left(75+116\,\delta\,\epsilon\right)+3\,h^2\,\delta^2\,\left(49+230\,\delta\,\epsilon\right)+h\,\delta\,\left(125+1432\,\delta\,\epsilon\right)\right)+$$
$$e^3\,\delta^3\,\left(57+26\,h^4\,\delta^4+1704\,\delta\,\epsilon+h^3\,\delta^3\,\left(129+322\,\delta\,\epsilon\right)+h^2\,\delta^2\,\left(237+1648\,\delta\,\epsilon\right)+h\,\delta\,\left(191+2880\,\delta\,\epsilon\right)\right)\Big)+6\,l^2$$
$$(\alpha+e\,\delta)\,\left(\beta+f\,\delta\right)\,\left(1+h\,\delta\right)^3\,\left(\alpha\,\left(\beta\,\left(1+h^2\,\delta^2+9\,\delta\,\epsilon+h\,\delta\,\left(2-27\,\delta\,\epsilon\right)\right)+f\,\delta\,\left(1+h^2\,\delta^2+45\,\delta\,\epsilon+h\,\delta\,\left(2+9\,\delta\,\epsilon\right)\right)\right)+$$
$$e\,\delta\,\left(\beta\,\left(1+h^2\,\delta^2+45\,\delta\,\epsilon+h\,\delta\,\left(2+9\,\delta\,\epsilon\right)\right)+f\,\delta\,\left(1+h^2\,\delta^2+81\,\delta\,\epsilon+h\,\delta\,\left(2+45\,\delta\,\epsilon\right)\right)\right)\right)\Big)\,\hbar^6+$$

$$\frac{1}{3\,\left(1+h\,\delta\right)^8}\,l\,(\alpha+e\,\delta)\,\left(\beta+f\,\delta\right)\,\epsilon^2\,\Big(f^2\,\delta\,\left(2\,e\,\alpha\,\delta\,\left(9+6\,h^3\,\delta^3+560\,\delta\,\epsilon+3\,h^2\,\delta^2\,\left(7+38\,\delta\,\epsilon\right)+4\,h\,\delta\,\left(6+131\,\delta\,\epsilon\right)\right)+$$
$$\alpha^2\,\left(6+3\,h^3\,\delta^3+290\,\delta\,\epsilon+12\,h^2\,\delta^2\,\left(1+2\,\delta\,\epsilon\right)+h\,\delta\,\left(15+164\,\delta\,\epsilon\right)\right)+$$
$$e^2\,\delta^2\,\left(9\,h^3\,\delta^3+6\,h^2\,\delta^2\,(5+64\,\delta\,\epsilon)+2\,\left(6+505\,\delta\,\epsilon\right)+h\,\delta\,\left(33+1244\,\delta\,\epsilon\right)\right)\Big)+$$
$$\beta\,\Big(e^2\,\beta\,\delta\,\left(6+3\,h^3\,\delta^3+290\,\delta\,\epsilon+12\,h^2\,\delta^2\,\left(1+2\,\delta\,\epsilon\right)+h\,\delta\,\left(15+164\,\delta\,\epsilon\right)\right)+$$
$$2\,e\,\Big(18\,l^2\,\delta^2\,\left(1+h\,\delta\right)^3\,\epsilon+\alpha\,\beta\,\left(3+56\,\delta\,\epsilon+2\,h\,\delta\,\left(3-62\,\delta\,\epsilon\right)+3\,h^2\,\delta^2\,\left(1-10\,\delta\,\epsilon\right)\right)\Big)+\alpha\,\Big(2\,\left(\alpha\,\beta+18\,l^2\,\delta\right)\,\epsilon+$$
$$h^3\,\left(-3\,\alpha\,\beta\,\delta^2+36\,l^2\,\delta^4\,\epsilon\right)+6\,h^2\,\delta\,\left(18\,l^2\,\delta^2\,\epsilon+\alpha\,\beta\,\left(-1+16\,\delta\,\epsilon\right)\right)+h\,\left(108\,l^2\,\delta^2\,\epsilon-\alpha\,\beta\,\left(3+52\,\delta\,\epsilon\right)\right)\Big)\Big)+$$
$$2\,f\,\Big(\alpha^2\,\beta\,\left(3+56\,\delta\,\epsilon+2\,h\,\delta\,\left(3-62\,\delta\,\epsilon\right)+3\,h^2\,\delta^2\,\left(1-10\,\delta\,\epsilon\right)\right)+$$
$$2\,\alpha\,\delta\,\Big(9\,l^2\,\delta\,\left(1+h\,\delta\right)^3\,\epsilon+e\,\beta\,\left(6+3\,h^3\,\delta^3+218\,\delta\,\epsilon+12\,h^2\,\delta^2\,\left(1-4\,\delta\,\epsilon\right)+5\,h\,\delta\,\left(3+4\,\delta\,\epsilon\right)\right)\Big)+e\,\delta^2\,$$
$$\Big(18\,l^2\,\delta\,\left(1+h\,\delta\right)^3\,\epsilon+e\,\beta\,\left(9+6\,h^3\,\delta^3+560\,\delta\,\epsilon+3\,h^2\,\delta^2\,\left(7+38\,\delta\,\epsilon\right)+4\,h\,\delta\,\left(6+131\,\delta\,\epsilon\right)\right)\Big)\Big)\Big)\,\hbar^7+\frac{1}{24\,\left(1+h\,\delta\right)^{10}}$$
$$(\alpha+e\,\delta)\,\left(\beta+f\,\delta\right)\,\epsilon^2\,\Big(f^3\,\delta\,\Big(\alpha^3\,\left(12+3\,h^4\,\delta^4+744\,\delta\,\epsilon+3\,h^3\,\delta^3\,\left(6+11\,\delta\,\epsilon\right)+13\,h^2\,\delta^2\,\left(3+20\,\delta\,\epsilon\right)+2\,h\,\delta\,\left(18+373\,\delta\,\epsilon\right)\right)+$$
$$e\,\alpha^2\,\delta\,\left(60+21\,h^4\,\delta^4+4464\,\delta\,\epsilon+h^3\,\delta^3\,\left(114+431\,\delta\,\epsilon\right)+h^2\,\delta^2\,\left(225+2776\,\delta\,\epsilon\right)+2\,h\,\delta\,\left(96+3067\,\delta\,\epsilon\right)\right)+$$
$$e^3\,\delta^3\,\left(48+27\,h^4\,\delta^4+5136\,\delta\,\epsilon+h^3\,\delta^3\,\left(126+1445\,\delta\,\epsilon\right)+3\,h^2\,\delta^2\,\left(73+2192\,\delta\,\epsilon\right)+2\,h\,\delta\,\left(84+5021\,\delta\,\epsilon\right)\right)+$$
$$e^2\,\alpha\,\delta^2\,\left(96+45\,h^4\,\delta^4+8496\,\delta\,\epsilon+h^3\,\delta^3\,\left(222+1483\,\delta\,\epsilon\right)+2\,h\,\delta\,\left(162+7175\,\delta\,\epsilon\right)+h^2\,\delta^2\,\left(405+8012\,\delta\,\epsilon\right)\right)\Big)+$$
$$\beta^2\,\Big(e^3\,\beta\,\delta\,\left(12+3\,h^4\,\delta^4+744\,\delta\,\epsilon+3\,h^3\,\delta^3\,\left(6+11\,\delta\,\epsilon\right)+13\,h^2\,\delta^2\,\left(3+20\,\delta\,\epsilon\right)+2\,h\,\delta\,\left(18+373\,\delta\,\epsilon\right)\right)+$$
$$e^2\,\Big(48\,l^2\,\delta^2\,\left(1+h\,\delta\right)^3\,\left(13+5\,h\,\delta\right)\,\epsilon+\alpha\,\beta\,\left(12-3\,h^4\,\delta^4+408\,\delta\,\epsilon+h^2\,\delta^2\,\left(9-472\,\delta\,\epsilon\right)+2\,h\,\delta\,\left(12-337\,\delta\,\epsilon\right)-$$
$$h^3\,\delta^3\,(6+65\,\delta\,\epsilon)\right)\Big)+\alpha^2\,\Big(48\,l^2\,\epsilon-6\,h\,\left(\alpha\,\beta+32\,l^2\,\delta\right)\,\epsilon+3\,h^4\,\left(\alpha\,\beta\,\delta^2-112\,l^2\,\delta^4\,\epsilon\right)+$$
$$h^3\,\delta\,\left(-960\,l^2\,\delta^2\,\epsilon+\alpha\,\beta\,\left(6-131\,\delta\,\epsilon\right)\right)+h^2\,\left(-864\,l^2\,\delta^2\,\epsilon+\alpha\,\beta\,\left(3+88\,\delta\,\epsilon\right)\right)\Big)+$$
$$e\,\alpha\,\Big(24\,\left(\alpha\,\beta+28\,l^2\,\delta\right)\,\epsilon-3\,h^4\,\left(\alpha\,\beta\,\delta^3+32\,l^2\,\delta^5\,\epsilon\right)+h^3\,\delta^2\,\left(384\,l^2\,\delta^2\,\epsilon+\alpha\,\beta\,\left(-18+131\,\delta\,\epsilon\right)\right)-$$
$$2\,h\,\left(-960\,l^2\,\delta^2\,\epsilon+\alpha\,\beta\,\left(6+173\,\delta\,\epsilon\right)\right)+h^2\,\delta\,\left(1728\,l^2\,\delta^2\,\epsilon+\alpha\,\beta\,\left(-27+436\,\delta\,\epsilon\right)\right)\Big)\Big)+$$
$$f\,\beta\,\Big(e^3\,\beta\,\delta^2\,\left(60+21\,h^4\,\delta^4+4464\,\delta\,\epsilon+h^3\,\delta^3\,\left(114+431\,\delta\,\epsilon\right)+h^2\,\delta^2\,\left(225+2776\,\delta\,\epsilon\right)+2\,h\,\delta\,\left(96+3067\,\delta\,\epsilon\right)\right)-$$
$$3\,e\,\alpha\,\Big(-64\,l^2\,\delta^2\,\left(1+h\,\delta\right)^3\,\left(13+5\,h\,\delta\right)\,\epsilon+\alpha\,\beta\,\left(7\,h^4\,\delta^4+h^3\,\delta^3\,\left(22-111\,\delta\,\epsilon\right)-8\,\left(1+38\,\delta\,\epsilon\right)+$$
$$h^2\,\delta^2\,\left(15+224\,\delta\,\epsilon\right)+2\,h\,\delta\,\left(-4+353\,\delta\,\epsilon\right)\right)\Big)+3\,e^2\,\delta\,\Big(32\,l^2\,\delta^2\,\left(1+h\,\delta\right)^3\,\left(19+11\,h\,\delta\right)\,\epsilon+$$
$$\alpha\,\beta\,\left(h^4\,\delta^4+h^2\,\delta^2\,\left(69-524\,\delta\,\epsilon\right)+h^3\,\delta^3\,\left(22-149\,\delta\,\epsilon\right)+2\,h\,\delta\,\left(38+187\,\delta\,\epsilon\right)+4\,\left(7+356\,\delta\,\epsilon\right)\right)\Big)+$$
$$\alpha^2\,\Big(24\,\left(\alpha\,\beta+28\,l^2\,\delta\right)\,\epsilon-3\,h^4\,\left(\alpha\,\beta\,\delta^3+32\,l^2\,\delta^5\,\epsilon\right)+h^3\,\delta^2\,\left(384\,l^2\,\delta^2\,\epsilon+\alpha\,\beta\,\left(-18+131\,\delta\,\epsilon\right)\right)-$$
$$2\,h\,\left(-960\,l^2\,\delta^2\,\epsilon+\alpha\,\beta\,\left(6+173\,\delta\,\epsilon\right)\right)+h^2\,\delta\,\left(1728\,l^2\,\delta^2\,\epsilon+\alpha\,\beta\,\left(-27+436\,\delta\,\epsilon\right)\right)\Big)\Big)+$$
$$f^2\,\Big(\alpha^3\,\beta\,\left(12-3\,h^4\,\delta^4+408\,\delta\,\epsilon+h^2\,\delta^2\,\left(9-472\,\delta\,\epsilon\right)+2\,h\,\delta\,\left(12-337\,\delta\,\epsilon\right)-h^3\,\delta^3\,(6+65\,\delta\,\epsilon)\right)+$$
$$3\,\alpha^2\,\delta\,\Big(16\,l^2\,\delta\,\left(1+h\,\delta\right)^3\,\left(13+5\,h\,\delta\right)\,\epsilon+$$
$$e\,\beta\,\left(h^4\,\delta^4+h^2\,\delta^2\,\left(69-524\,\delta\,\epsilon\right)+h^3\,\delta^3\,\left(22-149\,\delta\,\epsilon\right)+2\,h\,\delta\,\left(38+187\,\delta\,\epsilon\right)+4\,\left(7+356\,\delta\,\epsilon\right)\right)\Big)+$$
$$3\,e\,\alpha\,\delta^2\,\Big(32\,l^2\,\delta\,\left(1+h\,\delta\right)^3\,\left(19+11\,h\,\delta\right)\,\epsilon+e\,\beta\,\left(17\,h^4\,\delta^4+7\,h^3\,\delta^3\,\left(14+\delta\,\epsilon\right)+3\,h^2\,\delta^2\,\left(67+408\,\delta\,\epsilon\right)+$$
$$8\,\left(7+470\,\delta\,\epsilon\right)+2\,h\,\delta\,\left(88+2151\,\delta\,\epsilon\right)\right)\Big)+e^2\,\delta^3\,\Big(48\,l^2\,\delta\,\left(1+h\,\delta\right)^3\,\left(25+17\,h\,\delta\right)\,\epsilon+$$
$$e\,\beta\,\left(96+45\,h^4\,\delta^4+8496\,\delta\,\epsilon+h^3\,\delta^3\,\left(222+1483\,\delta\,\epsilon\right)+2\,h\,\delta\,\left(162+7175\,\delta\,\epsilon\right)+h^2\,\delta^2\,\left(405+8012\,\delta\,\epsilon\right)\right)\Big)\Big)\Big)\,\hbar^8+$$

$$\frac{1}{12\,\left(1+h\,\delta\right)^9}\,l\,(\alpha+e\,\delta)^2\,\left(\beta+f\,\delta\right)^2\,\Big(16\,l^2\,(\alpha+e\,\delta)\,\left(\beta+f\,\delta\right)\,\left(1+h\,\delta\right)^3+$$
$$\beta^2\,\left(h\,\alpha^2\,\left(-14+61\,h\,\delta\right)+e^2\,\delta\,\left(216+178\,h\,\delta+37\,h^2\,\delta^2\right)-2\,e\,\alpha\,\left(-18+98\,h\,\delta+41\,h^2\,\delta^2\right)\right)+$$
$$2\,f\,\beta\,\left(2\,e\,\alpha\,\delta\,\left(132+10\,h\,\delta-47\,h^2\,\delta^2\right)+\alpha^2\,\left(18-98\,h\,\delta-41\,h^2\,\delta^2\right)+e^2\,\delta^2\,\left(426+478\,h\,\delta+127\,h^2\,\delta^2\right)\right)+$$
$$f^2\,\delta\,\left(\alpha^2\,\left(216+178\,h\,\delta+37\,h^2\,\delta^2\right)+2\,e\,\alpha\,\delta\,\left(426+478\,h\,\delta+127\,h^2\,\delta^2\right)+e^2\,\delta^2\,\left(816+1138\,h\,\delta+397\,h^2\,\delta^2\right)\right)\Big)$$
$$\epsilon^3\,\hbar^9+\frac{1}{12\,\left(1+h\,\delta\right)^{11}}\,(\alpha+e\,\delta)^2$$
$$\left(\beta+f\,\delta\right)^2$$
$$\left(\beta\,\left(-h\,\alpha+e\,\left(2+h\,\delta\right)\right)+\right.$$
$$\left.f\,\left(\alpha\,\left(2+h\,\delta\right)+e\,\delta\,\left(4+3\,h\,\delta\right)\right)\right)$$

$$\left(12\,\mathsf{l}^2\,(\alpha+\mathsf{e}\,\delta)\,(\beta+\mathsf{f}\,\delta)\,(1+\mathsf{h}\,\delta)^3+\beta^2\,\left(\mathsf{h}\,\alpha^2\,\left(-2+7\,\mathsf{h}\,\delta\right)+\mathsf{e}\,\alpha\,\left(6-19\,\mathsf{h}\,\delta-7\,\mathsf{h}^2\,\delta^2\right)+\mathsf{e}^2\,\delta\,\left(24+19\,\mathsf{h}\,\delta+4\,\mathsf{h}^2\,\delta^2\right)\right)+\right.$$
$$\mathsf{f}\,\beta\,\left(\alpha^2\,\left(6-19\,\mathsf{h}\,\delta-7\,\mathsf{h}^2\,\delta^2\right)+2\,\mathsf{e}\,\alpha\,\delta\,\left(33+8\,\mathsf{h}\,\delta-7\,\mathsf{h}^2\,\delta^2\right)+\mathsf{e}^2\,\delta^2\,\left(96+107\,\mathsf{h}\,\delta+29\,\mathsf{h}^2\,\delta^2\right)\right)+$$
$$\left.\mathsf{f}^2\,\delta\,\left(\alpha^2\,\left(24+19\,\mathsf{h}\,\delta+4\,\mathsf{h}^2\,\delta^2\right)+\mathsf{e}\,\alpha\,\delta\,\left(96+107\,\mathsf{h}\,\delta+29\,\mathsf{h}^2\,\delta^2\right)+\mathsf{e}^2\,\delta^2\,\left(90+124\,\mathsf{h}\,\delta+43\,\mathsf{h}^2\,\delta^2\right)\right)\right)\epsilon^3\,\hbar^{10}$$