

(\* In Classes/17-1350-AKT/170321-g1Invariant@170320.nb \*)

$$\Lambda 1 = \frac{1}{2 \text{Log}[t_1]} \omega^4 (-1 + t_1) \left( \alpha^2 \beta^2 + 4 \alpha \beta \delta + 2 \delta^2 - 4 \alpha \beta \delta^2 - 4 \delta^3 + 2 \delta^4 + 4 \alpha \beta l_1 + 4 \delta l_1 - 8 \alpha \beta \delta l_1 - \right. \\ \left. 12 \delta^2 l_1 + 4 \alpha \beta \delta^2 l_1 + 12 \delta^3 l_1 - 4 \delta^4 l_1 - \alpha^2 \beta^2 t_1 - 4 \alpha \beta \delta t_1 - 2 \delta^2 t_1 + 8 \alpha \beta \delta^2 t_1 + \right. \\ \left. 8 \delta^3 t_1 - 6 \delta^4 t_1 + 8 \alpha \beta \delta l_1 t_1 + 12 \delta^2 l_1 t_1 - 8 \alpha \beta \delta^2 l_1 t_1 - 24 \delta^3 l_1 t_1 + 12 \delta^4 l_1 t_1 - \right. \\ \left. 4 \alpha \beta \delta^2 t_1^2 - 4 \delta^3 t_1^2 + 6 \delta^4 t_1^2 + 4 \alpha \beta \delta^2 l_1 t_1^2 + 12 \delta^3 l_1 t_1^2 - 12 \delta^4 l_1 t_1^2 - 2 \delta^4 t_1^3 + 4 \delta^4 l_1 t_1^3 + \right. \\ \left. \alpha^2 \delta f_0^2 (2 - \delta + \delta t_1) + 2 \alpha f_0 (\alpha \beta + 2 \delta - 2 \delta^2 + 2 \delta^2 t_1 + 2 \delta l_1 (1 - \delta + \delta t_1)^2) + \right. \\ \left. \delta e_0^2 (\beta + \delta f_0) (\beta (2 - \delta + \delta t_1) + \delta f_0 (4 - 3 \delta + 3 \delta t_1)) + \right. \\ \left. 2 e_0 (\alpha \delta^2 f_0^2 (3 - 2 \delta + 2 \delta t_1) + \beta (\alpha \beta + 2 \delta - 2 \delta^2 + 2 \delta^2 t_1 + 2 \delta l_1 (1 - \delta + \delta t_1)^2) + \right. \\ \left. 2 \delta f_0 (\delta l_1 (1 - \delta + \delta t_1)^2 + (2 - \delta + \delta t_1) (\alpha \beta + \delta - \delta^2 + \delta^2 t_1)) \right);$$

(\* In GWU \*)

$$\Lambda 2 = \left( (t - 1) (2 (\alpha \beta + \delta \mu)^2 - \alpha^2 \beta^2) - 4 e_k l_k f_k \delta^2 \mu^2 - \right. \\ \left. \delta (1 + \mu) (f_k^2 \alpha^2 + e_k^2 \beta^2) - e_k^2 f_k^2 \delta^3 (1 + 3 \mu) - 2 (\alpha \beta + 2 \delta \mu + e_k f_k \delta^2 (1 + 2 \mu) + 2 l_k \delta \mu^2) \right. \\ \left. (f_k \alpha + e_k \beta) - 4 (l_k \mu^2 + e_k f_k \delta (1 + \mu)) (\alpha \beta + \delta \mu) \right) (1 + t) / 4;$$

$$\text{Simplify}[\Lambda 1 == \frac{2 \omega^4 (1 - t) \Lambda 2}{(1 + t) \text{Log}[t]} /. \{x_{-0|1|k} \Rightarrow x, \mu \rightarrow 1 + (t - 1) \delta\}]$$

True

$$\text{lhs} = \text{Factor} \left[ \frac{2 \text{Log}[t]}{(1 - t) \omega^4} \Lambda 1 /. \{x_{-0|1} \Rightarrow x\} \right]$$

$$-4 l \alpha \beta - 2 f \alpha^2 \beta - 2 e \alpha \beta^2 - \alpha^2 \beta^2 + t \alpha^2 \beta^2 - 4 l \delta - 4 f \alpha \delta - 4 f l \alpha \delta - 2 f^2 \alpha^2 \delta - 4 e \beta \delta - \\ 4 e l \beta \delta - 4 \alpha \beta \delta - 8 e f \alpha \beta \delta + 8 l \alpha \beta \delta + 4 t \alpha \beta \delta - 8 l t \alpha \beta \delta - 2 e^2 \beta^2 \delta - 2 \delta^2 - 8 e f \delta^2 + \\ 12 l \delta^2 - 4 e f l \delta^2 + 2 t \delta^2 - 12 l t \delta^2 + 4 f \alpha \delta^2 - 6 e f^2 \alpha \delta^2 + 8 f l \alpha \delta^2 - 4 f t \alpha \delta^2 - \\ 8 f l t \alpha \delta^2 + f^2 \alpha^2 \delta^2 - f^2 t \alpha^2 \delta^2 + 4 e \beta \delta^2 - 6 e^2 f \beta \delta^2 + 8 e l \beta \delta^2 - 4 e t \beta \delta^2 - 8 e l t \beta \delta^2 + \\ 4 \alpha \beta \delta^2 + 4 e f \alpha \beta \delta^2 - 4 l \alpha \beta \delta^2 - 8 t \alpha \beta \delta^2 - 4 e f t \alpha \beta \delta^2 + 8 l t \alpha \beta \delta^2 + 4 t^2 \alpha \beta \delta^2 - \\ 4 l t^2 \alpha \beta \delta^2 + e^2 \beta^2 \delta^2 - e^2 t \beta^2 \delta^2 + 4 \delta^3 + 12 e f \delta^3 - 4 e^2 f^2 \delta^3 - 12 l \delta^3 + 8 e f l \delta^3 - \\ 8 t \delta^3 - 12 e f t \delta^3 + 24 l t \delta^3 - 8 e f l t \delta^3 + 4 t^2 \delta^3 - 12 l t^2 \delta^3 + 4 e f^2 \alpha \delta^3 - 4 f l \alpha \delta^3 - \\ 4 e f^2 t \alpha \delta^3 + 8 f l t \alpha \delta^3 - 4 f l t^2 \alpha \delta^3 + 4 e^2 f \beta \delta^3 - 4 e l \beta \delta^3 - 4 e^2 f t \beta \delta^3 + 8 e l t \beta \delta^3 - \\ 4 e l t^2 \beta \delta^3 - 2 \delta^4 - 4 e f \delta^4 + 3 e^2 f^2 \delta^4 + 4 l \delta^4 - 4 e f l \delta^4 + 6 t \delta^4 + 8 e f t \delta^4 - 3 e^2 f^2 t \delta^4 - \\ 12 l t \delta^4 + 8 e f l t \delta^4 - 6 t^2 \delta^4 - 4 e f t^2 \delta^4 + 12 l t^2 \delta^4 - 4 e f l t^2 \delta^4 + 2 t^3 \delta^4 - 4 l t^3 \delta^4$$

$$\text{rhs} = \text{Factor} \left[ \frac{4 \Lambda 2}{1 + t} /. \{x_{-k} \Rightarrow x, \mu \rightarrow 1 + (t - 1) \delta\} \right]$$

$$-4 l \alpha \beta - 2 f \alpha^2 \beta - 2 e \alpha \beta^2 - \alpha^2 \beta^2 + t \alpha^2 \beta^2 - 4 l \delta - 4 f \alpha \delta - 4 f l \alpha \delta - 2 f^2 \alpha^2 \delta - 4 e \beta \delta - \\ 4 e l \beta \delta - 4 \alpha \beta \delta - 8 e f \alpha \beta \delta + 8 l \alpha \beta \delta + 4 t \alpha \beta \delta - 8 l t \alpha \beta \delta - 2 e^2 \beta^2 \delta - 2 \delta^2 - 8 e f \delta^2 + \\ 12 l \delta^2 - 4 e f l \delta^2 + 2 t \delta^2 - 12 l t \delta^2 + 4 f \alpha \delta^2 - 6 e f^2 \alpha \delta^2 + 8 f l \alpha \delta^2 - 4 f t \alpha \delta^2 - \\ 8 f l t \alpha \delta^2 + f^2 \alpha^2 \delta^2 - f^2 t \alpha^2 \delta^2 + 4 e \beta \delta^2 - 6 e^2 f \beta \delta^2 + 8 e l \beta \delta^2 - 4 e t \beta \delta^2 - 8 e l t \beta \delta^2 + \\ 4 \alpha \beta \delta^2 + 4 e f \alpha \beta \delta^2 - 4 l \alpha \beta \delta^2 - 8 t \alpha \beta \delta^2 - 4 e f t \alpha \beta \delta^2 + 8 l t \alpha \beta \delta^2 + 4 t^2 \alpha \beta \delta^2 - \\ 4 l t^2 \alpha \beta \delta^2 + e^2 \beta^2 \delta^2 - e^2 t \beta^2 \delta^2 + 4 \delta^3 + 12 e f \delta^3 - 4 e^2 f^2 \delta^3 - 12 l \delta^3 + 8 e f l \delta^3 - \\ 8 t \delta^3 - 12 e f t \delta^3 + 24 l t \delta^3 - 8 e f l t \delta^3 + 4 t^2 \delta^3 - 12 l t^2 \delta^3 + 4 e f^2 \alpha \delta^3 - 4 f l \alpha \delta^3 - \\ 4 e f^2 t \alpha \delta^3 + 8 f l t \alpha \delta^3 - 4 f l t^2 \alpha \delta^3 + 4 e^2 f \beta \delta^3 - 4 e l \beta \delta^3 - 4 e^2 f t \beta \delta^3 + 8 e l t \beta \delta^3 - \\ 4 e l t^2 \beta \delta^3 - 2 \delta^4 - 4 e f \delta^4 + 3 e^2 f^2 \delta^4 + 4 l \delta^4 - 4 e f l \delta^4 + 6 t \delta^4 + 8 e f t \delta^4 - 3 e^2 f^2 t \delta^4 - \\ 12 l t \delta^4 + 8 e f l t \delta^4 - 6 t^2 \delta^4 - 4 e f t^2 \delta^4 + 12 l t^2 \delta^4 - 4 e f l t^2 \delta^4 + 2 t^3 \delta^4 - 4 l t^3 \delta^4$$

$$\text{Simplify}[\text{lhs} / \text{rhs}]$$

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$\Delta 2 / . \delta \rightarrow 0$

$$\frac{1}{4} (1+t) \left( (-1+t) \alpha^2 \beta^2 - 2 \alpha \beta (\beta \mathbf{e}_k + \alpha \mathbf{f}_k) - 4 \alpha \beta \mu^2 \mathbf{1}_k \right)$$