Johnson-Freyd: Advanced integration by parts: the BV formalism

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Video available @ Fields!

Def "QFT is that part of mathematics about evaluating oscillating integrals".

$$\langle f\rangle = \underbrace{\int}_{\mathcal{X}} f(x)\exp\left(\frac{i}{\hbar}S(x)\right)dx$$

space of fields. observable planck action

$\mathcal{X}$ is typically
* ∞-dim
* super
* stacky

no measure on $\mathcal{X}$

Compare w/ when integrals do make sense: $\mathcal{X}=\mathbb{A}^n_{x_1,\dots x_n}$

<u>Main idea 1:</u>

$$\int_{\mathbb{A}_n}\frac{\partial}{\partial x_i}\left(g(x)\exp(S/\hbar)\right) = \text{boundary term.}$$

$$||$$

so $\left\langle \frac{\partial g}{\partial x_i} + \hbar^{-1}g(x)\frac{\partial S}{\partial x_i}\right\rangle = 0$ if bdry terms vanish.

These are "Ward identities"

Package into homological algebra: 2-term complex:

$$\mathcal{T} = \mathcal{O}^{\oplus n} \longrightarrow \mathcal{O}$$

$$(g_1 \dots g_n)=\vec{g} \longmapsto \vec{g}S+\hbar\,\mathrm{div}\,\vec{g}$$

$\int$ factors through $H_0(\mathcal{T} \to \mathcal{O})$, which hopefully is small.

Implementation: If $A^n = \mathbb{R}^n$ and S is real and grows not too slowly and f grows not too quickly and $\hbar$ is pure imaginary. Then bndry term is $O(\hbar^\infty)$.

Also, if S has no crit. pts. in supp(f), same.
This suggests working in P.S. in $\hbar$, $\mathbb{R}[[\hbar]]$.

... Assume S has one crit. pt. at 0, and f compact support there. Also $S(0)=0$.

$$S = \sum \tfrac{1}{2} a_{ij} x_i x_j + b(x) \qquad \eta = a^{-1}$$
$$b(x) = O(x^3)$$

$$f = f(0) + f_i^{(1)} x_i + \tfrac{1}{2} f_{ij}^{(2)} x_i x_j + \dots$$

Exercise $\exists\, g_i(x)$ s.t. $f = f(0) + g_i(x) \frac{\partial S}{\partial x_i}$

Moral so far: I should learn how to do basic perturbation theory using Ward identities!
[Does this have implications to $\mathbb{O}(\dots)$?]

(131126a) Crainic arXiv:math/0403266 on Homological perturbations: A Homological Homotopy Equivalence (HHE) is a pair of complexes with quasi-isomorphisms $(L,b) \underset{p}{\overset{i}{\rightleftarrows}} (M,b)$, with a homotopy h between $1 = 1_M$ and ip, so $ip = 1 + bh + hb$. A perturbation is $\delta\colon M \to M$ with $\deg b = \deg \delta$ and $(b+\delta)^2 = 0$; it is small if $(1-\delta h)^{-1}$ exists. **Claim.** $(L,b_1) \underset{p_1}{\overset{i_1}{\rightleftarrows}} (M, b+\delta)$ is again an HHE, with $A := (1-\delta h)^{-1}\delta$, $b_1 := b + pAi$, $i_1 := i + hAi$, $p_1 := p + pAh$, and with $h_1 := h + hAh$.

Could this be related to $\mathbb{O}(\dots)$?

again an HHE, with $A := (1-\delta h)^{-1}\delta$, $b_1 := b + pAi$, $i_1 := i + hAi$, $p_1 := p + pAh$, and with $h_1 := h + hAh$.

Potential key: Using Ward identities, all integration can be reduced to a single one, which comes out transcendental. Is there a similar fact for $\oslash$?

Potential key: What is "translation invariance" for $\oslash$-integration? Could it be that this only applies to the "constant term" of a "complete" $\oslash$-integral?