Henriques @ Colloq: Representations of based loop groups

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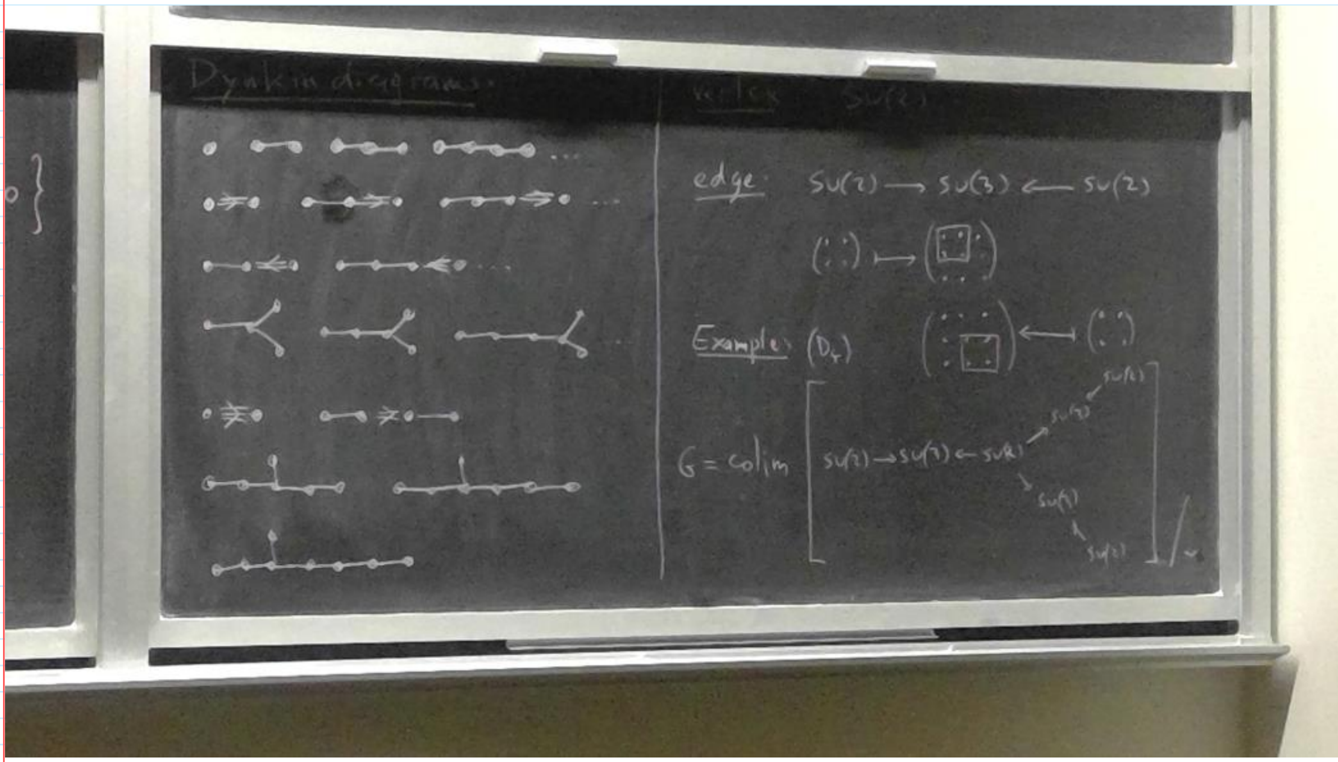


Based loop group

$$\Omega G = \{\gamma : [0,1] \to G : \gamma \in C^\infty \quad \begin{array}{l}\gamma(0)=\gamma(1)=e \\ \gamma^{(k)}(0)=\gamma^{(k)}(1)=0\end{array}\}$$

as opposed to free loop groups

$$LG = \{\gamma : S^1 \to G : \gamma \in C^\infty\}$$

$LSU(n)$: Dynkin diagram is n-gon.

$G \to LG$ by constant loops.

this gives $(n-1)$ $SU(2)$'s. The last one is

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \longmapsto \begin{pmatrix} d & & & zc \\ & 1 & & \\ & & 1 & \\ z^{-1}b & & & a \end{pmatrix} \quad \text{where } z \in S^1$$

This really gives $\widetilde{LG}$, a central extension of LG

$\mathrm{Rep}^k(LG)$: {level k positive energy reps of $\widetilde{LG}$}

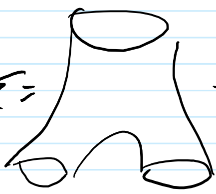
<u>Definition</u> (Segal?) A unitary rep of $\widetilde{LG}$ is positive energy if it admits an extension

$$\begin{tikzcd} \widetilde{LG} \arrow[r] \arrow[d] & U(H) \\ S^1 \ltimes \widetilde{LG} \arrow[ur, dashed] & \end{tikzcd}$$

rotating the loop.

s.t. the decomposition of H into irreps of the new S^1 consists only of positive weights.

Fusion Product, depends on a complex structure on

$\Sigma =$  $\to$ A functor $F_\Sigma : \mathrm{Rep}^k(LG) \times \mathrm{Rep}^k(LG) \xrightarrow{\boxtimes} \mathrm{Rep}^k(LG)$

by

$$H_1 \boxtimes H_2 = \mathrm{Ind}_{\mathrm{Map}_{\mathrm{hol}}(\Sigma, G_{\mathbb{C}})}^{\widetilde{LG}_{\mathbb{C}}} (H_1 \otimes H_2)$$

there's a map

$$H_1 \otimes H_2 \longrightarrow H_1 \boxtimes H_2$$