

Pensieve header: The loose \$gl\_n\$ version 2 (failed).

```

 $\chi_{cond\_} := \text{If}[\text{TrueQ}@\text{cond}, 1, 0];$ 
 $\mathbf{B}[\mathbf{0}, \_ ] = \mathbf{0}; \mathbf{B}[\_, \mathbf{0}] = \mathbf{0};$ 
 $\mathbf{B}[\_ * \mathbf{x} : (\mathbf{e} \mid \mathbf{h}) \_ \_ , \mathbf{y}\_] := \text{Expand}[\mathbf{c} \mathbf{B}[\mathbf{x}, \mathbf{y}]];$ 
 $\mathbf{B}[\mathbf{y}\_ , \_ * \mathbf{x} : (\mathbf{e} \mid \mathbf{h}) \_ \_ ] := \text{Expand}[\mathbf{c} \mathbf{B}[\mathbf{y}, \mathbf{x}]];$ 
 $\mathbf{B}[\mathbf{x}\_ \text{Plus}, \mathbf{y}\_] := \mathbf{B}[\#, \mathbf{y}] \& /@ \mathbf{x};$ 
 $\mathbf{B}[\mathbf{x}\_ , \mathbf{y}\_ \text{Plus}] := \mathbf{B}[\mathbf{x}, \#] \& /@ \mathbf{y};$ 

```

```

 $\mathbf{B}[\mathbf{e}_{i,j}, \mathbf{e}_{j,i}] := \chi_{i < j} \mathbf{h}_{i,j} - \chi_{j < i} \mathbf{h}_{j,i};$ 
 $\mathbf{B}[\mathbf{h}_{i,j}, \mathbf{e}_{i,j}] := 2 \mathbf{e}_{i,j};$ 
 $\mathbf{B}[\mathbf{h}_{j,i}, \mathbf{e}_{i,j}] := -2 \mathbf{e}_{i,j};$ 

 $\mathbf{B}[\mathbf{e}_{i,j}, \mathbf{e}_{k,l}] := \eta \chi_{j=k} \mathbf{e}_{i,l} - \eta \chi_{l=i} \mathbf{e}_{k,j};$ 
 $\mathbf{B}[\mathbf{h}_{i,j}, \mathbf{e}_{k,l}] := \text{Expand}[\eta (\chi_{i=k} \mathbf{e}_{i,l} - \chi_{i=l} \mathbf{e}_{k,i} - \chi_{j=k} \mathbf{e}_{j,l} + \chi_{j=l} \mathbf{e}_{k,j})];$ 
 $\mathbf{B}[\mathbf{h}\_ \_ , \mathbf{h}\_ \_ ] = \mathbf{0};$ 

```

```

 $\mathbf{B}[\mathbf{x}\_ , \mathbf{x}\_] = \mathbf{0};$ 
 $\mathbf{B}[\mathbf{y}\_ , \mathbf{x}\_] := \text{Expand}[-\mathbf{B}[\mathbf{x}, \mathbf{y}]];$ 

```

```

 $\text{Basis}[n\_ ] := \text{Union}@\text{Flatten}@\text{Table}[\{\mathbf{e}_{i,j}, \mathbf{e}_{j,i}, \mathbf{h}_{i,j}\}, \{\mathbf{i}, n-1\}, \{\mathbf{j}, \mathbf{i}+1, n\}];$ 

```

**Basis[4]**

{e<sub>1,2</sub>, e<sub>1,3</sub>, e<sub>1,4</sub>, e<sub>2,1</sub>, e<sub>2,3</sub>, e<sub>2,4</sub>, e<sub>3,1</sub>, e<sub>3,2</sub>, e<sub>3,4</sub>, e<sub>4,1</sub>, e<sub>4,2</sub>, e<sub>4,3</sub>, h<sub>1,2</sub>, h<sub>1,3</sub>, h<sub>1,4</sub>, h<sub>2,3</sub>, h<sub>2,4</sub>, h<sub>3,4</sub>}

n = 3;

Table[

```

  {x, y} → B[x, y],
  {x, Basis[n]}, {y, Basis[n]}
] // MatrixForm

```

$$\begin{pmatrix}
 \{e_{1,2}, e_{1,2}\} \rightarrow \mathbf{0} & \{e_{1,2}, e_{1,3}\} \rightarrow \mathbf{0} & \{e_{1,2}, e_{2,1}\} \rightarrow \mathbf{h}_{1,2} & \{e_{1,2}, e_{2,3}\} \rightarrow \eta e_{1,3} & \{e_{1,2}, e_{3,1}\} \rightarrow \eta e_{3,2} \\
 \{e_{1,3}, e_{1,2}\} \rightarrow \mathbf{0} & \{e_{1,3}, e_{1,3}\} \rightarrow \mathbf{0} & \{e_{1,3}, e_{2,1}\} \rightarrow -\eta e_{2,3} & \{e_{1,3}, e_{2,3}\} \rightarrow \mathbf{0} & \{e_{1,3}, e_{3,1}\} \rightarrow \eta e_{3,2} \\
 \{e_{2,1}, e_{1,2}\} \rightarrow -\mathbf{h}_{1,2} & \{e_{2,1}, e_{1,3}\} \rightarrow \eta e_{2,3} & \{e_{2,1}, e_{2,1}\} \rightarrow \mathbf{0} & \{e_{2,1}, e_{2,3}\} \rightarrow \mathbf{0} & \{e_{2,1}, e_{3,1}\} \rightarrow \eta e_{3,2} \\
 \{e_{2,3}, e_{1,2}\} \rightarrow -\eta e_{1,3} & \{e_{2,3}, e_{1,3}\} \rightarrow \mathbf{0} & \{e_{2,3}, e_{2,1}\} \rightarrow \mathbf{0} & \{e_{2,3}, e_{2,3}\} \rightarrow \mathbf{0} & \{e_{2,3}, e_{3,1}\} \rightarrow \eta e_{3,2} \\
 \{e_{3,1}, e_{1,2}\} \rightarrow \eta e_{3,2} & \{e_{3,1}, e_{1,3}\} \rightarrow -\mathbf{h}_{1,3} & \{e_{3,1}, e_{2,1}\} \rightarrow \mathbf{0} & \{e_{3,1}, e_{2,3}\} \rightarrow -\eta e_{2,1} & \{e_{3,1}, e_{3,1}\} \rightarrow \mathbf{0} \\
 \{e_{3,2}, e_{1,2}\} \rightarrow \mathbf{0} & \{e_{3,2}, e_{1,3}\} \rightarrow -\eta e_{1,2} & \{e_{3,2}, e_{2,1}\} \rightarrow \eta e_{3,1} & \{e_{3,2}, e_{2,3}\} \rightarrow -\mathbf{h}_{2,3} & \{e_{3,2}, e_{3,1}\} \rightarrow \eta e_{3,2} \\
 \{h_{1,2}, e_{1,2}\} \rightarrow 2 e_{1,2} & \{h_{1,2}, e_{1,3}\} \rightarrow \eta e_{1,3} & \{h_{1,2}, e_{2,1}\} \rightarrow -2 e_{2,1} & \{h_{1,2}, e_{2,3}\} \rightarrow -\eta e_{2,3} & \{h_{1,2}, e_{3,1}\} \rightarrow \eta e_{3,2} \\
 \{h_{1,3}, e_{1,2}\} \rightarrow \eta e_{1,2} & \{h_{1,3}, e_{1,3}\} \rightarrow 2 e_{1,3} & \{h_{1,3}, e_{2,1}\} \rightarrow -\eta e_{2,1} & \{h_{1,3}, e_{2,3}\} \rightarrow \eta e_{2,3} & \{h_{1,3}, e_{3,1}\} \rightarrow \eta e_{3,2} \\
 \{h_{2,3}, e_{1,2}\} \rightarrow -\eta e_{1,2} & \{h_{2,3}, e_{1,3}\} \rightarrow \eta e_{1,3} & \{h_{2,3}, e_{2,1}\} \rightarrow \eta e_{2,1} & \{h_{2,3}, e_{2,3}\} \rightarrow 2 e_{2,3} & \{h_{2,3}, e_{3,1}\} \rightarrow \eta e_{3,2}
 \end{pmatrix}$$

n = 4;

Union@Table[

```

  {x, y} = t; B[x, y] + B[y, x],
  {t, Tuples[Basis[n], 2]}
]
{0}

```

```

n = 3;
DeleteCases[Table[
  ({x, y, z} = t) → B[x, B[y, z]] + B[y, B[z, x]] + B[z, B[x, y]] /. η → 0,
  {t, Tuples[Basis[n], 3]}
], _ → 0]
{}

```

```

n = 3;
DeleteCases[Table[
  ({x, y, z} = t) → B[x, B[y, z]] + B[y, B[z, x]] + B[z, B[x, y]] /. η → 1,
  {t, Tuples[Basis[n], 3]}
], _ → 0]
{
  {e1,2, e2,3, e3,1} → h1,2 - h1,3 + h2,3, {e1,2, e3,1, e2,3} → -h1,2 + h1,3 - h2,3,
  {e1,3, e2,1, e3,2} → h1,2 - h1,3 + h2,3, {e1,3, e3,2, e2,1} → -h1,2 + h1,3 - h2,3,
  {e2,1, e1,3, e3,2} → -h1,2 + h1,3 - h2,3, {e2,1, e3,2, e1,3} → h1,2 - h1,3 + h2,3,
  {e2,3, e1,2, e3,1} → -h1,2 + h1,3 - h2,3, {e2,3, e3,1, e1,2} → h1,2 - h1,3 + h2,3,
  {e3,1, e1,2, e2,3} → h1,2 - h1,3 + h2,3, {e3,1, e2,3, e1,2} → -h1,2 + h1,3 - h2,3,
  {e3,2, e1,3, e2,1} → h1,2 - h1,3 + h2,3, {e3,2, e2,1, e1,3} → -h1,2 + h1,3 - h2,3}

```

```

n = 3;
DeleteCases[Table[
  ({x, y, z} = t) → B[x, B[y, z]] + B[y, B[z, x]] + B[z, B[x, y]],
  {t, Tuples[Basis[n], 3]}
], _ → 0]
{
  {e1,2, e1,3, e2,1} → η e1,3 - η² e1,3, {e1,2, e1,3, e3,1} → -η e1,2 + η² e1,2,
  {e1,2, e2,1, e1,3} → -η e1,3 + η² e1,3, {e1,2, e2,1, e2,3} → η e2,3 - η² e2,3,
  {e1,2, e2,1, e3,1} → η e3,1 - η² e3,1, {e1,2, e2,1, e3,2} → -η e3,2 + η² e3,2,
  {e1,2, e2,3, e2,1} → -η e2,3 + η² e2,3, {e1,2, e2,3, e3,1} → η h1,2 - η h1,3 + η h2,3,
  {e1,2, e2,3, e3,2} → η e1,2 - η² e1,2, {e1,2, e2,3, h1,2} → -2 η e1,3 + 2 η² e1,3,
  {e1,2, e2,3, h1,3} → 2 η e1,3 - 2 η² e1,3, {e1,2, e2,3, h2,3} → -2 η e1,3 + 2 η² e1,3,
  {e1,2, e3,1, e1,3} → η e1,2 - η² e1,2, {e1,2, e3,1, e2,1} → -η e3,1 + η² e3,1,
  {e1,2, e3,1, e2,3} → -η h1,2 + η h1,3 - η h2,3, {e1,2, e3,1, h1,2} → 2 η e3,2 - 2 η² e3,2,
  {e1,2, e3,1, h1,3} → -2 η e3,2 + 2 η² e3,2, {e1,2, e3,1, h2,3} → 2 η e3,2 - 2 η² e3,2,
  {e1,2, e3,2, e2,1} → η e3,2 - η² e3,2, {e1,2, e3,2, e2,3} → -η e1,2 + η² e1,2,
  {e1,2, h1,2, e2,3} → 2 η e1,3 - 2 η² e1,3, {e1,2, h1,2, e3,1} → -2 η e3,2 + 2 η² e3,2,
  {e1,2, h1,3, e2,3} → -2 η e1,3 + 2 η² e1,3, {e1,2, h1,3, e3,1} → 2 η e3,2 - 2 η² e3,2,
  {e1,2, h2,3, e2,3} → 2 η e1,3 - 2 η² e1,3, {e1,2, h2,3, e3,1} → -2 η e3,2 + 2 η² e3,2,
  {e1,3, e1,2, e2,1} → -η e1,3 + η² e1,3, {e1,3, e1,2, e3,1} → η e1,2 - η² e1,2,
  {e1,3, e2,1, e1,2} → η e1,3 - η² e1,3, {e1,3, e2,1, e3,1} → -η e2,1 + η² e2,1,
  {e1,3, e2,1, e3,2} → η h1,2 - η h1,3 + η h2,3, {e1,3, e2,1, h1,2} → -2 η e2,3 + 2 η² e2,3,
  {e1,3, e2,1, h1,3} → 2 η e2,3 - 2 η² e2,3, {e1,3, e2,1, h2,3} → -2 η e2,3 + 2 η² e2,3,
  {e1,3, e2,3, e3,1} → η e2,3 - η² e2,3, {e1,3, e2,3, e3,2} → -η e1,3 + η² e1,3,
  {e1,3, e3,1, e1,2} → -η e1,2 + η² e1,2, {e1,3, e3,1, e2,1} → η e2,1 - η² e2,1,
  {e1,3, e3,1, e2,3} → -η e2,3 + η² e2,3, {e1,3, e3,1, e3,2} → η e3,2 - η² e3,2,
  {e1,3, e3,2, e2,1} → -η h1,2 + η h1,3 - η h2,3, {e1,3, e3,2, e2,3} → η e1,3 - η² e1,3,
  {e1,3, e3,2, e3,1} → -η e3,2 + η² e3,2, {e1,3, e3,2, h1,2} → 2 η e1,2 - 2 η² e1,2,
  {e1,3, e3,2, h1,3} → -2 η e1,2 + 2 η² e1,2, {e1,3, e3,2, h2,3} → 2 η e1,2 - 2 η² e1,2,
  {e1,3, h1,2, e2,1} → 2 η e2,3 - 2 η² e2,3, {e1,3, h1,2, e3,2} → -2 η e1,2 + 2 η² e1,2,

```

$$\begin{aligned}
 \{e_{1,3}, h_{1,3}, e_{2,1}\} &\rightarrow -2\eta e_{2,3} + 2\eta^2 e_{2,3}, \{e_{1,3}, h_{1,3}, e_{3,2}\} \rightarrow 2\eta e_{1,2} - 2\eta^2 e_{1,2}, \\
 \{e_{1,3}, h_{2,3}, e_{2,1}\} &\rightarrow 2\eta e_{2,3} - 2\eta^2 e_{2,3}, \{e_{1,3}, h_{2,3}, e_{3,2}\} \rightarrow -2\eta e_{1,2} + 2\eta^2 e_{1,2}, \\
 \{e_{2,1}, e_{1,2}, e_{1,3}\} &\rightarrow \eta e_{1,3} - \eta^2 e_{1,3}, \{e_{2,1}, e_{1,2}, e_{2,3}\} \rightarrow -\eta e_{2,3} + \eta^2 e_{2,3}, \\
 \{e_{2,1}, e_{1,2}, e_{3,1}\} &\rightarrow -\eta e_{3,1} + \eta^2 e_{3,1}, \{e_{2,1}, e_{1,2}, e_{3,2}\} \rightarrow \eta e_{3,2} - \eta^2 e_{3,2}, \\
 \{e_{2,1}, e_{1,3}, e_{1,2}\} &\rightarrow -\eta e_{1,3} + \eta^2 e_{1,3}, \{e_{2,1}, e_{1,3}, e_{3,1}\} \rightarrow \eta e_{2,1} - \eta^2 e_{2,1}, \\
 \{e_{2,1}, e_{1,3}, e_{3,2}\} &\rightarrow -\eta h_{1,2} + \eta h_{1,3} - \eta h_{2,3}, \{e_{2,1}, e_{1,3}, h_{1,2}\} \rightarrow 2\eta e_{2,3} - 2\eta^2 e_{2,3}, \\
 \{e_{2,1}, e_{1,3}, h_{1,3}\} &\rightarrow -2\eta e_{2,3} + 2\eta^2 e_{2,3}, \{e_{2,1}, e_{1,3}, h_{2,3}\} \rightarrow 2\eta e_{2,3} - 2\eta^2 e_{2,3}, \\
 \{e_{2,1}, e_{2,3}, e_{1,2}\} &\rightarrow \eta e_{2,3} - \eta^2 e_{2,3}, \{e_{2,1}, e_{2,3}, e_{3,2}\} \rightarrow -\eta e_{2,1} + \eta^2 e_{2,1}, \\
 \{e_{2,1}, e_{3,1}, e_{1,2}\} &\rightarrow \eta e_{3,1} - \eta^2 e_{3,1}, \{e_{2,1}, e_{3,1}, e_{1,3}\} \rightarrow -\eta e_{2,1} + \eta^2 e_{2,1}, \\
 \{e_{2,1}, e_{3,2}, e_{1,2}\} &\rightarrow -\eta e_{3,2} + \eta^2 e_{3,2}, \{e_{2,1}, e_{3,2}, e_{1,3}\} \rightarrow \eta h_{1,2} - \eta h_{1,3} + \eta h_{2,3}, \\
 \{e_{2,1}, e_{3,2}, e_{2,3}\} &\rightarrow \eta e_{2,1} - \eta^2 e_{2,1}, \{e_{2,1}, e_{3,2}, h_{1,2}\} \rightarrow -2\eta e_{3,1} + 2\eta^2 e_{3,1}, \\
 \{e_{2,1}, e_{3,2}, h_{1,3}\} &\rightarrow 2\eta e_{3,1} - 2\eta^2 e_{3,1}, \{e_{2,1}, e_{3,2}, h_{2,3}\} \rightarrow -2\eta e_{3,1} + 2\eta^2 e_{3,1}, \\
 \{e_{2,1}, h_{1,2}, e_{1,3}\} &\rightarrow -2\eta e_{2,3} + 2\eta^2 e_{2,3}, \{e_{2,1}, h_{1,2}, e_{3,2}\} \rightarrow 2\eta e_{3,1} - 2\eta^2 e_{3,1}, \\
 \{e_{2,1}, h_{1,3}, e_{1,3}\} &\rightarrow 2\eta e_{2,3} - 2\eta^2 e_{2,3}, \{e_{2,1}, h_{1,3}, e_{3,2}\} \rightarrow -2\eta e_{3,1} + 2\eta^2 e_{3,1}, \\
 \{e_{2,1}, h_{2,3}, e_{1,3}\} &\rightarrow -2\eta e_{2,3} + 2\eta^2 e_{2,3}, \{e_{2,1}, h_{2,3}, e_{3,2}\} \rightarrow 2\eta e_{3,1} - 2\eta^2 e_{3,1}, \\
 \{e_{2,3}, e_{1,2}, e_{2,1}\} &\rightarrow \eta e_{2,3} - \eta^2 e_{2,3}, \{e_{2,3}, e_{1,2}, e_{3,1}\} \rightarrow -\eta h_{1,2} + \eta h_{1,3} - \eta h_{2,3}, \\
 \{e_{2,3}, e_{1,2}, e_{3,2}\} &\rightarrow -\eta e_{1,2} + \eta^2 e_{1,2}, \{e_{2,3}, e_{1,2}, h_{1,2}\} \rightarrow 2\eta e_{1,3} - 2\eta^2 e_{1,3}, \\
 \{e_{2,3}, e_{1,2}, h_{1,3}\} &\rightarrow -2\eta e_{1,3} + 2\eta^2 e_{1,3}, \{e_{2,3}, e_{1,2}, h_{2,3}\} \rightarrow 2\eta e_{1,3} - 2\eta^2 e_{1,3}, \\
 \{e_{2,3}, e_{1,3}, e_{3,1}\} &\rightarrow -\eta e_{2,3} + \eta^2 e_{2,3}, \{e_{2,3}, e_{1,3}, e_{3,2}\} \rightarrow \eta e_{1,3} - \eta^2 e_{1,3}, \\
 \{e_{2,3}, e_{2,1}, e_{1,2}\} &\rightarrow -\eta e_{2,3} + \eta^2 e_{2,3}, \{e_{2,3}, e_{2,1}, e_{3,2}\} \rightarrow \eta e_{2,1} - \eta^2 e_{2,1}, \\
 \{e_{2,3}, e_{3,1}, e_{1,2}\} &\rightarrow \eta h_{1,2} - \eta h_{1,3} + \eta h_{2,3}, \{e_{2,3}, e_{3,1}, e_{1,3}\} \rightarrow \eta e_{2,3} - \eta^2 e_{2,3}, \\
 \{e_{2,3}, e_{3,1}, e_{3,2}\} &\rightarrow -\eta e_{3,1} + \eta^2 e_{3,1}, \{e_{2,3}, e_{3,1}, h_{1,2}\} \rightarrow -2\eta e_{2,1} + 2\eta^2 e_{2,1}, \\
 \{e_{2,3}, e_{3,1}, h_{1,3}\} &\rightarrow 2\eta e_{2,1} - 2\eta^2 e_{2,1}, \{e_{2,3}, e_{3,1}, h_{2,3}\} \rightarrow -2\eta e_{2,1} + 2\eta^2 e_{2,1}, \\
 \{e_{2,3}, e_{3,2}, e_{1,2}\} &\rightarrow \eta e_{1,2} - \eta^2 e_{1,2}, \{e_{2,3}, e_{3,2}, e_{1,3}\} \rightarrow -\eta e_{1,3} + \eta^2 e_{1,3}, \\
 \{e_{2,3}, e_{3,2}, e_{2,1}\} &\rightarrow -\eta e_{2,1} + \eta^2 e_{2,1}, \{e_{2,3}, e_{3,2}, e_{3,1}\} \rightarrow \eta e_{3,1} - \eta^2 e_{3,1}, \\
 \{e_{2,3}, h_{1,2}, e_{1,2}\} &\rightarrow -2\eta e_{1,3} + 2\eta^2 e_{1,3}, \{e_{2,3}, h_{1,2}, e_{3,1}\} \rightarrow 2\eta e_{2,1} - 2\eta^2 e_{2,1}, \\
 \{e_{2,3}, h_{1,3}, e_{1,2}\} &\rightarrow 2\eta e_{1,3} - 2\eta^2 e_{1,3}, \{e_{2,3}, h_{1,3}, e_{3,1}\} \rightarrow -2\eta e_{2,1} + 2\eta^2 e_{2,1}, \\
 \{e_{2,3}, h_{2,3}, e_{1,2}\} &\rightarrow -2\eta e_{1,3} + 2\eta^2 e_{1,3}, \{e_{2,3}, h_{2,3}, e_{3,1}\} \rightarrow 2\eta e_{2,1} - 2\eta^2 e_{2,1}, \\
 \{e_{3,1}, e_{1,2}, e_{1,3}\} &\rightarrow -\eta e_{1,2} + \eta^2 e_{1,2}, \{e_{3,1}, e_{1,2}, e_{2,1}\} \rightarrow \eta e_{3,1} - \eta^2 e_{3,1}, \\
 \{e_{3,1}, e_{1,2}, e_{2,3}\} &\rightarrow \eta h_{1,2} - \eta h_{1,3} + \eta h_{2,3}, \{e_{3,1}, e_{1,2}, h_{1,2}\} \rightarrow -2\eta e_{3,2} + 2\eta^2 e_{3,2}, \\
 \{e_{3,1}, e_{1,2}, h_{1,3}\} &\rightarrow 2\eta e_{3,2} - 2\eta^2 e_{3,2}, \{e_{3,1}, e_{1,2}, h_{2,3}\} \rightarrow -2\eta e_{3,2} + 2\eta^2 e_{3,2}, \\
 \{e_{3,1}, e_{1,3}, e_{1,2}\} &\rightarrow \eta e_{1,2} - \eta^2 e_{1,2}, \{e_{3,1}, e_{1,3}, e_{2,1}\} \rightarrow -\eta e_{2,1} + \eta^2 e_{2,1}, \\
 \{e_{3,1}, e_{1,3}, e_{2,3}\} &\rightarrow \eta e_{2,3} - \eta^2 e_{2,3}, \{e_{3,1}, e_{1,3}, e_{3,2}\} \rightarrow -\eta e_{3,2} + \eta^2 e_{3,2}, \\
 \{e_{3,1}, e_{2,1}, e_{1,2}\} &\rightarrow -\eta e_{3,1} + \eta^2 e_{3,1}, \{e_{3,1}, e_{2,1}, e_{1,3}\} \rightarrow \eta e_{2,1} - \eta^2 e_{2,1}, \\
 \{e_{3,1}, e_{2,3}, e_{1,2}\} &\rightarrow -\eta h_{1,2} + \eta h_{1,3} - \eta h_{2,3}, \{e_{3,1}, e_{2,3}, e_{1,3}\} \rightarrow -\eta e_{2,3} + \eta^2 e_{2,3}, \\
 \{e_{3,1}, e_{2,3}, e_{3,2}\} &\rightarrow \eta e_{3,1} - \eta^2 e_{3,1}, \{e_{3,1}, e_{2,3}, h_{1,2}\} \rightarrow 2\eta e_{2,1} - 2\eta^2 e_{2,1}, \\
 \{e_{3,1}, e_{2,3}, h_{1,3}\} &\rightarrow -2\eta e_{2,1} + 2\eta^2 e_{2,1}, \{e_{3,1}, e_{2,3}, h_{2,3}\} \rightarrow 2\eta e_{2,1} - 2\eta^2 e_{2,1}, \\
 \{e_{3,1}, e_{3,2}, e_{1,3}\} &\rightarrow \eta e_{3,2} - \eta^2 e_{3,2}, \{e_{3,1}, e_{3,2}, e_{2,3}\} \rightarrow -\eta e_{3,1} + \eta^2 e_{3,1}, \\
 \{e_{3,1}, h_{1,2}, e_{1,2}\} &\rightarrow 2\eta e_{3,2} - 2\eta^2 e_{3,2}, \{e_{3,1}, h_{1,2}, e_{2,3}\} \rightarrow -2\eta e_{2,1} + 2\eta^2 e_{2,1}, \\
 \{e_{3,1}, h_{1,3}, e_{1,2}\} &\rightarrow -2\eta e_{3,2} + 2\eta^2 e_{3,2}, \{e_{3,1}, h_{1,3}, e_{2,3}\} \rightarrow 2\eta e_{2,1} - 2\eta^2 e_{2,1}, \\
 \{e_{3,1}, h_{2,3}, e_{1,2}\} &\rightarrow 2\eta e_{3,2} - 2\eta^2 e_{3,2}, \{e_{3,1}, h_{2,3}, e_{2,3}\} \rightarrow -2\eta e_{2,1} + 2\eta^2 e_{2,1}, \\
 \{e_{3,2}, e_{1,2}, e_{2,1}\} &\rightarrow -\eta e_{3,2} + \eta^2 e_{3,2}, \{e_{3,2}, e_{1,2}, e_{2,3}\} \rightarrow \eta e_{1,2} - \eta^2 e_{1,2}, \\
 \{e_{3,2}, e_{1,3}, e_{2,1}\} &\rightarrow \eta h_{1,2} - \eta h_{1,3} + \eta h_{2,3}, \{e_{3,2}, e_{1,3}, e_{2,3}\} \rightarrow -\eta e_{1,3} + \eta^2 e_{1,3}, \\
 \{e_{3,2}, e_{1,3}, e_{3,1}\} &\rightarrow \eta e_{3,2} - \eta^2 e_{3,2}, \{e_{3,2}, e_{1,3}, h_{1,2}\} \rightarrow -2\eta e_{1,2} + 2\eta^2 e_{1,2}, \\
 \{e_{3,2}, e_{1,3}, h_{1,3}\} &\rightarrow 2\eta e_{1,2} - 2\eta^2 e_{1,2}, \{e_{3,2}, e_{1,3}, h_{2,3}\} \rightarrow -2\eta e_{1,2} + 2\eta^2 e_{1,2}, \\
 \{e_{3,2}, e_{2,1}, e_{1,2}\} &\rightarrow \eta e_{3,2} - \eta^2 e_{3,2}, \{e_{3,2}, e_{2,1}, e_{1,3}\} \rightarrow -\eta h_{1,2} + \eta h_{1,3} - \eta h_{2,3}, \\
 \{e_{3,2}, e_{2,1}, e_{2,3}\} &\rightarrow -\eta e_{2,1} + \eta^2 e_{2,1}, \{e_{3,2}, e_{2,1}, h_{1,2}\} \rightarrow 2\eta e_{3,1} - 2\eta^2 e_{3,1}, \\
 \{e_{3,2}, e_{2,1}, h_{1,3}\} &\rightarrow -2\eta e_{3,1} + 2\eta^2 e_{3,1}, \{e_{3,2}, e_{2,1}, h_{2,3}\} \rightarrow 2\eta e_{3,1} - 2\eta^2 e_{3,1}, \\
 \{e_{3,2}, e_{2,3}, e_{1,2}\} &\rightarrow -\eta e_{1,2} + \eta^2 e_{1,2}, \{e_{3,2}, e_{2,3}, e_{1,3}\} \rightarrow \eta e_{1,3} - \eta^2 e_{1,3},
 \end{aligned}$$

$$\begin{aligned}
\{e_{3,2}, e_{2,3}, e_{2,1}\} &\rightarrow \eta e_{2,1} - \eta^2 e_{2,1}, \{e_{3,2}, e_{2,3}, e_{3,1}\} \rightarrow -\eta e_{3,1} + \eta^2 e_{3,1}, \\
\{e_{3,2}, e_{3,1}, e_{1,3}\} &\rightarrow -\eta e_{3,2} + \eta^2 e_{3,2}, \{e_{3,2}, e_{3,1}, e_{2,3}\} \rightarrow \eta e_{3,1} - \eta^2 e_{3,1}, \\
\{e_{3,2}, h_{1,2}, e_{1,3}\} &\rightarrow 2\eta e_{1,2} - 2\eta^2 e_{1,2}, \{e_{3,2}, h_{1,2}, e_{2,1}\} \rightarrow -2\eta e_{3,1} + 2\eta^2 e_{3,1}, \\
\{e_{3,2}, h_{1,3}, e_{1,3}\} &\rightarrow -2\eta e_{1,2} + 2\eta^2 e_{1,2}, \{e_{3,2}, h_{1,3}, e_{2,1}\} \rightarrow 2\eta e_{3,1} - 2\eta^2 e_{3,1}, \\
\{e_{3,2}, h_{2,3}, e_{1,3}\} &\rightarrow 2\eta e_{1,2} - 2\eta^2 e_{1,2}, \{e_{3,2}, h_{2,3}, e_{2,1}\} \rightarrow -2\eta e_{3,1} + 2\eta^2 e_{3,1}, \\
\{h_{1,2}, e_{1,2}, e_{2,3}\} &\rightarrow -2\eta e_{1,3} + 2\eta^2 e_{1,3}, \{h_{1,2}, e_{1,2}, e_{3,1}\} \rightarrow 2\eta e_{3,2} - 2\eta^2 e_{3,2}, \\
\{h_{1,2}, e_{1,3}, e_{2,1}\} &\rightarrow -2\eta e_{2,3} + 2\eta^2 e_{2,3}, \{h_{1,2}, e_{1,3}, e_{3,2}\} \rightarrow 2\eta e_{1,2} - 2\eta^2 e_{1,2}, \\
\{h_{1,2}, e_{2,1}, e_{1,3}\} &\rightarrow 2\eta e_{2,3} - 2\eta^2 e_{2,3}, \{h_{1,2}, e_{2,1}, e_{3,2}\} \rightarrow -2\eta e_{3,1} + 2\eta^2 e_{3,1}, \\
\{h_{1,2}, e_{2,3}, e_{1,2}\} &\rightarrow 2\eta e_{1,3} - 2\eta^2 e_{1,3}, \{h_{1,2}, e_{2,3}, e_{3,1}\} \rightarrow -2\eta e_{2,1} + 2\eta^2 e_{2,1}, \\
\{h_{1,2}, e_{3,1}, e_{1,2}\} &\rightarrow -2\eta e_{3,2} + 2\eta^2 e_{3,2}, \{h_{1,2}, e_{3,1}, e_{2,3}\} \rightarrow 2\eta e_{2,1} - 2\eta^2 e_{2,1}, \\
\{h_{1,2}, e_{3,2}, e_{1,3}\} &\rightarrow -2\eta e_{1,2} + 2\eta^2 e_{1,2}, \{h_{1,2}, e_{3,2}, e_{2,1}\} \rightarrow 2\eta e_{3,1} - 2\eta^2 e_{3,1}, \\
\{h_{1,3}, e_{1,2}, e_{2,3}\} &\rightarrow 2\eta e_{1,3} - 2\eta^2 e_{1,3}, \{h_{1,3}, e_{1,2}, e_{3,1}\} \rightarrow -2\eta e_{3,2} + 2\eta^2 e_{3,2}, \\
\{h_{1,3}, e_{1,3}, e_{2,1}\} &\rightarrow 2\eta e_{2,3} - 2\eta^2 e_{2,3}, \{h_{1,3}, e_{1,3}, e_{3,2}\} \rightarrow -2\eta e_{1,2} + 2\eta^2 e_{1,2}, \\
\{h_{1,3}, e_{2,1}, e_{1,3}\} &\rightarrow -2\eta e_{2,3} + 2\eta^2 e_{2,3}, \{h_{1,3}, e_{2,1}, e_{3,2}\} \rightarrow 2\eta e_{3,1} - 2\eta^2 e_{3,1}, \\
\{h_{1,3}, e_{2,3}, e_{1,2}\} &\rightarrow -2\eta e_{1,3} + 2\eta^2 e_{1,3}, \{h_{1,3}, e_{2,3}, e_{3,1}\} \rightarrow 2\eta e_{2,1} - 2\eta^2 e_{2,1}, \\
\{h_{1,3}, e_{3,1}, e_{1,2}\} &\rightarrow 2\eta e_{3,2} - 2\eta^2 e_{3,2}, \{h_{1,3}, e_{3,1}, e_{2,3}\} \rightarrow -2\eta e_{2,1} + 2\eta^2 e_{2,1}, \\
\{h_{1,3}, e_{3,2}, e_{1,3}\} &\rightarrow 2\eta e_{1,2} - 2\eta^2 e_{1,2}, \{h_{1,3}, e_{3,2}, e_{2,1}\} \rightarrow -2\eta e_{3,1} + 2\eta^2 e_{3,1}, \\
\{h_{2,3}, e_{1,2}, e_{2,3}\} &\rightarrow -2\eta e_{1,3} + 2\eta^2 e_{1,3}, \{h_{2,3}, e_{1,2}, e_{3,1}\} \rightarrow 2\eta e_{3,2} - 2\eta^2 e_{3,2}, \\
\{h_{2,3}, e_{1,3}, e_{2,1}\} &\rightarrow -2\eta e_{2,3} + 2\eta^2 e_{2,3}, \{h_{2,3}, e_{1,3}, e_{3,2}\} \rightarrow 2\eta e_{1,2} - 2\eta^2 e_{1,2}, \\
\{h_{2,3}, e_{2,1}, e_{1,3}\} &\rightarrow 2\eta e_{2,3} - 2\eta^2 e_{2,3}, \{h_{2,3}, e_{2,1}, e_{3,2}\} \rightarrow -2\eta e_{3,1} + 2\eta^2 e_{3,1}, \\
\{h_{2,3}, e_{2,3}, e_{1,2}\} &\rightarrow 2\eta e_{1,3} - 2\eta^2 e_{1,3}, \{h_{2,3}, e_{2,3}, e_{3,1}\} \rightarrow -2\eta e_{2,1} + 2\eta^2 e_{2,1}, \\
\{h_{2,3}, e_{3,1}, e_{1,2}\} &\rightarrow -2\eta e_{3,2} + 2\eta^2 e_{3,2}, \{h_{2,3}, e_{3,1}, e_{2,3}\} \rightarrow 2\eta e_{2,1} - 2\eta^2 e_{2,1}, \\
\{h_{2,3}, e_{3,2}, e_{1,3}\} &\rightarrow -2\eta e_{1,2} + 2\eta^2 e_{1,2}, \{h_{2,3}, e_{3,2}, e_{2,1}\} \rightarrow 2\eta e_{3,1} - 2\eta^2 e_{3,1}
\end{aligned}$$