

Pensieve header: Implementing and verifying $\$gl_n^\epsilon\$$; with highlighted centre. Continues pensieve://2017-01/.

$$h_{\text{today}} = h_{\text{old}} - \epsilon g; h_{\text{old}} = h_{\text{today}} + \epsilon g.$$

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Xcond := If[TrueQ@cond, 1, 0];
B[0, _] = 0; B[_, 0] = 0;
B[c_*x : (e | f | g | h)__, y_] := Expand[c B[x, y]];
B[y_, c_*x : (e | f | g | h)__) := Expand[c B[y, x]];
B[x_Plus, y_] := B[#, y] & /@ x;
B[x_, y_Plus] := B[x, #] & /@ y;
P[0, _] = 0; P[_, 0] = 0;
P[c_*x : (e | f | g | h)__, y_] := Expand[c P[x, y]];
P[y_, c_*x : (e | f | g | h)__) := Expand[c P[y, x]];
P[x_Plus, y_] := P[#, y] & /@ x;
P[x_, y_Plus] := P[x, #] & /@ y;

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P[ei,j, fk,l] := Xj=k∧i=l; P[fk,l, ei,j] := Xj=k∧i=l;
P[(e | f)__, h_] = 0; P[h_, (e | f)__) = 0;
P[gi, hj] := 2 Xi=j; P[hj, gi] := 2 Xi=j; P[hi, hj] := -4 ε Xi=j;
P[(e | g)__, (e | g)__) = 0;
P[(f | g)__, (f | g)__) = 0;

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B[h_, _] = 0;
B[g_, g_] = 0;

B[ei,j, ek,l] := Xj=k ei,l - Xl=i ek,j;
B[fi,j, fk,l] := ε Xj=k fi,l - ε Xl=i fk,j;
B[ei,j, fk,l] := Expand[
  Xj=k (ε Xi<l ei,l + Xi=l (hi + 2 ε gi) / 2 + Xi>l fi,l) -
  Xl=i (ε Xk<j ek,j + Xk=j (hj + 2 ε gj) / 2 + Xk>j fk,j)];

B[gi, ej,k] := (Xi=j - Xi=k) ej,k;
B[gi, fj,k] := (Xi=j - Xi=k) fj,k;

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B[y_, x_] := Expand[-B[x, y]];

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Basis+[n_] := Union@Flatten@{
  Table[ei,j, {i, n - 1}, {j, i + 1, n}],
  Table[gi, {i, n}]
};
Basis-[n_] := Union@Flatten@{
  Table[fi,j, {i, 2, n}, {j, i - 1}],
  Table[hi, {i, n}]
};
Basis[n_] := Join[Basis+[n], Basis-[n]];

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Basis[4]

{g₁, g₂, g₃, g₄, e_{1,2}, e_{1,3}, e_{1,4}, e_{2,3}, e_{2,4}, e_{3,4}, h₁, h₂, h₃, h₄, f_{2,1}, f_{3,1}, f_{3,2}, f_{4,1}, f_{4,2}, f_{4,3}}

n = 3;

Table[

{x, y} → B[x, y],

{x, Basis[n]}, {y, Basis[n]}

] // **MatrixForm**

{g ₁ , g ₁ } → 0	{g ₁ , g ₂ } → 0	{g ₁ , g ₃ } → 0	{g ₁ , e _{1,2} } → e _{1,2}
{g ₂ , g ₁ } → 0	{g ₂ , g ₂ } → 0	{g ₂ , g ₃ } → 0	{g ₂ , e _{1,2} } → -e _{1,2}
{g ₃ , g ₁ } → 0	{g ₃ , g ₂ } → 0	{g ₃ , g ₃ } → 0	{g ₃ , e _{1,2} } → 0
{e _{1,2} , g ₁ } → -e _{1,2}	{e _{1,2} , g ₂ } → e _{1,2}	{e _{1,2} , g ₃ } → 0	{e _{1,2} , e _{1,2} } → 0
{e _{1,3} , g ₁ } → -e _{1,3}	{e _{1,3} , g ₂ } → 0	{e _{1,3} , g ₃ } → e _{1,3}	{e _{1,3} , e _{1,2} } → 0
{e _{2,3} , g ₁ } → 0	{e _{2,3} , g ₂ } → -e _{2,3}	{e _{2,3} , g ₃ } → e _{2,3}	{e _{2,3} , e _{1,2} } → -e _{1,3}
{h ₁ , g ₁ } → 0	{h ₁ , g ₂ } → 0	{h ₁ , g ₃ } → 0	{h ₁ , e _{1,2} } → 0
{h ₂ , g ₁ } → 0	{h ₂ , g ₂ } → 0	{h ₂ , g ₃ } → 0	{h ₂ , e _{1,2} } → 0
{h ₃ , g ₁ } → 0	{h ₃ , g ₂ } → 0	{h ₃ , g ₃ } → 0	{h ₃ , e _{1,2} } → 0
{f _{2,1} , g ₁ } → f _{2,1}	{f _{2,1} , g ₂ } → -f _{2,1}	{f _{2,1} , g ₃ } → 0	{f _{2,1} , e _{1,2} } → -ε g ₁ + ε g ₂ - $\frac{h_1}{2}$ + $\frac{h_2}{2}$
{f _{3,1} , g ₁ } → f _{3,1}	{f _{3,1} , g ₂ } → 0	{f _{3,1} , g ₃ } → -f _{3,1}	{f _{3,1} , e _{1,2} } → f _{3,2}
{f _{3,2} , g ₁ } → 0	{f _{3,2} , g ₂ } → f _{3,2}	{f _{3,2} , g ₃ } → -f _{3,2}	{f _{3,2} , e _{1,2} } → 0

n = 4;

Union@**Table**[

{x, y} = t; B[x, y] + B[y, x],

{t, Tuples[Basis[n], 2]}

]

{0}

n = 3;

DeleteCases[**Flatten**@**Table**[

{x, y, z} → P[B[x, y], z] + P[y, B[x, z]],

{x, Basis[n]}, {y, Basis[n]}, {z, Basis[n]}

], _ → 0]

{}

n = 3;

DeleteCases[**Table**[

{(x, y, z) = t} → B[x, B[y, z]] + B[y, B[z, x]] + B[z, B[x, y]],

{t, Tuples[Basis[n], 3]}

], _ → 0]

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