Pensieve header: Testing Gaussian pairing, plus g1 logos games.

$$\mathcal{O}\left(e^{\delta ef} \,\middle|\, fe\right) = \mathcal{O}\left(e^{\delta\partial_\alpha\partial_\beta} e^{\beta e} e^{\alpha f}\Big|_{\alpha=\beta=0} \,\middle|\, fe \right) = e^{\delta\partial_\alpha\partial_\beta}\,\mathcal{O}\left(e^{\beta e} e^{\alpha f} \,\middle|\, fe\right)\Big|_{\alpha=\beta=0}$$

$$= e^{\delta\partial_\alpha\partial_\beta}\,\mathcal{O}\left(e^{-\alpha\beta h+\beta e+\alpha f} \,\middle|\, ef\right)\Big|_{\alpha=\beta=0} = \mathcal{O}\left(e^{\delta\partial_\alpha\partial_\beta} e^{-\alpha\beta h+\beta e+\alpha f}\Big|_{\alpha=\beta=0} \,\middle|\, ef\right)$$

(170211) Gaussian pairing:

$$\left\langle \exp\left(\frac{x\subset}{2} + \sum_{i\in I} i\bullet\!\!-\right) \,\middle|\, \exp\left(\frac{\supset y}{2} + \sum_{j\in J} -\!\!\bullet j\right)\right\rangle =$$

$$\exp\left(\log\left(\frac{1}{1-xy}\right)\bigcirc + \sum_{i\in I, j\in J}\frac{i\bullet\!\!-\!\!\bullet j}{1-xy} + \sum_{i_{1,2}\in I}\frac{{}^{i_1}_{i_2}\!\!\,\overset{\bullet}{\underset{\bullet}{|}}\, y}{1-xy} + \sum_{j_{1,2}\in J}\frac{x\,\overset{\bullet}{\underset{\bullet}{|}}\,{}^{j_1}_{j_2}}{1-xy}\right)$$

```
DP_{D_α_, D_β_}[P_][f_] :=
  Total[CoefficientRules[P, {D_α, D_β}] /. ({m_, n_} → c_) :> c D[f, {α, m}, {β, n}]]
```

```
With[{n = 10},
  DP_{D_α, D_β}[Normal@Series[ⅇ^(δ D_α D_β), {δ, 0, n}]][ⅇ^(-α β h + β e + α e)] /. {α | β → 0}
 ] /. {e | f → 0}
```

$1 - h\,\delta + h^2\,\delta^2 - h^3\,\delta^3 + h^4\,\delta^4 - h^5\,\delta^5 + h^6\,\delta^6 - h^7\,\delta^7 + h^8\,\delta^8 - h^9\,\delta^9 + h^{10}\,\delta^{10}$

```
Series[1/(1 + h δ), {δ, 0, 10}]
```

$1 - h\,\delta + h^2\,\delta^2 - h^3\,\delta^3 + h^4\,\delta^4 - h^5\,\delta^5 + h^6\,\delta^6 - h^7\,\delta^7 + h^8\,\delta^8 - h^9\,\delta^9 + h^{10}\,\delta^{10} + O[\delta]^{11}$

```
With[{n = 5}, DP_{D_α, D_β}[Normal@Series[ⅇ^(t D_α D_β), {t, 0, n}]][ⅇ^(-α β h)]] // Simplify
```

$$\frac{1}{120}\,e^{-h\alpha\beta}\left(120 - 120\,h\,t - 25\,h^9\,t^5\,\alpha^4\,\beta^4 + h^{10}\,t^5\,\alpha^5\,\beta^5 + 120\,h^2\,t\,(t+\alpha\beta) + 5\,h^8\,t^4\,\alpha^3\,\beta^3\,(40\,t+\alpha\beta) - 120\,h^3\,t^2\,(t+2\,\alpha\beta) - 40\,h^7\,t^4\,\alpha^2\,\beta^2\,(15\,t+2\,\alpha\beta) + 60\,h^4\,t^2\,(2\,t^2+6\,t\,\alpha\beta+\alpha^2\beta^2) + 20\,h^6\,t^3\,\alpha\beta\,(30\,t^2+18\,t\,\alpha\beta+\alpha^2\beta^2) - 60\,h^5\,t^3\,(2\,t^2+8\,t\,\alpha\beta+3\,\alpha^2\beta^2)\right)$$

```
Series[ⅇ^(-(1+h t)^-1 α β h)/(1 + h t), {t, 0, 5}] // Normal // Simplify
```

$$\frac{1}{120}\,e^{-h\alpha\beta}\left(120 - 120\,h\,t - 25\,h^9\,t^5\,\alpha^4\,\beta^4 + h^{10}\,t^5\,\alpha^5\,\beta^5 + 120\,h^2\,t\,(t+\alpha\beta) + 5\,h^8\,t^4\,\alpha^3\,\beta^3\,(40\,t+\alpha\beta) - 120\,h^3\,t^2\,(t+2\,\alpha\beta) - 40\,h^7\,t^4\,\alpha^2\,\beta^2\,(15\,t+2\,\alpha\beta) + 60\,h^4\,t^2\,(2\,t^2+6\,t\,\alpha\beta+\alpha^2\beta^2) + 20\,h^6\,t^3\,\alpha\beta\,(30\,t^2+18\,t\,\alpha\beta+\alpha^2\beta^2) - 60\,h^5\,t^3\,(2\,t^2+8\,t\,\alpha\beta+3\,\alpha^2\beta^2)\right)$$

```
With[{ψ = ⅇ^(-(1+h t)^-1 α β h)/(1 + h t)}, {∂_t ψ - ∂_{α,β} ψ, ψ /. t → 0}]
```

$\{0,\ e^{-h\alpha\beta}\}$

```
oldΛ = 1/(2 μ^4) (-b α^2 β^2 + u^2 β^2 δ (2 + b δ) + w^2 δ (α + u δ) (α (2 + b δ) + u δ (4 + 3 b δ)) -
     4 b α β δ μ + 4 c α β μ^2 - 2 b δ^2 μ^2 + 4 c δ μ^3 + 2 u β (α β + 2 δ μ (1 + c μ)) +
     2 w (α^2 β + 2 α δ (u β (2 + b δ) + μ (1 + c μ)) + u δ^2 (u β (3 + 2 b δ) + 2 μ (2 + b δ + c μ))));
```

```
Λl = Simplify[oldΛ /. {δ → 0, μ → 1, b → h, c → l, u → e, w → f}]
```

$\frac{1}{2}\,\alpha\,\beta\,(4\,l + 2\,f\,\alpha + 2\,e\,\beta - h\,\alpha\,\beta)$

```
Λq = Factor[oldΛ /. {b → h, c → l, u → e, w → f, α → 0, β → 0, μ → 1 + h δ}]
```

$$\frac{1}{2\left(1+h\,\delta\right)^4}\,\delta\left(4\,l+8\,e\,f\,\delta-2\,h\,\delta+4\,e\,f\,l\,\delta+12\,h\,l\,\delta+4\,e^2\,f^2\,\delta^2+12\,e\,f\,h\,\delta^2-4\,h^2\,\delta^2+8\,e\,f\,h\,l\,\delta^2+12\,h^2\,l\,\delta^2+3\,e^2\,f^2\,h\,\delta^3+4\,e\,f\,h^2\,\delta^3-2\,h^3\,\delta^3+4\,e\,f\,h^2\,l\,\delta^3+4\,h^3\,l\,\delta^3\right)$$

$$\Lambda p = \text{Factor}\left[\Lambda l + \left(1+h\,\delta\right)^{-1}\delta\left(\partial_{\alpha,\beta}\Lambda l\right)+\frac{1}{2}\left(1+h\,\delta\right)^{-2}\delta^2\left(\partial_{\alpha,\beta,\alpha,\beta}\Lambda l\right)\ /.\ \left\{\alpha\to\left(1+h\,\delta\right)^{-1}\delta\,e,\ \beta\to\left(1+h\,\delta\right)^{-1}\delta\,f\right\}\right]$$

$$\frac{1}{2\left(1+h\,\delta\right)^4}\,\delta\left(4\,l+8\,e\,f\,\delta-2\,h\,\delta+4\,e\,f\,l\,\delta+12\,h\,l\,\delta+4\,e^2\,f^2\,\delta^2+12\,e\,f\,h\,\delta^2-4\,h^2\,\delta^2+8\,e\,f\,h\,l\,\delta^2+12\,h^2\,l\,\delta^2+3\,e^2\,f^2\,h\,\delta^3+4\,e\,f\,h^2\,\delta^3-2\,h^3\,\delta^3+4\,e\,f\,h^2\,l\,\delta^3+4\,h^3\,l\,\delta^3\right)$$

```
Λp == Λq
```

True