## Dancso visit summary

January 5, 2017 8:30 AM

## Some genus g excerpts:

(131104) Humbert's thesis pp 22: The relations of  $t_n^1$ :  $[v_i, w_j] = \langle v, w \rangle t_{ij}$ ,  $[v_i, t_{jk}] = 0$ ,  $[x_i, y_i] = -\sum_{j \neq i} t_{ij}$ . Imply centrality of  $\sum_j v_j$  and  $t_{ij} = t_{ji}$ ,  $[v_i + v_j, t_{ij}] = 0$ ,  $[t_{ij}, t_{kl}] = 0$ , and  $[t_{ij}, t_{ik} + t_{jk}] = 0$ . Canonical forms?

(150217) Enriquez:  $B_n^1$  is  $\langle \sigma_i, X_1^{\pm} \rangle \mod (\sigma_1^{\pm 1} X_1^{\pm})^2 = (X_1^{\pm} \sigma_1^{\pm 1})^2$ ,  $[X_1^{\pm}, \sigma_i] = 1$  for  $i \ge 2$ ,  $[X_1^{-}, (X_2^{+})^{-1}] = \sigma_1^2, X_1^{\pm} \cdots X_n^{\pm} = 1$ , and braid relations, where  $X_{i+1}^{\pm} = \sigma_i^{\pm 1} X_i^{\pm} \sigma_i^{\pm 1}$ .

Elliptic Braids.  $PB_n^1 := \pi_1(C_n^1)$  is generated by  $\sigma_{ij}, X_i, Y_j$ , with  $PB_n$  relations and  $(X_i, X_j) = 1 = (Y_i, Y_j), (X_i, Y_j) = \sigma_{ij}^{-1}, (X_iX_j, \sigma_{ij}) = 1 = (Y_iY_j, \sigma_{ij}), and <math>\prod X_i$  and  $\prod Y_j$  are central. [Bez] implies  $\mathcal{A}(PB_n^1) = \langle x_i, y_j \rangle / ([x_i, x_j] = [y_i, y_j] = [x_i + x_j, [x_i, y_j]] = [y_i + y_j, [x_i, y_j]] = [x_i, \sum y_j] = [y_j, \sum x_i] = 0, [x_i, y_j] = [x_j, y_i]),$  and [CEE] construct a Taylor expansion using *sophisticated* iterated integrals. [En2] relates this to *Elliptic Associators*.

(150224b) Fadell-Neuwirth: For 0 < r < n,  $m \ge 0$ , and  $M = \sum_{g\ge 1}^2 |D^2$ ,  $1 \rightarrow PB_{n-r}(M \setminus \underline{m+r}) \rightarrow PB_n(M \setminus \underline{m}) \rightarrow PB_r(M \setminus \underline{m}) \rightarrow 1$  is exact. (150224a) Surface braids: Bardakov, Bellingeri, Birman, Funar, Gervais, Gonzalez-Meneses, Guaschi, Juan-Pineda.

(150131) Katz 5.1:  $R\mathcal{A}^{s}(|_{h}\uparrow^{n}) \hookrightarrow \mathcal{A}^{s}(|_{h}\uparrow^{n})/C$ . *R*: nothing on last strand.  $|_{h}$ : a handle line.

(150130b) Katz 5.2:  $L\mathcal{A}_1^{s \downarrow}(\uparrow^n) \cong \mathcal{A}_1^{s \downarrow}(\uparrow^n)/C$ .  $\Box_1$ : elliptic.  $\downarrow$ : strutless. *s*: skeleton-connected. *L*: only lonely vertices on last strand. *C*: closed surface.

(150130a) **Q.** Why is  $PB^g$  related to non-tangential differential operators on Fun( $g^g$ )?

(141224) Katz points (BBS:Katz-141224, arXiv:1412.7848): • Cheptea-Habiro-Massuyeau's arXiv:math/0701277 has a Clifford-like relation in sec. 8 (earlier, in Habiro's arXiv: math/0001185, fig. 48). • LMO for Lagrangian cobordisms partially interprets leg-gluing in  $\mathcal{B}$ . •  $\mathcal{B}^{g}$ -grading: # of trivalent vertices (excluding univalents).

(150123) gr  $(PB_n^0 \to PB_n^1)$  is 0:  $\mathcal{A}^{pb,0} \to \mathcal{A}^{pb,1}$  for a degree mismatch. Likely  $[PB_n^1, PB_n^1] \supseteq PB_n^0$ .

(13112b) A map  $\mathcal{A}^{w}(\uparrow \) \to \mathcal{A}^{u}(\uparrow \circledast)$  arises in deducing wheeling from the full Duflo (not  $\alpha^{-1}$  for  $\alpha$  is not well-defined!?); There's a pairing  $\mathcal{A}^{w}(\uparrow \) \otimes \mathcal{A}^{u}(\uparrow) \to \mathcal{A}^{u}(\uparrow)$ . Topological meaning?

Comment. There might be good related stuff hidden in Kricker's papers and blackboard shots! Especially,



Idea for a (modest) paper (which should at least serve to motivate us to complete the statement and proof of the "w-genus g" theorem of <u>http://drorbn.net/bbs/show?shot=Dancso-170104-152331.jpg</u>):

Title. Finite type invariants of certain 2-knots in certain 4-manifolds.

Abstract. We construct a homomophic expansion (a universal finite type invariant) for a certain class of 2-knots in a certain class of 4-manifolds (details and small print inside), using some novel techniques which include certain 4D surgeries and a related "Aarhus integral", and a certain diagrammatic analog of Heisenberg algebras. Almost certainly, this paper will not be interesting in the long term, as it is clear that our results should be subsumed within a higher level of understanding of 2D in 4D and of w-knot and v-knot techniques. Yet to gain this higher level understanding we must start somewhere, and right now, this is here.

Some after-visit thoughts are in monoblog.

We should hear Alekseev's talk at MSRI, and read <u>https://arxiv.org/abs/1611.05581</u>. Also some\_\_\_\_\_\_ LD16 talks.