

Pensieve header: Implementing and verifying  $\$gl_n^\wedge\epsilon\$, version 3 (includes the derived series and the LCS of the commutator algebra).$

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Xcond_ := If[TrueQ@cond, 1, 0];
B[0, _] = 0; B[_, 0] = 0;
B[c_ * x : (e | f | g | h)_, y_] := Expand[c B[x, y]];
B[y_, c_ * x : (e | f | g | h)_] := Expand[c B[y, x]];
B[x_Plus, y_] := B[#, y] & /@ x;
B[x_, y_Plus] := B[x, #] & /@ y;
P[0, _] = 0; P[_, 0] = 0;
P[c_ * x : (e | f | g | h)_, y_] := Expand[c P[x, y]];
P[y_, c_ * x : (e | f | g | h)_] := Expand[c P[y, x]];
P[x_Plus, y_] := P[#, y] & /@ x;
P[x_, y_Plus] := P[x, #] & /@ y;

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P[e_{i,j}, f_{k,l}] := X_{j=k \wedge l=i}; P[f_{k,l}, e_{i,j}] := X_{j=k \wedge l=i};
P[e_, h_] = 0; P[h_, e_] = 0;
P[g_, f_] = 0; P[f_, g_] = 0;
P[g_i, h_j] := 2 X_{i=j}; P[h_j, g_i] := 2 X_{i=j};
P[(e | g)_, (e | g)_] = 0;
P[(f | h)_, (f | h)_] = 0;

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B[(h | g)_, (h | g)_] = 0;

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B[e_{i,j}, e_{k,l}] := X_{j=k} e_{i,l} - X_{l=i} e_{k,j};
B[f_{i,j}, f_{k,l}] := \epsilon X_{j=k} f_{i,l} - \epsilon X_{l=i} f_{k,j};
B[e_{i,j}, f_{k,l}] := Expand[
  X_{j=k} (\epsilon X_{i<l} e_{i,l} + X_{i=l} (h_i + \epsilon g_i) / 2 + X_{i>l} f_{i,l}) -
  X_{l=i} (\epsilon X_{k<j} e_{k,j} + X_{k=j} (h_j + \epsilon g_j) / 2 + X_{k>j} f_{k,j})];
B[g_i, e_{j,k}] := (X_{i=j} - X_{i=k}) e_{j,k};
B[h_i, e_{j,k}] := \epsilon (X_{i=j} - X_{i=k}) e_{j,k};
B[g_i, f_{j,k}] := (X_{i=j} - X_{i=k}) f_{j,k};
B[h_i, f_{j,k}] := \epsilon (X_{i=j} - X_{i=k}) f_{j,k};

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B[y_, x_] := Expand[-B[x, y]];

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Basis^+[n_] := Union@Flatten@{
  Table[e_{i,j}, {i, n-1}, {j, i+1, n}],
  Table[g_i, {i, n}]
};
Basis^-[n_] := Union@Flatten@{
  Table[f_{i,j}, {i, 2, n}, {j, i-1}],
  Table[h_i, {i, n}]
};
Basis[n_] := Join[Basis^+[n], Basis^-[n]];

```

**Basis**[4]

{g<sub>1</sub>, g<sub>2</sub>, g<sub>3</sub>, g<sub>4</sub>, e<sub>1,2</sub>, e<sub>1,3</sub>, e<sub>1,4</sub>, e<sub>2,3</sub>, e<sub>2,4</sub>, e<sub>3,4</sub>, h<sub>1</sub>, h<sub>2</sub>, h<sub>3</sub>, h<sub>4</sub>, f<sub>2,1</sub>, f<sub>3,1</sub>, f<sub>3,2</sub>, f<sub>4,1</sub>, f<sub>4,2</sub>, f<sub>4,3</sub>}

n = 3;

**Table**[

{x, y} → B[x, y],

{x, Basis[n]}, {y, Basis[n]}

] // **MatrixForm**

{g <sub>1</sub> , g <sub>1</sub> } → 0	{g <sub>1</sub> , g <sub>2</sub> } → 0	{g <sub>1</sub> , g <sub>3</sub> } → 0	{g <sub>1</sub> , e <sub>1,2</sub> } → e <sub>1,2</sub>	
{g <sub>2</sub> , g <sub>1</sub> } → 0	{g <sub>2</sub> , g <sub>2</sub> } → 0	{g <sub>2</sub> , g <sub>3</sub> } → 0	{g <sub>2</sub> , e <sub>1,2</sub> } → -e <sub>1,2</sub>	
{g <sub>3</sub> , g <sub>1</sub> } → 0	{g <sub>3</sub> , g <sub>2</sub> } → 0	{g <sub>3</sub> , g <sub>3</sub> } → 0	{g <sub>3</sub> , e <sub>1,2</sub> } → 0	{
{e <sub>1,2</sub> , g <sub>1</sub> } → -e <sub>1,2</sub>	{e <sub>1,2</sub> , g <sub>2</sub> } → e <sub>1,2</sub>	{e <sub>1,2</sub> , g <sub>3</sub> } → 0	{e <sub>1,2</sub> , e <sub>1,2</sub> } → 0	
{e <sub>1,3</sub> , g <sub>1</sub> } → -e <sub>1,3</sub>	{e <sub>1,3</sub> , g <sub>2</sub> } → 0	{e <sub>1,3</sub> , g <sub>3</sub> } → e <sub>1,3</sub>	{e <sub>1,3</sub> , e <sub>1,2</sub> } → 0	
{e <sub>2,3</sub> , g <sub>1</sub> } → 0	{e <sub>2,3</sub> , g <sub>2</sub> } → -e <sub>2,3</sub>	{e <sub>2,3</sub> , g <sub>3</sub> } → e <sub>2,3</sub>	{e <sub>2,3</sub> , e <sub>1,2</sub> } → -e <sub>1,3</sub>	
{h <sub>1</sub> , g <sub>1</sub> } → 0	{h <sub>1</sub> , g <sub>2</sub> } → 0	{h <sub>1</sub> , g <sub>3</sub> } → 0	{h <sub>1</sub> , e <sub>1,2</sub> } → e e <sub>1,2</sub>	{
{h <sub>2</sub> , g <sub>1</sub> } → 0	{h <sub>2</sub> , g <sub>2</sub> } → 0	{h <sub>2</sub> , g <sub>3</sub> } → 0	{h <sub>2</sub> , e <sub>1,2</sub> } → -e e <sub>1,2</sub>	
{h <sub>3</sub> , g <sub>1</sub> } → 0	{h <sub>3</sub> , g <sub>2</sub> } → 0	{h <sub>3</sub> , g <sub>3</sub> } → 0	{h <sub>3</sub> , e <sub>1,2</sub> } → 0	{t
{f <sub>2,1</sub> , g <sub>1</sub> } → f <sub>2,1</sub>	{f <sub>2,1</sub> , g <sub>2</sub> } → -f <sub>2,1</sub>	{f <sub>2,1</sub> , g <sub>3</sub> } → 0	{f <sub>2,1</sub> , e <sub>1,2</sub> } → - $\frac{e g_1}{2} + \frac{e g_2}{2} - \frac{h_1}{2} + \frac{h_2}{2}$	{t
{f <sub>3,1</sub> , g <sub>1</sub> } → f <sub>3,1</sub>	{f <sub>3,1</sub> , g <sub>2</sub> } → 0	{f <sub>3,1</sub> , g <sub>3</sub> } → -f <sub>3,1</sub>	{f <sub>3,1</sub> , e <sub>1,2</sub> } → f <sub>3,2</sub>	{f <sub>3,1</sub> , e <sub>1</sub>
{f <sub>3,2</sub> , g <sub>1</sub> } → 0	{f <sub>3,2</sub> , g <sub>2</sub> } → f <sub>3,2</sub>	{f <sub>3,2</sub> , g <sub>3</sub> } → -f <sub>3,2</sub>	{f <sub>3,2</sub> , e <sub>1,2</sub> } → 0	{f.

n = 4;

**Union**@**Table**[

{x, y} = t; B[x, y] + B[y, x],

{t, Tuples[Basis[n], 2]}

]

{}

n = 3;

**DeleteCases**[**Flatten**@**Table**[

{x, y, z} → P[B[x, y], z] + P[y, B[x, z]],

{x, Basis[n]}, {y, Basis[n]}, {z, Basis[n]}

], \_ → 0]

{}

n = 3;

**DeleteCases**[**Table**[

{(x, y, z) = t} → B[x, B[y, z]] + B[y, B[z, x]] + B[z, B[x, y]],

{t, Tuples[Basis[n], 3]}

], \_ → 0]

{}

## The Derived Series (DS).

```
DS[n_, k_, 1] := DS[n, k, 1] = Flatten[Table[εj Basis[n], {j, 0, k}]];
DS[n_, k_, p_] /; p > 1 := DS[n, k, p] = DeleteCases[
  Union@Flatten@Outer[B, DS[n, k, p - 1], DS[n, k, p - 1]] /. εj /. j > k => 0, 0]
```

**DS[2, 1, 1]**

{g<sub>1</sub>, g<sub>2</sub>, e<sub>1,2</sub>, h<sub>1</sub>, h<sub>2</sub>, f<sub>2,1</sub>, ε g<sub>1</sub>, ε g<sub>2</sub>, ε e<sub>1,2</sub>, ε h<sub>1</sub>, ε h<sub>2</sub>, ε f<sub>2,1</sub>}

**DS[2, 1, 2]**

{ $\frac{\epsilon g_1}{2} - \frac{\epsilon g_2}{2} + \frac{h_1}{2} - \frac{h_2}{2}$ ,  $-\frac{\epsilon g_1}{2} + \frac{\epsilon g_2}{2} - \frac{h_1}{2} + \frac{h_2}{2}$ ,  $\frac{\epsilon h_1}{2} - \frac{\epsilon h_2}{2}$ ,  
 $-\frac{\epsilon h_1}{2} + \frac{\epsilon h_2}{2}$ , -e<sub>1,2</sub>, e<sub>1,2</sub>, -ε e<sub>1,2</sub>, ε e<sub>1,2</sub>, -f<sub>2,1</sub>, f<sub>2,1</sub>, -ε f<sub>2,1</sub>, ε f<sub>2,1</sub>}

**DS[2, 1, 3]**

{ $\frac{\epsilon g_1}{2} - \frac{\epsilon g_2}{2} + \frac{h_1}{2} - \frac{h_2}{2}$ ,  $-\frac{\epsilon g_1}{2} + \frac{\epsilon g_2}{2} - \frac{h_1}{2} + \frac{h_2}{2}$ ,  
 $\frac{\epsilon h_1}{2} - \frac{\epsilon h_2}{2}$ ,  $-\frac{\epsilon h_1}{2} + \frac{\epsilon h_2}{2}$ , -2 ε e<sub>1,2</sub>, 2 ε e<sub>1,2</sub>, -2 ε f<sub>2,1</sub>, 2 ε f<sub>2,1</sub>}

**DS[2, 1, 4]**

{}

**DS[2, 2, 1]**

{g<sub>1</sub>, g<sub>2</sub>, e<sub>1,2</sub>, h<sub>1</sub>, h<sub>2</sub>, f<sub>2,1</sub>, ε g<sub>1</sub>, ε g<sub>2</sub>, ε e<sub>1,2</sub>,  
 ε h<sub>1</sub>, ε h<sub>2</sub>, ε f<sub>2,1</sub>, ε<sup>2</sup> g<sub>1</sub>, ε<sup>2</sup> g<sub>2</sub>, ε<sup>2</sup> e<sub>1,2</sub>, ε<sup>2</sup> h<sub>1</sub>, ε<sup>2</sup> h<sub>2</sub>, ε<sup>2</sup> f<sub>2,1</sub>}

**DS[2, 2, 2]**

{ $\frac{\epsilon g_1}{2} - \frac{\epsilon g_2}{2} + \frac{h_1}{2} - \frac{h_2}{2}$ ,  $-\frac{\epsilon g_1}{2} + \frac{\epsilon g_2}{2} - \frac{h_1}{2} + \frac{h_2}{2}$ ,  $\frac{\epsilon^2 g_1}{2} - \frac{\epsilon^2 g_2}{2} + \frac{\epsilon h_1}{2} - \frac{\epsilon h_2}{2}$ ,  
 $-\frac{1}{2} \epsilon^2 g_1 + \frac{\epsilon^2 g_2}{2} - \frac{\epsilon h_1}{2} + \frac{\epsilon h_2}{2}$ ,  $\frac{\epsilon^2 h_1}{2} - \frac{\epsilon^2 h_2}{2}$ ,  $-\frac{1}{2} \epsilon^2 h_1 + \frac{\epsilon^2 h_2}{2}$ , -e<sub>1,2</sub>, e<sub>1,2</sub>,  
 -ε e<sub>1,2</sub>, ε e<sub>1,2</sub>, -ε<sup>2</sup> e<sub>1,2</sub>, ε<sup>2</sup> e<sub>1,2</sub>, -f<sub>2,1</sub>, f<sub>2,1</sub>, -ε f<sub>2,1</sub>, ε f<sub>2,1</sub>, -ε<sup>2</sup> f<sub>2,1</sub>, ε<sup>2</sup> f<sub>2,1</sub>}

**DS[2, 2, 3]**

{ $\frac{\epsilon g_1}{2} - \frac{\epsilon g_2}{2} + \frac{h_1}{2} - \frac{h_2}{2}$ ,  $-\frac{\epsilon g_1}{2} + \frac{\epsilon g_2}{2} - \frac{h_1}{2} + \frac{h_2}{2}$ ,  $\frac{\epsilon^2 g_1}{2} - \frac{\epsilon^2 g_2}{2} + \frac{\epsilon h_1}{2} - \frac{\epsilon h_2}{2}$ ,  
 $-\frac{1}{2} \epsilon^2 g_1 + \frac{\epsilon^2 g_2}{2} - \frac{\epsilon h_1}{2} + \frac{\epsilon h_2}{2}$ ,  $\frac{\epsilon^2 h_1}{2} - \frac{\epsilon^2 h_2}{2}$ ,  $-\frac{1}{2} \epsilon^2 h_1 + \frac{\epsilon^2 h_2}{2}$ , -2 ε e<sub>1,2</sub>,  
 2 ε e<sub>1,2</sub>, -2 ε<sup>2</sup> e<sub>1,2</sub>, 2 ε<sup>2</sup> e<sub>1,2</sub>, -2 ε f<sub>2,1</sub>, 2 ε f<sub>2,1</sub>, -2 ε<sup>2</sup> f<sub>2,1</sub>, 2 ε<sup>2</sup> f<sub>2,1</sub>}

**DS[2, 2, 4]**

{2 ε<sup>2</sup> h<sub>1</sub> - 2 ε<sup>2</sup> h<sub>2</sub>, -2 ε<sup>2</sup> h<sub>1</sub> + 2 ε<sup>2</sup> h<sub>2</sub>, -4 ε<sup>2</sup> e<sub>1,2</sub>, 4 ε<sup>2</sup> e<sub>1,2</sub>, -4 ε<sup>2</sup> f<sub>2,1</sub>, 4 ε<sup>2</sup> f<sub>2,1</sub>}

**DS[2, 2, 5]**

{}

**DS[2, 3, 1]**

$$\{g_1, g_2, e_{1,2}, h_1, h_2, f_{2,1}, \epsilon g_1, \epsilon g_2, \epsilon e_{1,2}, \epsilon h_1, \epsilon h_2, \epsilon f_{2,1}, \epsilon^2 g_1, \epsilon^2 g_2, \epsilon^2 e_{1,2}, \epsilon^2 h_1, \epsilon^2 h_2, \epsilon^2 f_{2,1}, \epsilon^3 g_1, \epsilon^3 g_2, \epsilon^3 e_{1,2}, \epsilon^3 h_1, \epsilon^3 h_2, \epsilon^3 f_{2,1}\}$$
**DS[2, 3, 2]**

$$\left\{ \frac{\epsilon g_1}{2} - \frac{\epsilon g_2}{2} + \frac{h_1}{2} - \frac{h_2}{2}, -\frac{\epsilon g_1}{2} + \frac{\epsilon g_2}{2} - \frac{h_1}{2} + \frac{h_2}{2}, \frac{\epsilon^2 g_1}{2} - \frac{\epsilon^2 g_2}{2} + \frac{\epsilon h_1}{2} - \frac{\epsilon h_2}{2}, \right. \\ \left. -\frac{1}{2} \epsilon^2 g_1 + \frac{\epsilon^2 g_2}{2} - \frac{\epsilon h_1}{2} + \frac{\epsilon h_2}{2}, \frac{\epsilon^3 g_1}{2} - \frac{\epsilon^3 g_2}{2} + \frac{\epsilon^2 h_1}{2} - \frac{\epsilon^2 h_2}{2}, -\frac{1}{2} \epsilon^3 g_1 + \frac{\epsilon^3 g_2}{2} - \frac{\epsilon^2 h_1}{2} + \frac{\epsilon^2 h_2}{2}, \right. \\ \left. \frac{\epsilon^3 h_1}{2} - \frac{\epsilon^3 h_2}{2}, -\frac{1}{2} \epsilon^3 h_1 + \frac{\epsilon^3 h_2}{2}, -e_{1,2}, e_{1,2}, -\epsilon e_{1,2}, \epsilon e_{1,2}, -\epsilon^2 e_{1,2}, \epsilon^2 e_{1,2}, \right. \\ \left. -\epsilon^3 e_{1,2}, \epsilon^3 e_{1,2}, -f_{2,1}, f_{2,1}, -\epsilon f_{2,1}, \epsilon f_{2,1}, -\epsilon^2 f_{2,1}, \epsilon^2 f_{2,1}, -\epsilon^3 f_{2,1}, \epsilon^3 f_{2,1} \right\}$$
**DS[2, 3, 3]**

$$\left\{ \frac{\epsilon g_1}{2} - \frac{\epsilon g_2}{2} + \frac{h_1}{2} - \frac{h_2}{2}, -\frac{\epsilon g_1}{2} + \frac{\epsilon g_2}{2} - \frac{h_1}{2} + \frac{h_2}{2}, \frac{\epsilon^2 g_1}{2} - \frac{\epsilon^2 g_2}{2} + \frac{\epsilon h_1}{2} - \frac{\epsilon h_2}{2}, \right. \\ \left. -\frac{1}{2} \epsilon^2 g_1 + \frac{\epsilon^2 g_2}{2} - \frac{\epsilon h_1}{2} + \frac{\epsilon h_2}{2}, \frac{\epsilon^3 g_1}{2} - \frac{\epsilon^3 g_2}{2} + \frac{\epsilon^2 h_1}{2} - \frac{\epsilon^2 h_2}{2}, -\frac{1}{2} \epsilon^3 g_1 + \frac{\epsilon^3 g_2}{2} - \frac{\epsilon^2 h_1}{2} + \frac{\epsilon^2 h_2}{2}, \right. \\ \left. \frac{\epsilon^3 h_1}{2} - \frac{\epsilon^3 h_2}{2}, -\frac{1}{2} \epsilon^3 h_1 + \frac{\epsilon^3 h_2}{2}, -2 \epsilon e_{1,2}, 2 \epsilon e_{1,2}, -2 \epsilon^2 e_{1,2}, 2 \epsilon^2 e_{1,2}, \right. \\ \left. -2 \epsilon^3 e_{1,2}, 2 \epsilon^3 e_{1,2}, -2 \epsilon f_{2,1}, 2 \epsilon f_{2,1}, -2 \epsilon^2 f_{2,1}, 2 \epsilon^2 f_{2,1}, -2 \epsilon^3 f_{2,1}, 2 \epsilon^3 f_{2,1} \right\}$$
**DS[2, 3, 4]**

$$\{2 \epsilon^3 g_1 - 2 \epsilon^3 g_2 + 2 \epsilon^2 h_1 - 2 \epsilon^2 h_2, -2 \epsilon^3 g_1 + 2 \epsilon^3 g_2 - 2 \epsilon^2 h_1 + 2 \epsilon^2 h_2, 2 \epsilon^3 h_1 - 2 \epsilon^3 h_2, \\ -2 \epsilon^3 h_1 + 2 \epsilon^3 h_2, -4 \epsilon^2 e_{1,2}, 4 \epsilon^2 e_{1,2}, -4 \epsilon^3 e_{1,2}, 4 \epsilon^3 e_{1,2}, -4 \epsilon^2 f_{2,1}, 4 \epsilon^2 f_{2,1}, -4 \epsilon^3 f_{2,1}, 4 \epsilon^3 f_{2,1}\}$$
**DS[2, 3, 5]**

{}

**DS[2, 10, 7]**

{}

**DS[3, 1, 1]**

$$\{g_1, g_2, g_3, e_{1,2}, e_{1,3}, e_{2,3}, h_1, h_2, h_3, f_{2,1}, f_{3,1}, f_{3,2}, \epsilon g_1, \epsilon g_2, \epsilon g_3, \epsilon e_{1,2}, \epsilon e_{1,3}, \epsilon e_{2,3}, \epsilon h_1, \epsilon h_2, \epsilon h_3, \epsilon f_{2,1}, \epsilon f_{3,1}, \epsilon f_{3,2}\}$$

**DS[3, 1, 2]**

$$\left\{ \frac{\epsilon g_1}{2} - \frac{\epsilon g_2}{2} + \frac{h_1}{2} - \frac{h_2}{2}, -\frac{\epsilon g_1}{2} + \frac{\epsilon g_2}{2} - \frac{h_1}{2} + \frac{h_2}{2}, \frac{\epsilon h_1}{2} - \frac{\epsilon h_2}{2}, -\frac{\epsilon h_1}{2} + \frac{\epsilon h_2}{2}, \frac{\epsilon g_1}{2} - \frac{\epsilon g_3}{2} + \frac{h_1}{2} - \frac{h_3}{2}, \right. \\ \left. \frac{\epsilon g_2}{2} - \frac{\epsilon g_3}{2} + \frac{h_2}{2} - \frac{h_3}{2}, -\frac{\epsilon g_1}{2} + \frac{\epsilon g_3}{2} - \frac{h_1}{2} + \frac{h_3}{2}, -\frac{\epsilon g_2}{2} + \frac{\epsilon g_3}{2} - \frac{h_2}{2} + \frac{h_3}{2}, \frac{\epsilon h_1}{2} - \frac{\epsilon h_3}{2}, \frac{\epsilon h_2}{2} - \frac{\epsilon h_3}{2}, \right. \\ \left. -\frac{\epsilon h_1}{2} + \frac{\epsilon h_3}{2}, -\frac{\epsilon h_2}{2} + \frac{\epsilon h_3}{2}, -e_{1,2}, e_{1,2}, -e_{1,2}, e_{1,2}, -e_{1,3}, e_{1,3}, -e_{1,3}, e_{1,3}, -e_{2,3}, e_{2,3}, \right. \\ \left. -e_{2,3}, e_{2,3}, -f_{2,1}, f_{2,1}, -f_{2,1}, e_{2,1}, -f_{3,1}, f_{3,1}, -f_{3,1}, e_{3,1}, -f_{3,2}, f_{3,2}, -e_{3,2}, e_{3,2} \right\}$$

**DS[3, 1, 3]**

$$\left\{ \frac{\epsilon g_1}{2} - \frac{\epsilon g_2}{2} + \frac{h_1}{2} - \frac{h_2}{2}, -\frac{\epsilon g_1}{2} + \frac{\epsilon g_2}{2} - \frac{h_1}{2} + \frac{h_2}{2}, \frac{\epsilon h_1}{2} - \frac{\epsilon h_2}{2}, -\frac{\epsilon h_1}{2} + \frac{\epsilon h_2}{2}, \frac{\epsilon g_1}{2} - \frac{\epsilon g_3}{2} + \frac{h_1}{2} - \frac{h_3}{2}, \right. \\ \left. \frac{\epsilon g_2}{2} - \frac{\epsilon g_3}{2} + \frac{h_2}{2} - \frac{h_3}{2}, -\frac{\epsilon g_1}{2} + \frac{\epsilon g_3}{2} - \frac{h_1}{2} + \frac{h_3}{2}, -\frac{\epsilon g_2}{2} + \frac{\epsilon g_3}{2} - \frac{h_2}{2} + \frac{h_3}{2}, \frac{\epsilon h_1}{2} - \frac{\epsilon h_3}{2}, \frac{\epsilon h_2}{2} - \frac{\epsilon h_3}{2}, \right. \\ \left. -\frac{\epsilon h_1}{2} + \frac{\epsilon h_3}{2}, -\frac{\epsilon h_2}{2} + \frac{\epsilon h_3}{2}, -2e_{1,2}, -e_{1,2}, e_{1,2}, 2e_{1,2}, -e_{1,3}, e_{1,3}, -2e_{1,3}, -e_{1,3}, \right. \\ \left. e_{1,3}, 2e_{1,3}, -2e_{2,3}, -e_{2,3}, e_{2,3}, 2e_{2,3}, -f_{2,1}, f_{2,1}, -2e_{2,1}, -e_{2,1}, e_{2,1}, \right. \\ \left. 2e_{2,1}, -2e_{3,1}, -e_{3,1}, e_{3,1}, 2e_{3,1}, -f_{3,2}, f_{3,2}, -2e_{3,2}, -e_{3,2}, e_{3,2}, 2e_{3,2} \right\}$$

**DS[3, 1, 4]**

$$\left\{ \epsilon h_1 - \epsilon h_2, \frac{\epsilon h_1}{2} - \frac{\epsilon h_2}{2}, -\frac{\epsilon h_1}{2} + \frac{\epsilon h_2}{2}, -\epsilon h_1 + \epsilon h_2, \epsilon h_1 - \epsilon h_3, \epsilon h_2 - \epsilon h_3, \right. \\ \left. \frac{\epsilon h_1}{2} - \frac{\epsilon h_3}{2}, \frac{\epsilon h_2}{2} - \frac{\epsilon h_3}{2}, -\frac{\epsilon h_1}{2} + \frac{\epsilon h_3}{2}, -\frac{\epsilon h_2}{2} + \frac{\epsilon h_3}{2}, -\epsilon h_1 + \epsilon h_3, -\epsilon h_2 + \epsilon h_3, \right. \\ \left. -e_{1,2}, e_{1,2}, -2e_{1,3}, -e_{1,3}, e_{1,3}, 2e_{1,3}, -e_{2,3}, e_{2,3}, -2e_{2,1}, \right. \\ \left. -e_{2,1}, e_{2,1}, 2e_{2,1}, -e_{3,1}, e_{3,1}, -2e_{3,2}, -e_{3,2}, e_{3,2}, 2e_{3,2} \right\}$$

**DS[3, 1, 5]**

{}

**DS[5, 1, 5]**

$$\left\{ \frac{\epsilon h_1}{2} - \frac{\epsilon h_2}{2}, -\frac{\epsilon h_1}{2} + \frac{\epsilon h_2}{2}, \frac{\epsilon h_2}{2} - \frac{\epsilon h_3}{2}, -\frac{\epsilon h_2}{2} + \frac{\epsilon h_3}{2}, \frac{\epsilon h_3}{2} - \frac{\epsilon h_4}{2}, \right. \\ \left. -\frac{\epsilon h_3}{2} + \frac{\epsilon h_4}{2}, \frac{\epsilon h_1}{2} - \frac{\epsilon h_5}{2}, \frac{\epsilon h_4}{2} - \frac{\epsilon h_5}{2}, -\frac{\epsilon h_1}{2} + \frac{\epsilon h_5}{2}, -\frac{\epsilon h_4}{2} + \frac{\epsilon h_5}{2}, -e_{1,4}, \right. \\ \left. e_{1,4}, -e_{1,5}, e_{1,5}, -e_{2,5}, e_{2,5}, -e_{2,1}, e_{2,1}, -e_{3,1}, e_{3,1}, \right. \\ \left. -e_{3,2}, e_{3,2}, -e_{4,2}, e_{4,2}, -e_{4,3}, e_{4,3}, -e_{5,3}, e_{5,3}, -e_{5,4}, e_{5,4} \right\}$$

**DS[5, 1, 6]**

{}

**DS[5, 2, 6]**

{}

DS[5, 3, 6]

$$\left\{ \begin{aligned} & \epsilon^3 h_1 - \epsilon^3 h_2, \frac{\epsilon^3 h_1}{2} - \frac{\epsilon^3 h_2}{2}, -\frac{1}{2} \epsilon^3 h_1 + \frac{\epsilon^3 h_2}{2}, -\epsilon^3 h_1 + \epsilon^3 h_2, 2 \epsilon^3 h_1 - 2 \epsilon^3 h_3, \epsilon^3 h_1 - \epsilon^3 h_3, \\ & \epsilon^3 h_2 - \epsilon^3 h_3, \frac{\epsilon^3 h_1}{2} - \frac{\epsilon^3 h_3}{2}, \frac{\epsilon^3 h_2}{2} - \frac{\epsilon^3 h_3}{2}, -\frac{1}{2} \epsilon^3 h_1 + \frac{\epsilon^3 h_3}{2}, -\frac{1}{2} \epsilon^3 h_2 + \frac{\epsilon^3 h_3}{2}, -\epsilon^3 h_1 + \epsilon^3 h_3, \\ & -\epsilon^3 h_2 + \epsilon^3 h_3, -2 \epsilon^3 h_1 + 2 \epsilon^3 h_3, 2 \epsilon^3 h_1 - 2 \epsilon^3 h_4, 2 \epsilon^3 h_2 - 2 \epsilon^3 h_4, \epsilon^3 h_1 - \epsilon^3 h_4, \epsilon^3 h_2 - \epsilon^3 h_4, \\ & \epsilon^3 h_3 - \epsilon^3 h_4, \frac{\epsilon^3 h_1}{2} - \frac{\epsilon^3 h_4}{2}, \frac{\epsilon^3 h_2}{2} - \frac{\epsilon^3 h_4}{2}, \frac{\epsilon^3 h_3}{2} - \frac{\epsilon^3 h_4}{2}, -\frac{1}{2} \epsilon^3 h_1 + \frac{\epsilon^3 h_4}{2}, -\frac{1}{2} \epsilon^3 h_2 + \frac{\epsilon^3 h_4}{2}, \\ & -\frac{1}{2} \epsilon^3 h_3 + \frac{\epsilon^3 h_4}{2}, -\epsilon^3 h_1 + \epsilon^3 h_4, -\epsilon^3 h_2 + \epsilon^3 h_4, -\epsilon^3 h_3 + \epsilon^3 h_4, -2 \epsilon^3 h_1 + 2 \epsilon^3 h_4, \\ & -2 \epsilon^3 h_2 + 2 \epsilon^3 h_4, 2 \epsilon^3 h_2 - 2 \epsilon^3 h_5, 2 \epsilon^3 h_3 - 2 \epsilon^3 h_5, \epsilon^3 h_1 - \epsilon^3 h_5, \epsilon^3 h_2 - \epsilon^3 h_5, \epsilon^3 h_3 - \epsilon^3 h_5, \\ & \epsilon^3 h_4 - \epsilon^3 h_5, \frac{\epsilon^3 h_1}{2} - \frac{\epsilon^3 h_5}{2}, \frac{\epsilon^3 h_2}{2} - \frac{\epsilon^3 h_5}{2}, \frac{\epsilon^3 h_3}{2} - \frac{\epsilon^3 h_5}{2}, \frac{\epsilon^3 h_4}{2} - \frac{\epsilon^3 h_5}{2}, -\frac{1}{2} \epsilon^3 h_1 + \frac{\epsilon^3 h_5}{2}, \\ & -\frac{1}{2} \epsilon^3 h_2 + \frac{\epsilon^3 h_5}{2}, -\frac{1}{2} \epsilon^3 h_3 + \frac{\epsilon^3 h_5}{2}, -\frac{1}{2} \epsilon^3 h_4 + \frac{\epsilon^3 h_5}{2}, -\epsilon^3 h_1 + \epsilon^3 h_5, -\epsilon^3 h_2 + \epsilon^3 h_5, \\ & -\epsilon^3 h_3 + \epsilon^3 h_5, -\epsilon^3 h_4 + \epsilon^3 h_5, -2 \epsilon^3 h_2 + 2 \epsilon^3 h_5, -2 \epsilon^3 h_3 + 2 \epsilon^3 h_5, -\epsilon^3 e_{1,2}, \epsilon^3 e_{1,2}, \\ & -\epsilon^3 e_{1,3}, \epsilon^3 e_{1,3}, -\epsilon^3 e_{1,4}, \epsilon^3 e_{1,4}, -2 \epsilon^3 e_{1,5}, -\epsilon^3 e_{1,5}, \epsilon^3 e_{1,5}, 2 \epsilon^3 e_{1,5}, -\epsilon^3 e_{2,3}, \epsilon^3 e_{2,3}, \\ & -\epsilon^3 e_{2,4}, \epsilon^3 e_{2,4}, -\epsilon^3 e_{2,5}, \epsilon^3 e_{2,5}, -\epsilon^3 e_{3,4}, \epsilon^3 e_{3,4}, -\epsilon^3 e_{3,5}, \epsilon^3 e_{3,5}, -\epsilon^3 e_{4,5}, \epsilon^3 e_{4,5}, \\ & -2 \epsilon^3 f_{2,1}, -\epsilon^3 f_{2,1}, \epsilon^3 f_{2,1}, 2 \epsilon^3 f_{2,1}, -\epsilon^3 f_{3,1}, \epsilon^3 f_{3,1}, -2 \epsilon^3 f_{3,2}, -\epsilon^3 f_{3,2}, \epsilon^3 f_{3,2}, \\ & 2 \epsilon^3 f_{3,2}, -\epsilon^3 f_{4,1}, \epsilon^3 f_{4,1}, -\epsilon^3 f_{4,2}, \epsilon^3 f_{4,2}, -2 \epsilon^3 f_{4,3}, -\epsilon^3 f_{4,3}, \epsilon^3 f_{4,3}, 2 \epsilon^3 f_{4,3}, \\ & -\epsilon^3 f_{5,1}, \epsilon^3 f_{5,1}, -\epsilon^3 f_{5,2}, \epsilon^3 f_{5,2}, -\epsilon^3 f_{5,3}, \epsilon^3 f_{5,3}, -2 \epsilon^3 f_{5,4}, -\epsilon^3 f_{5,4}, \epsilon^3 f_{5,4}, 2 \epsilon^3 f_{5,4} \end{aligned} \right\}$$

DS[5, 3, 7]

{}

## The LCS of the Commutator Algebra (LCSCA).

```
LCSCA[n_, k_, 1] := LCSCA[n, k, 1] = DS[n, k, 2];
LCSCA[n_, k_, p_] /; p > 1 := LCSCA[n, k, p] = DeleteCases[
  Union@Flatten@Outer[B, LCSCA[n, k, p - 1], LCSCA[n, k, 1]] /.  $\epsilon^{j-}$  /; j > k :> 0, 0];
```

LCSCA[2, 1, 1]

$$\left\{ \begin{aligned} & \frac{\epsilon g_1}{2} - \frac{\epsilon g_2}{2} + \frac{h_1}{2} - \frac{h_2}{2}, -\frac{\epsilon g_1}{2} + \frac{\epsilon g_2}{2} - \frac{h_1}{2} + \frac{h_2}{2}, \frac{\epsilon h_1}{2} - \frac{\epsilon h_2}{2}, \\ & -\frac{\epsilon h_1}{2} + \frac{\epsilon h_2}{2}, -e_{1,2}, e_{1,2}, -\epsilon e_{1,2}, \epsilon e_{1,2}, -f_{2,1}, f_{2,1}, -\epsilon f_{2,1}, \epsilon f_{2,1} \end{aligned} \right\}$$

LCSCA[2, 1, 2]

$$\left\{ \begin{aligned} & \frac{\epsilon g_1}{2} - \frac{\epsilon g_2}{2} + \frac{h_1}{2} - \frac{h_2}{2}, -\frac{\epsilon g_1}{2} + \frac{\epsilon g_2}{2} - \frac{h_1}{2} + \frac{h_2}{2}, \\ & \frac{\epsilon h_1}{2} - \frac{\epsilon h_2}{2}, -\frac{\epsilon h_1}{2} + \frac{\epsilon h_2}{2}, -2 \epsilon e_{1,2}, 2 \epsilon e_{1,2}, -2 \epsilon f_{2,1}, 2 \epsilon f_{2,1} \end{aligned} \right\}$$

LCSCA[2, 1, 3]

$$\{ \epsilon h_1 - \epsilon h_2, -\epsilon h_1 + \epsilon h_2, -2 \epsilon e_{1,2}, 2 \epsilon e_{1,2}, -2 \epsilon f_{2,1}, 2 \epsilon f_{2,1} \}$$

**LCSCA[2, 1, 4]**

$$\{\epsilon h_1 - \epsilon h_2, -\epsilon h_1 + \epsilon h_2\}$$

**LCSCA[2, 1, 5]**

$$\{\}$$

**LCSCA[2, 2, 1]**

$$\left\{ \frac{\epsilon g_1}{2} - \frac{\epsilon g_2}{2} + \frac{h_1}{2} - \frac{h_2}{2}, -\frac{\epsilon g_1}{2} + \frac{\epsilon g_2}{2} - \frac{h_1}{2} + \frac{h_2}{2}, \frac{\epsilon^2 g_1}{2} - \frac{\epsilon^2 g_2}{2} + \frac{\epsilon h_1}{2} - \frac{\epsilon h_2}{2}, \right. \\ \left. -\frac{1}{2} \epsilon^2 g_1 + \frac{\epsilon^2 g_2}{2} - \frac{\epsilon h_1}{2} + \frac{\epsilon h_2}{2}, \frac{\epsilon^2 h_1}{2} - \frac{\epsilon^2 h_2}{2}, -\frac{1}{2} \epsilon^2 h_1 + \frac{\epsilon^2 h_2}{2}, -e_{1,2}, e_{1,2}, \right. \\ \left. -\epsilon e_{1,2}, \epsilon e_{1,2}, -\epsilon^2 e_{1,2}, \epsilon^2 e_{1,2}, -f_{2,1}, f_{2,1}, -\epsilon f_{2,1}, \epsilon f_{2,1}, -\epsilon^2 f_{2,1}, \epsilon^2 f_{2,1} \right\}$$

**LCSCA[2, 2, 6]**

$$\{2 \epsilon^2 h_1 - 2 \epsilon^2 h_2, -2 \epsilon^2 h_1 + 2 \epsilon^2 h_2\}$$

**LCSCA[2, 2, 7]**

$$\{\}$$

**LCSCA[3, 3, 1]**

$$\left\{ \frac{\epsilon g_1}{2} - \frac{\epsilon g_2}{2} + \frac{h_1}{2} - \frac{h_2}{2}, -\frac{\epsilon g_1}{2} + \frac{\epsilon g_2}{2} - \frac{h_1}{2} + \frac{h_2}{2}, \frac{\epsilon^2 g_1}{2} - \frac{\epsilon^2 g_2}{2} + \frac{\epsilon h_1}{2} - \frac{\epsilon h_2}{2}, \right. \\ \left. -\frac{1}{2} \epsilon^2 g_1 + \frac{\epsilon^2 g_2}{2} - \frac{\epsilon h_1}{2} + \frac{\epsilon h_2}{2}, \frac{\epsilon^3 g_1}{2} - \frac{\epsilon^3 g_2}{2} + \frac{\epsilon^2 h_1}{2} - \frac{\epsilon^2 h_2}{2}, -\frac{1}{2} \epsilon^3 g_1 + \frac{\epsilon^3 g_2}{2} - \frac{\epsilon^2 h_1}{2} + \frac{\epsilon^2 h_2}{2}, \right. \\ \frac{\epsilon^3 h_1}{2} - \frac{\epsilon^3 h_2}{2}, -\frac{1}{2} \epsilon^3 h_1 + \frac{\epsilon^3 h_2}{2}, \frac{\epsilon g_1}{2} - \frac{\epsilon g_3}{2} + \frac{h_1}{2} - \frac{h_3}{2}, \frac{\epsilon g_2}{2} - \frac{\epsilon g_3}{2} + \frac{h_2}{2} - \frac{h_3}{2}, -\frac{\epsilon g_1}{2} + \frac{\epsilon g_3}{2} - \frac{h_1}{2} + \frac{h_3}{2}, \\ \left. -\frac{\epsilon g_2}{2} + \frac{\epsilon g_3}{2} - \frac{h_2}{2} + \frac{h_3}{2}, \frac{\epsilon^2 g_1}{2} - \frac{\epsilon^2 g_3}{2} + \frac{\epsilon h_1}{2} - \frac{\epsilon h_3}{2}, \frac{\epsilon^2 g_2}{2} - \frac{\epsilon^2 g_3}{2} + \frac{\epsilon h_2}{2} - \frac{\epsilon h_3}{2}, \right. \\ \left. -\frac{1}{2} \epsilon^2 g_1 + \frac{\epsilon^2 g_3}{2} - \frac{\epsilon h_1}{2} + \frac{\epsilon h_3}{2}, -\frac{1}{2} \epsilon^2 g_2 + \frac{\epsilon^2 g_3}{2} - \frac{\epsilon h_2}{2} + \frac{\epsilon h_3}{2}, \frac{\epsilon^3 g_1}{2} - \frac{\epsilon^3 g_3}{2} + \frac{\epsilon^2 h_1}{2} - \frac{\epsilon^2 h_3}{2}, \right. \\ \frac{\epsilon^3 g_2}{2} - \frac{\epsilon^3 g_3}{2} + \frac{\epsilon^2 h_2}{2} - \frac{\epsilon^2 h_3}{2}, -\frac{1}{2} \epsilon^3 g_1 + \frac{\epsilon^3 g_3}{2} - \frac{\epsilon^2 h_1}{2} + \frac{\epsilon^2 h_3}{2}, -\frac{1}{2} \epsilon^3 g_2 + \frac{\epsilon^3 g_3}{2} - \frac{\epsilon^2 h_2}{2} + \frac{\epsilon^2 h_3}{2}, \\ \left. \frac{\epsilon^3 h_1}{2} - \frac{\epsilon^3 h_3}{2}, \frac{\epsilon^3 h_2}{2} - \frac{\epsilon^3 h_3}{2}, -\frac{1}{2} \epsilon^3 h_1 + \frac{\epsilon^3 h_3}{2}, -\frac{1}{2} \epsilon^3 h_2 + \frac{\epsilon^3 h_3}{2}, -e_{1,2}, e_{1,2}, -\epsilon e_{1,2}, \right. \\ \left. \epsilon e_{1,2}, -\epsilon^2 e_{1,2}, \epsilon^2 e_{1,2}, -\epsilon^3 e_{1,2}, \epsilon^3 e_{1,2}, -e_{1,3}, e_{1,3}, -\epsilon e_{1,3}, \epsilon e_{1,3}, -\epsilon^2 e_{1,3}, \epsilon^2 e_{1,3}, \right. \\ \left. -\epsilon^3 e_{1,3}, \epsilon^3 e_{1,3}, -e_{2,3}, e_{2,3}, -\epsilon e_{2,3}, \epsilon e_{2,3}, -\epsilon^2 e_{2,3}, \epsilon^2 e_{2,3}, -\epsilon^3 e_{2,3}, \epsilon^3 e_{2,3}, -f_{2,1}, f_{2,1}, \right. \\ \left. -\epsilon f_{2,1}, \epsilon f_{2,1}, -\epsilon^2 f_{2,1}, \epsilon^2 f_{2,1}, -\epsilon^3 f_{2,1}, \epsilon^3 f_{2,1}, -f_{3,1}, f_{3,1}, -\epsilon f_{3,1}, \epsilon f_{3,1}, -\epsilon^2 f_{3,1}, \right. \\ \left. \epsilon^2 f_{3,1}, -\epsilon^3 f_{3,1}, \epsilon^3 f_{3,1}, -f_{3,2}, f_{3,2}, -\epsilon f_{3,2}, \epsilon f_{3,2}, -\epsilon^2 f_{3,2}, \epsilon^2 f_{3,2}, -\epsilon^3 f_{3,2}, \epsilon^3 f_{3,2} \right\}$$

**LCSCA[3, 3, 12]**

$$\left\{ 4 \epsilon^3 h_1 - 4 \epsilon^3 h_2, 2 \epsilon^3 h_1 - 2 \epsilon^3 h_2, \epsilon^3 h_1 - \epsilon^3 h_2, \frac{\epsilon^3 h_1}{2} - \frac{\epsilon^3 h_2}{2}, -\frac{1}{2} \epsilon^3 h_1 + \frac{\epsilon^3 h_2}{2}, \right. \\ \left. -\epsilon^3 h_1 + \epsilon^3 h_2, -2 \epsilon^3 h_1 + 2 \epsilon^3 h_2, -4 \epsilon^3 h_1 + 4 \epsilon^3 h_2, 4 \epsilon^3 h_1 - 4 \epsilon^3 h_3, 4 \epsilon^3 h_2 - 4 \epsilon^3 h_3, \right. \\ \left. 2 \epsilon^3 h_1 - 2 \epsilon^3 h_3, 2 \epsilon^3 h_2 - 2 \epsilon^3 h_3, \epsilon^3 h_1 - \epsilon^3 h_3, \epsilon^3 h_2 - \epsilon^3 h_3, \frac{\epsilon^3 h_1}{2} - \frac{\epsilon^3 h_3}{2}, \right. \\ \left. \frac{\epsilon^3 h_2}{2} - \frac{\epsilon^3 h_3}{2}, -\frac{1}{2} \epsilon^3 h_1 + \frac{\epsilon^3 h_3}{2}, -\frac{1}{2} \epsilon^3 h_2 + \frac{\epsilon^3 h_3}{2}, -\epsilon^3 h_1 + \epsilon^3 h_3, -\epsilon^3 h_2 + \epsilon^3 h_3, \right. \\ \left. -2 \epsilon^3 h_1 + 2 \epsilon^3 h_3, -2 \epsilon^3 h_2 + 2 \epsilon^3 h_3, -4 \epsilon^3 h_1 + 4 \epsilon^3 h_3, -4 \epsilon^3 h_2 + 4 \epsilon^3 h_3 \right\}$$

**LCSCA[3, 3, 13]**

{}

**LCSCA[4, 4, 1]**

$$\left\{ \frac{\epsilon g_1}{2} - \frac{\epsilon g_2}{2} + \frac{h_1}{2} - \frac{h_2}{2}, -\frac{\epsilon g_1}{2} + \frac{\epsilon g_2}{2} - \frac{h_1}{2} + \frac{h_2}{2}, \frac{\epsilon^2 g_1}{2} - \frac{\epsilon^2 g_2}{2} + \frac{\epsilon h_1}{2} - \frac{\epsilon h_2}{2}, \right. \\ \left. -\frac{1}{2} \epsilon^2 g_1 + \frac{\epsilon^2 g_2}{2} - \frac{\epsilon h_1}{2} + \frac{\epsilon h_2}{2}, \frac{\epsilon^3 g_1}{2} - \frac{\epsilon^3 g_2}{2} + \frac{\epsilon^2 h_1}{2} - \frac{\epsilon^2 h_2}{2}, -\frac{1}{2} \epsilon^3 g_1 + \frac{\epsilon^3 g_2}{2} - \frac{\epsilon^2 h_1}{2} + \frac{\epsilon^2 h_2}{2}, \right. \\ \left. \frac{\epsilon^4 g_1}{2} - \frac{\epsilon^4 g_2}{2} + \frac{\epsilon^3 h_1}{2} - \frac{\epsilon^3 h_2}{2}, -\frac{1}{2} \epsilon^4 g_1 + \frac{\epsilon^4 g_2}{2} - \frac{\epsilon^3 h_1}{2} + \frac{\epsilon^3 h_2}{2}, \frac{\epsilon^4 h_1}{2} - \frac{\epsilon^4 h_2}{2}, -\frac{1}{2} \epsilon^4 h_1 + \frac{\epsilon^4 h_2}{2}, \right. \\ \left. \frac{\epsilon g_1}{2} - \frac{\epsilon g_3}{2} + \frac{h_1}{2} - \frac{h_3}{2}, \frac{\epsilon g_2}{2} - \frac{\epsilon g_3}{2} + \frac{h_2}{2} - \frac{h_3}{2}, -\frac{\epsilon g_1}{2} + \frac{\epsilon g_3}{2} - \frac{h_1}{2} + \frac{h_3}{2}, -\frac{\epsilon g_2}{2} + \frac{\epsilon g_3}{2} - \frac{h_2}{2} + \frac{h_3}{2}, \right. \\ \left. \frac{\epsilon^2 g_1}{2} - \frac{\epsilon^2 g_3}{2} + \frac{\epsilon h_1}{2} - \frac{\epsilon h_3}{2}, \frac{\epsilon^2 g_2}{2} - \frac{\epsilon^2 g_3}{2} + \frac{\epsilon h_2}{2} - \frac{\epsilon h_3}{2}, -\frac{1}{2} \epsilon^2 g_1 + \frac{\epsilon^2 g_3}{2} - \frac{\epsilon h_1}{2} + \frac{\epsilon h_3}{2}, \right. \\ \left. -\frac{1}{2} \epsilon^2 g_2 + \frac{\epsilon^2 g_3}{2} - \frac{\epsilon h_2}{2} + \frac{\epsilon h_3}{2}, \frac{\epsilon^3 g_1}{2} - \frac{\epsilon^3 g_3}{2} + \frac{\epsilon^2 h_1}{2} - \frac{\epsilon^2 h_3}{2}, \frac{\epsilon^3 g_2}{2} - \frac{\epsilon^3 g_3}{2} + \frac{\epsilon^2 h_2}{2} - \frac{\epsilon^2 h_3}{2}, \right. \\ \left. -\frac{1}{2} \epsilon^3 g_1 + \frac{\epsilon^3 g_3}{2} - \frac{\epsilon^2 h_1}{2} + \frac{\epsilon^2 h_3}{2}, -\frac{1}{2} \epsilon^3 g_2 + \frac{\epsilon^3 g_3}{2} - \frac{\epsilon^2 h_2}{2} + \frac{\epsilon^2 h_3}{2}, \frac{\epsilon^4 g_1}{2} - \frac{\epsilon^4 g_3}{2} + \frac{\epsilon^3 h_1}{2} - \frac{\epsilon^3 h_3}{2}, \right. \\ \left. \frac{\epsilon^4 g_2}{2} - \frac{\epsilon^4 g_3}{2} + \frac{\epsilon^3 h_2}{2} - \frac{\epsilon^3 h_3}{2}, -\frac{1}{2} \epsilon^4 g_1 + \frac{\epsilon^4 g_3}{2} - \frac{\epsilon^3 h_1}{2} + \frac{\epsilon^3 h_3}{2}, -\frac{1}{2} \epsilon^4 g_2 + \frac{\epsilon^4 g_3}{2} - \frac{\epsilon^3 h_2}{2} + \frac{\epsilon^3 h_3}{2}, \right. \\ \left. \frac{\epsilon^4 h_1}{2} - \frac{\epsilon^4 h_3}{2}, \frac{\epsilon^4 h_2}{2} - \frac{\epsilon^4 h_3}{2}, -\frac{1}{2} \epsilon^4 h_1 + \frac{\epsilon^4 h_3}{2}, -\frac{1}{2} \epsilon^4 h_2 + \frac{\epsilon^4 h_3}{2}, \frac{\epsilon g_1}{2} - \frac{\epsilon g_4}{2} + \frac{h_1}{2} - \frac{h_4}{2}, \right. \\ \left. \frac{\epsilon g_2}{2} - \frac{\epsilon g_4}{2} + \frac{h_2}{2} - \frac{h_4}{2}, \frac{\epsilon g_3}{2} - \frac{\epsilon g_4}{2} + \frac{h_3}{2} - \frac{h_4}{2}, -\frac{\epsilon g_1}{2} + \frac{\epsilon g_4}{2} - \frac{h_1}{2} + \frac{h_4}{2}, -\frac{\epsilon g_2}{2} + \frac{\epsilon g_4}{2} - \frac{h_2}{2} + \frac{h_4}{2}, \right. \\ \left. -\frac{\epsilon g_3}{2} + \frac{\epsilon g_4}{2} - \frac{h_3}{2} + \frac{h_4}{2}, \frac{\epsilon^2 g_1}{2} - \frac{\epsilon^2 g_4}{2} + \frac{\epsilon h_1}{2} - \frac{\epsilon h_4}{2}, \frac{\epsilon^2 g_2}{2} - \frac{\epsilon^2 g_4}{2} + \frac{\epsilon h_2}{2} - \frac{\epsilon h_4}{2}, \right. \\ \left. \frac{\epsilon^2 g_3}{2} - \frac{\epsilon^2 g_4}{2} + \frac{\epsilon h_3}{2} - \frac{\epsilon h_4}{2}, -\frac{1}{2} \epsilon^2 g_1 + \frac{\epsilon^2 g_4}{2} - \frac{\epsilon h_1}{2} + \frac{\epsilon h_4}{2}, -\frac{1}{2} \epsilon^2 g_2 + \frac{\epsilon^2 g_4}{2} - \frac{\epsilon h_2}{2} + \frac{\epsilon h_4}{2}, \right. \\ \left. -\frac{1}{2} \epsilon^2 g_3 + \frac{\epsilon^2 g_4}{2} - \frac{\epsilon h_3}{2} + \frac{\epsilon h_4}{2}, \frac{\epsilon^3 g_1}{2} - \frac{\epsilon^3 g_4}{2} + \frac{\epsilon^2 h_1}{2} - \frac{\epsilon^2 h_4}{2}, \frac{\epsilon^3 g_2}{2} - \frac{\epsilon^3 g_4}{2} + \frac{\epsilon^2 h_2}{2} - \frac{\epsilon^2 h_4}{2}, \right. \\ \left. \frac{\epsilon^3 g_3}{2} - \frac{\epsilon^3 g_4}{2} + \frac{\epsilon^2 h_3}{2} - \frac{\epsilon^2 h_4}{2}, -\frac{1}{2} \epsilon^3 g_1 + \frac{\epsilon^3 g_4}{2} - \frac{\epsilon^2 h_1}{2} + \frac{\epsilon^2 h_4}{2}, -\frac{1}{2} \epsilon^3 g_2 + \frac{\epsilon^3 g_4}{2} - \frac{\epsilon^2 h_2}{2} + \frac{\epsilon^2 h_4}{2}, \right. \\ \left. -\frac{1}{2} \epsilon^3 g_3 + \frac{\epsilon^3 g_4}{2} - \frac{\epsilon^2 h_3}{2} + \frac{\epsilon^2 h_4}{2}, \frac{\epsilon^4 g_1}{2} - \frac{\epsilon^4 g_4}{2} + \frac{\epsilon^3 h_1}{2} - \frac{\epsilon^3 h_4}{2}, \frac{\epsilon^4 g_2}{2} - \frac{\epsilon^4 g_4}{2} + \frac{\epsilon^3 h_2}{2} - \frac{\epsilon^3 h_4}{2}, \right. \\ \left. -\frac{1}{2} \epsilon^4 g_1 + \frac{\epsilon^4 g_4}{2} - \frac{\epsilon^3 h_1}{2} + \frac{\epsilon^3 h_4}{2}, -\frac{1}{2} \epsilon^4 g_2 + \frac{\epsilon^4 g_4}{2} - \frac{\epsilon^3 h_2}{2} + \frac{\epsilon^3 h_4}{2}, \right. \\ \left. -\frac{1}{2} \epsilon^4 g_3 + \frac{\epsilon^4 g_4}{2} - \frac{\epsilon^3 h_3}{2} + \frac{\epsilon^3 h_4}{2}, \frac{\epsilon^4 h_1}{2} - \frac{\epsilon^4 h_4}{2}, \frac{\epsilon^4 h_2}{2} - \frac{\epsilon^4 h_4}{2}, \frac{\epsilon^4 h_3}{2} - \frac{\epsilon^4 h_4}{2} \right\}$$



$$\begin{aligned} & \frac{\epsilon^4 g_3}{2} - \frac{\epsilon^4 g_4}{2} + \frac{\epsilon^3 h_3}{2} - \frac{\epsilon^3 h_4}{2}, -\frac{1}{2} \epsilon^4 g_1 + \frac{\epsilon^4 g_4}{2} - \frac{\epsilon^3 h_1}{2} + \frac{\epsilon^3 h_4}{2}, -\frac{1}{2} \epsilon^4 g_2 + \frac{\epsilon^4 g_4}{2} - \frac{\epsilon^3 h_2}{2} + \frac{\epsilon^3 h_4}{2}, \\ & -\frac{1}{2} \epsilon^4 g_3 + \frac{\epsilon^4 g_4}{2} - \frac{\epsilon^3 h_3}{2} + \frac{\epsilon^3 h_4}{2}, \frac{\epsilon^4 h_1}{2} - \frac{\epsilon^4 h_4}{2}, \frac{\epsilon^4 h_2}{2} - \frac{\epsilon^4 h_4}{2}, \frac{\epsilon^4 h_3}{2} - \frac{\epsilon^4 h_4}{2}, -\frac{1}{2} \epsilon^4 h_1 + \frac{\epsilon^4 h_4}{2}, \\ & -\frac{1}{2} \epsilon^4 h_2 + \frac{\epsilon^4 h_4}{2}, -\frac{1}{2} \epsilon^4 h_3 + \frac{\epsilon^4 h_4}{2}, -e_{1,2}, e_{1,2}, -e_{1,2}, e_{1,2}, -e^2 e_{1,2}, e^2 e_{1,2}, -e^3 e_{1,2}, \\ & e^3 e_{1,2}, -e^4 e_{1,2}, e^4 e_{1,2}, -e_{1,3}, e_{1,3}, -e_{1,3}, e_{1,3}, -e^2 e_{1,3}, e^2 e_{1,3}, -e^3 e_{1,3}, e^3 e_{1,3}, \\ & -e^4 e_{1,3}, e^4 e_{1,3}, -e_{1,4}, e_{1,4}, -e_{1,4}, e_{1,4}, -e^2 e_{1,4}, e^2 e_{1,4}, -e^3 e_{1,4}, e^3 e_{1,4}, -e^4 e_{1,4}, \\ & e^4 e_{1,4}, -e_{2,3}, e_{2,3}, -e_{2,3}, e_{2,3}, -e^2 e_{2,3}, e^2 e_{2,3}, -e^3 e_{2,3}, e^3 e_{2,3}, -e^4 e_{2,3}, e^4 e_{2,3}, -e_{2,4}, \\ & e_{2,4}, -e_{2,4}, e_{2,4}, -e^2 e_{2,4}, e^2 e_{2,4}, -e^3 e_{2,4}, e^3 e_{2,4}, -e^4 e_{2,4}, e^4 e_{2,4}, -e_{3,4}, e_{3,4}, -e_{3,4}, \\ & e_{3,4}, -e^2 e_{3,4}, e^2 e_{3,4}, -e^3 e_{3,4}, e^3 e_{3,4}, -e^4 e_{3,4}, e^4 e_{3,4}, -f_{2,1}, f_{2,1}, -e_{2,1}, e_{2,1}, -e^2 f_{2,1}, \\ & e^2 f_{2,1}, -e^3 f_{2,1}, e^3 f_{2,1}, -e^4 f_{2,1}, e^4 f_{2,1}, -f_{3,1}, f_{3,1}, -e_{3,1}, e_{3,1}, -e^2 f_{3,1}, e^2 f_{3,1}, \\ & -e^3 f_{3,1}, e^3 f_{3,1}, -e^4 f_{3,1}, e^4 f_{3,1}, -f_{3,2}, f_{3,2}, -e_{3,2}, e_{3,2}, -e^2 f_{3,2}, e^2 f_{3,2}, -e^3 f_{3,2}, \\ & e^3 f_{3,2}, -e^4 f_{3,2}, e^4 f_{3,2}, -f_{4,1}, f_{4,1}, -e_{4,1}, e_{4,1}, -e^2 f_{4,1}, e^2 f_{4,1}, -e^3 f_{4,1}, e^3 f_{4,1}, \\ & -e^4 f_{4,1}, e^4 f_{4,1}, -f_{4,2}, f_{4,2}, -e_{4,2}, e_{4,2}, -e^2 f_{4,2}, e^2 f_{4,2}, -e^3 f_{4,2}, e^3 f_{4,2}, -e^4 f_{4,2}, \\ & e^4 f_{4,2}, -f_{4,3}, f_{4,3}, -e_{4,3}, e_{4,3}, -e^2 f_{4,3}, e^2 f_{4,3}, -e^3 f_{4,3}, e^3 f_{4,3}, -e^4 f_{4,3}, e^4 f_{4,3} \} \end{aligned}$$

**LCSCA[4, 4, 20]**

$$\begin{aligned} & \{ 8 \epsilon^4 h_1 - 8 \epsilon^4 h_2, 4 \epsilon^4 h_1 - 4 \epsilon^4 h_2, 2 \epsilon^4 h_1 - 2 \epsilon^4 h_2, \epsilon^4 h_1 - \epsilon^4 h_2, \frac{\epsilon^4 h_1}{2} - \frac{\epsilon^4 h_2}{2}, -\frac{1}{2} \epsilon^4 h_1 + \frac{\epsilon^4 h_2}{2}, \\ & -\epsilon^4 h_1 + \epsilon^4 h_2, -2 \epsilon^4 h_1 + 2 \epsilon^4 h_2, -4 \epsilon^4 h_1 + 4 \epsilon^4 h_2, -8 \epsilon^4 h_1 + 8 \epsilon^4 h_2, 8 \epsilon^4 h_2 - 8 \epsilon^4 h_3, 4 \epsilon^4 h_2 - 4 \epsilon^4 h_3, \\ & 2 \epsilon^4 h_2 - 2 \epsilon^4 h_3, \epsilon^4 h_2 - \epsilon^4 h_3, \frac{\epsilon^4 h_2}{2} - \frac{\epsilon^4 h_3}{2}, -\frac{1}{2} \epsilon^4 h_2 + \frac{\epsilon^4 h_3}{2}, -\epsilon^4 h_2 + \epsilon^4 h_3, -2 \epsilon^4 h_2 + 2 \epsilon^4 h_3, \\ & -4 \epsilon^4 h_2 + 4 \epsilon^4 h_3, -8 \epsilon^4 h_2 + 8 \epsilon^4 h_3, 8 \epsilon^4 h_3 - 8 \epsilon^4 h_4, 8 \epsilon^4 h_3 - 8 \epsilon^4 h_4, 4 \epsilon^4 h_3 - 4 \epsilon^4 h_4, \\ & 4 \epsilon^4 h_3 - 4 \epsilon^4 h_4, 2 \epsilon^4 h_3 - 2 \epsilon^4 h_4, 2 \epsilon^4 h_3 - 2 \epsilon^4 h_4, \epsilon^4 h_3 - \epsilon^4 h_4, \epsilon^4 h_3 - \epsilon^4 h_4, \frac{\epsilon^4 h_3}{2} - \frac{\epsilon^4 h_4}{2}, \\ & \frac{\epsilon^4 h_3}{2} - \frac{\epsilon^4 h_4}{2}, -\frac{1}{2} \epsilon^4 h_3 + \frac{\epsilon^4 h_4}{2}, -\frac{1}{2} \epsilon^4 h_3 + \frac{\epsilon^4 h_4}{2}, -\epsilon^4 h_3 + \epsilon^4 h_4, -\epsilon^4 h_3 + \epsilon^4 h_4, -2 \epsilon^4 h_3 + 2 \epsilon^4 h_4, \\ & -2 \epsilon^4 h_3 + 2 \epsilon^4 h_4, -4 \epsilon^4 h_3 + 4 \epsilon^4 h_4, -4 \epsilon^4 h_3 + 4 \epsilon^4 h_4, -8 \epsilon^4 h_3 + 8 \epsilon^4 h_4, -8 \epsilon^4 h_3 + 8 \epsilon^4 h_4 \} \end{aligned}$$

**LCSCA[4, 4, 21]**

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**LCSCA[5, 5, 1]**

$$\begin{aligned} & \left\{ \frac{\epsilon g_1}{2} - \frac{\epsilon g_2}{2} + \frac{h_1}{2} - \frac{h_2}{2}, -\frac{\epsilon g_1}{2} + \frac{\epsilon g_2}{2} - \frac{h_1}{2} + \frac{h_2}{2}, \frac{\epsilon^2 g_1}{2} - \frac{\epsilon^2 g_2}{2} + \frac{\epsilon h_1}{2} - \frac{\epsilon h_2}{2}, \right. \\ & -\frac{1}{2} \epsilon^2 g_1 + \frac{\epsilon^2 g_2}{2} - \frac{\epsilon h_1}{2} + \frac{\epsilon h_2}{2}, \frac{\epsilon^3 g_1}{2} - \frac{\epsilon^3 g_2}{2} + \frac{\epsilon^2 h_1}{2} - \frac{\epsilon^2 h_2}{2}, -\frac{1}{2} \epsilon^3 g_1 + \frac{\epsilon^3 g_2}{2} - \frac{\epsilon^2 h_1}{2} + \frac{\epsilon^2 h_2}{2}, \\ & \frac{\epsilon^4 g_1}{2} - \frac{\epsilon^4 g_2}{2} + \frac{\epsilon^3 h_1}{2} - \frac{\epsilon^3 h_2}{2}, -\frac{1}{2} \epsilon^4 g_1 + \frac{\epsilon^4 g_2}{2} - \frac{\epsilon^3 h_1}{2} + \frac{\epsilon^3 h_2}{2}, \frac{\epsilon^5 g_1}{2} - \frac{\epsilon^5 g_2}{2} + \frac{\epsilon^4 h_1}{2} - \frac{\epsilon^4 h_2}{2}, \\ & -\frac{1}{2} \epsilon^5 g_1 + \frac{\epsilon^5 g_2}{2} - \frac{\epsilon^4 h_1}{2} + \frac{\epsilon^4 h_2}{2}, \frac{\epsilon^5 h_1}{2} - \frac{\epsilon^5 h_2}{2}, -\frac{1}{2} \epsilon^5 h_1 + \frac{\epsilon^5 h_2}{2}, \frac{\epsilon g_1}{2} - \frac{\epsilon g_3}{2} + \frac{h_1}{2} - \frac{h_3}{2}, \\ & \frac{\epsilon g_2}{2} - \frac{\epsilon g_3}{2} + \frac{h_2}{2} - \frac{h_3}{2}, -\frac{\epsilon g_1}{2} + \frac{\epsilon g_3}{2} - \frac{h_1}{2} + \frac{h_3}{2}, -\frac{\epsilon g_2}{2} + \frac{\epsilon g_3}{2} - \frac{h_2}{2} + \frac{h_3}{2}, \frac{\epsilon^2 g_1}{2} - \frac{\epsilon^2 g_3}{2} + \frac{\epsilon h_1}{2} - \frac{\epsilon h_3}{2}, \\ & \frac{\epsilon^2 g_2}{2} - \frac{\epsilon^2 g_3}{2} + \frac{\epsilon h_2}{2} - \frac{\epsilon h_3}{2}, -\frac{1}{2} \epsilon^2 g_1 + \frac{\epsilon^2 g_3}{2} - \frac{\epsilon h_1}{2} + \frac{\epsilon h_3}{2}, -\frac{1}{2} \epsilon^2 g_2 + \frac{\epsilon^2 g_3}{2} - \frac{\epsilon h_2}{2} + \frac{\epsilon h_3}{2}, \end{aligned}$$



$$\begin{aligned}
 &-\frac{1}{2} \epsilon^3 g_2 + \frac{\epsilon^3 g_5}{2} - \frac{\epsilon^2 h_2}{2} + \frac{\epsilon^2 h_5}{2}, -\frac{1}{2} \epsilon^3 g_3 + \frac{\epsilon^3 g_5}{2} - \frac{\epsilon^2 h_3}{2} + \frac{\epsilon^2 h_5}{2}, -\frac{1}{2} \epsilon^3 g_4 + \frac{\epsilon^3 g_5}{2} - \frac{\epsilon^2 h_4}{2} + \frac{\epsilon^2 h_5}{2}, \\
 &\frac{\epsilon^4 g_1}{2} - \frac{\epsilon^4 g_5}{2} + \frac{\epsilon^3 h_1}{2} - \frac{\epsilon^3 h_5}{2}, \frac{\epsilon^4 g_2}{2} - \frac{\epsilon^4 g_5}{2} + \frac{\epsilon^3 h_2}{2} - \frac{\epsilon^3 h_5}{2}, \frac{\epsilon^4 g_3}{2} - \frac{\epsilon^4 g_5}{2} + \frac{\epsilon^3 h_3}{2} - \frac{\epsilon^3 h_5}{2}, \\
 &\frac{\epsilon^4 g_4}{2} - \frac{\epsilon^4 g_5}{2} + \frac{\epsilon^3 h_4}{2} - \frac{\epsilon^3 h_5}{2}, -\frac{1}{2} \epsilon^4 g_1 + \frac{\epsilon^4 g_5}{2} - \frac{\epsilon^3 h_1}{2} + \frac{\epsilon^3 h_5}{2}, -\frac{1}{2} \epsilon^4 g_2 + \frac{\epsilon^4 g_5}{2} - \frac{\epsilon^3 h_2}{2} + \frac{\epsilon^3 h_5}{2}, \\
 &-\frac{1}{2} \epsilon^4 g_3 + \frac{\epsilon^4 g_5}{2} - \frac{\epsilon^3 h_3}{2} + \frac{\epsilon^3 h_5}{2}, -\frac{1}{2} \epsilon^4 g_4 + \frac{\epsilon^4 g_5}{2} - \frac{\epsilon^3 h_4}{2} + \frac{\epsilon^3 h_5}{2}, \frac{\epsilon^5 g_1}{2} - \frac{\epsilon^5 g_5}{2} + \frac{\epsilon^4 h_1}{2} - \frac{\epsilon^4 h_5}{2}, \\
 &\frac{\epsilon^5 g_2}{2} - \frac{\epsilon^5 g_5}{2} + \frac{\epsilon^4 h_2}{2} - \frac{\epsilon^4 h_5}{2}, \frac{\epsilon^5 g_3}{2} - \frac{\epsilon^5 g_5}{2} + \frac{\epsilon^4 h_3}{2} - \frac{\epsilon^4 h_5}{2}, \frac{\epsilon^5 g_4}{2} - \frac{\epsilon^5 g_5}{2} + \frac{\epsilon^4 h_4}{2} - \frac{\epsilon^4 h_5}{2}, \\
 &-\frac{1}{2} \epsilon^5 g_1 + \frac{\epsilon^5 g_5}{2} - \frac{\epsilon^4 h_1}{2} + \frac{\epsilon^4 h_5}{2}, -\frac{1}{2} \epsilon^5 g_2 + \frac{\epsilon^5 g_5}{2} - \frac{\epsilon^4 h_2}{2} + \frac{\epsilon^4 h_5}{2}, -\frac{1}{2} \epsilon^5 g_3 + \frac{\epsilon^5 g_5}{2} - \frac{\epsilon^4 h_3}{2} + \frac{\epsilon^4 h_5}{2}, \\
 &-\frac{1}{2} \epsilon^5 g_4 + \frac{\epsilon^5 g_5}{2} - \frac{\epsilon^4 h_4}{2} + \frac{\epsilon^4 h_5}{2}, \frac{\epsilon^5 h_1}{2} - \frac{\epsilon^5 h_5}{2}, \frac{\epsilon^5 h_2}{2} - \frac{\epsilon^5 h_5}{2}, \frac{\epsilon^5 h_3}{2} - \frac{\epsilon^5 h_5}{2}, \frac{\epsilon^5 h_4}{2} - \frac{\epsilon^5 h_5}{2}, \\
 &-\frac{1}{2} \epsilon^5 h_1 + \frac{\epsilon^5 h_5}{2}, -\frac{1}{2} \epsilon^5 h_2 + \frac{\epsilon^5 h_5}{2}, -\frac{1}{2} \epsilon^5 h_3 + \frac{\epsilon^5 h_5}{2}, -\frac{1}{2} \epsilon^5 h_4 + \frac{\epsilon^5 h_5}{2}, -e_{1,2}, e_{1,2}, -e_{1,2}, \\
 &\in e_{1,2}, -e^2 e_{1,2}, e^2 e_{1,2}, -e^3 e_{1,2}, e^3 e_{1,2}, -e^4 e_{1,2}, e^4 e_{1,2}, -e^5 e_{1,2}, e^5 e_{1,2}, -e_{1,3}, e_{1,3}, -e_{1,3}, \\
 &\in e_{1,3}, -e^2 e_{1,3}, e^2 e_{1,3}, -e^3 e_{1,3}, e^3 e_{1,3}, -e^4 e_{1,3}, e^4 e_{1,3}, -e^5 e_{1,3}, e^5 e_{1,3}, -e_{1,4}, e_{1,4}, -e_{1,4}, \\
 &\in e_{1,4}, -e^2 e_{1,4}, e^2 e_{1,4}, -e^3 e_{1,4}, e^3 e_{1,4}, -e^4 e_{1,4}, e^4 e_{1,4}, -e^5 e_{1,4}, e^5 e_{1,4}, -e_{1,5}, e_{1,5}, -e_{1,5}, \\
 &\in e_{1,5}, -e^2 e_{1,5}, e^2 e_{1,5}, -e^3 e_{1,5}, e^3 e_{1,5}, -e^4 e_{1,5}, e^4 e_{1,5}, -e^5 e_{1,5}, e^5 e_{1,5}, -e_{2,3}, e_{2,3}, -e_{2,3}, \\
 &\in e_{2,3}, -e^2 e_{2,3}, e^2 e_{2,3}, -e^3 e_{2,3}, e^3 e_{2,3}, -e^4 e_{2,3}, e^4 e_{2,3}, -e^5 e_{2,3}, e^5 e_{2,3}, -e_{2,4}, e_{2,4}, -e_{2,4}, \\
 &\in e_{2,4}, -e^2 e_{2,4}, e^2 e_{2,4}, -e^3 e_{2,4}, e^3 e_{2,4}, -e^4 e_{2,4}, e^4 e_{2,4}, -e^5 e_{2,4}, e^5 e_{2,4}, -e_{2,5}, e_{2,5}, -e_{2,5}, \\
 &\in e_{2,5}, -e^2 e_{2,5}, e^2 e_{2,5}, -e^3 e_{2,5}, e^3 e_{2,5}, -e^4 e_{2,5}, e^4 e_{2,5}, -e^5 e_{2,5}, e^5 e_{2,5}, -e_{3,4}, e_{3,4}, \\
 &-e_{3,4}, \in e_{3,4}, -e^2 e_{3,4}, e^2 e_{3,4}, -e^3 e_{3,4}, e^3 e_{3,4}, -e^4 e_{3,4}, e^4 e_{3,4}, -e^5 e_{3,4}, e^5 e_{3,4}, -e_{3,5}, \\
 &\in e_{3,5}, -e_{3,5}, \in e_{3,5}, -e^2 e_{3,5}, e^2 e_{3,5}, -e^3 e_{3,5}, e^3 e_{3,5}, -e^4 e_{3,5}, e^4 e_{3,5}, -e^5 e_{3,5}, e^5 e_{3,5}, \\
 &-e_{4,5}, e_{4,5}, -e_{4,5}, \in e_{4,5}, -e^2 e_{4,5}, e^2 e_{4,5}, -e^3 e_{4,5}, e^3 e_{4,5}, -e^4 e_{4,5}, e^4 e_{4,5}, -e^5 e_{4,5}, e^5 e_{4,5}, \\
 &\in e_{4,5}, -f_{2,1}, f_{2,1}, -e_{f_{2,1}}, \in f_{2,1}, -e^2 f_{2,1}, e^2 f_{2,1}, -e^3 f_{2,1}, e^3 f_{2,1}, -e^4 f_{2,1}, e^4 f_{2,1}, \\
 &-e^5 f_{2,1}, e^5 f_{2,1}, -f_{3,1}, f_{3,1}, -e_{f_{3,1}}, \in f_{3,1}, -e^2 f_{3,1}, e^2 f_{3,1}, -e^3 f_{3,1}, e^3 f_{3,1}, -e^4 f_{3,1}, \\
 &\in e^4 f_{3,1}, -e^5 f_{3,1}, e^5 f_{3,1}, -f_{3,2}, f_{3,2}, -e_{f_{3,2}}, \in f_{3,2}, -e^2 f_{3,2}, e^2 f_{3,2}, -e^3 f_{3,2}, e^3 f_{3,2}, \\
 &-e^4 f_{3,2}, e^4 f_{3,2}, -e^5 f_{3,2}, e^5 f_{3,2}, -f_{4,1}, f_{4,1}, -e_{f_{4,1}}, \in f_{4,1}, -e^2 f_{4,1}, e^2 f_{4,1}, -e^3 f_{4,1}, \\
 &\in e^3 f_{4,1}, -e^4 f_{4,1}, e^4 f_{4,1}, -e^5 f_{4,1}, e^5 f_{4,1}, -f_{4,2}, f_{4,2}, -e_{f_{4,2}}, \in f_{4,2}, -e^2 f_{4,2}, e^2 f_{4,2}, \\
 &-e^3 f_{4,2}, e^3 f_{4,2}, -e^4 f_{4,2}, e^4 f_{4,2}, -e^5 f_{4,2}, e^5 f_{4,2}, -f_{4,3}, f_{4,3}, -e_{f_{4,3}}, \in f_{4,3}, -e^2 f_{4,3}, \\
 &\in e^2 f_{4,3}, -e^3 f_{4,3}, e^3 f_{4,3}, -e^4 f_{4,3}, e^4 f_{4,3}, -e^5 f_{4,3}, e^5 f_{4,3}, -f_{5,1}, f_{5,1}, -e_{f_{5,1}}, \in f_{5,1}, \\
 &-e^2 f_{5,1}, e^2 f_{5,1}, -e^3 f_{5,1}, e^3 f_{5,1}, -e^4 f_{5,1}, e^4 f_{5,1}, -e^5 f_{5,1}, e^5 f_{5,1}, -f_{5,2}, f_{5,2}, -e_{f_{5,2}}, \\
 &\in e_{f_{5,2}}, -e^2 f_{5,2}, e^2 f_{5,2}, -e^3 f_{5,2}, e^3 f_{5,2}, -e^4 f_{5,2}, e^4 f_{5,2}, -e^5 f_{5,2}, e^5 f_{5,2}, -f_{5,3}, f_{5,3}, \\
 &-e_{f_{5,3}}, \in e_{f_{5,3}}, -e^2 f_{5,3}, e^2 f_{5,3}, -e^3 f_{5,3}, e^3 f_{5,3}, -e^4 f_{5,3}, e^4 f_{5,3}, -e^5 f_{5,3}, e^5 f_{5,3}, -f_{5,4}, \\
 &\in e_{f_{5,4}}, -e_{f_{5,4}}, \in e_{f_{5,4}}, -e^2 f_{5,4}, e^2 f_{5,4}, -e^3 f_{5,4}, e^3 f_{5,4}, -e^4 f_{5,4}, e^4 f_{5,4}, -e^5 f_{5,4}, e^5 f_{5,4} \}
 \end{aligned}$$

LCSCA[5, 5, 30]

$$\begin{aligned}
& \left\{ 16 \epsilon^5 h_1 - 16 \epsilon^5 h_2, 8 \epsilon^5 h_1 - 8 \epsilon^5 h_2, 4 \epsilon^5 h_1 - 4 \epsilon^5 h_2, 2 \epsilon^5 h_1 - 2 \epsilon^5 h_2, \epsilon^5 h_1 - \epsilon^5 h_2, \frac{\epsilon^5 h_1}{2} - \frac{\epsilon^5 h_2}{2}, \right. \\
& -\frac{1}{2} \epsilon^5 h_1 + \frac{\epsilon^5 h_2}{2}, -\epsilon^5 h_1 + \epsilon^5 h_2, -2 \epsilon^5 h_1 + 2 \epsilon^5 h_2, -4 \epsilon^5 h_1 + 4 \epsilon^5 h_2, -8 \epsilon^5 h_1 + 8 \epsilon^5 h_2, \\
& -16 \epsilon^5 h_1 + 16 \epsilon^5 h_2, 16 \epsilon^5 h_2 - 16 \epsilon^5 h_3, 8 \epsilon^5 h_2 - 8 \epsilon^5 h_3, 4 \epsilon^5 h_2 - 4 \epsilon^5 h_3, 2 \epsilon^5 h_2 - 2 \epsilon^5 h_3, \\
& \epsilon^5 h_2 - \epsilon^5 h_3, \frac{\epsilon^5 h_2}{2} - \frac{\epsilon^5 h_3}{2}, -\frac{1}{2} \epsilon^5 h_2 + \frac{\epsilon^5 h_3}{2}, -\epsilon^5 h_2 + \epsilon^5 h_3, -2 \epsilon^5 h_2 + 2 \epsilon^5 h_3, -4 \epsilon^5 h_2 + 4 \epsilon^5 h_3, \\
& -8 \epsilon^5 h_2 + 8 \epsilon^5 h_3, -16 \epsilon^5 h_2 + 16 \epsilon^5 h_3, 16 \epsilon^5 h_3 - 16 \epsilon^5 h_4, 8 \epsilon^5 h_3 - 8 \epsilon^5 h_4, 4 \epsilon^5 h_3 - 4 \epsilon^5 h_4, \\
& 2 \epsilon^5 h_3 - 2 \epsilon^5 h_4, \epsilon^5 h_3 - \epsilon^5 h_4, \frac{\epsilon^5 h_3}{2} - \frac{\epsilon^5 h_4}{2}, -\frac{1}{2} \epsilon^5 h_3 + \frac{\epsilon^5 h_4}{2}, -\epsilon^5 h_3 + \epsilon^5 h_4, -2 \epsilon^5 h_3 + 2 \epsilon^5 h_4, \\
& -4 \epsilon^5 h_3 + 4 \epsilon^5 h_4, -8 \epsilon^5 h_3 + 8 \epsilon^5 h_4, -16 \epsilon^5 h_3 + 16 \epsilon^5 h_4, 16 \epsilon^5 h_1 - 16 \epsilon^5 h_5, 16 \epsilon^5 h_4 - 16 \epsilon^5 h_5, \\
& 8 \epsilon^5 h_1 - 8 \epsilon^5 h_5, 8 \epsilon^5 h_4 - 8 \epsilon^5 h_5, 4 \epsilon^5 h_1 - 4 \epsilon^5 h_5, 4 \epsilon^5 h_4 - 4 \epsilon^5 h_5, 2 \epsilon^5 h_1 - 2 \epsilon^5 h_5, 2 \epsilon^5 h_4 - 2 \epsilon^5 h_5, \\
& \epsilon^5 h_1 - \epsilon^5 h_5, \epsilon^5 h_4 - \epsilon^5 h_5, \frac{\epsilon^5 h_1}{2} - \frac{\epsilon^5 h_5}{2}, \frac{\epsilon^5 h_4}{2} - \frac{\epsilon^5 h_5}{2}, -\frac{1}{2} \epsilon^5 h_1 + \frac{\epsilon^5 h_5}{2}, -\frac{1}{2} \epsilon^5 h_4 + \frac{\epsilon^5 h_5}{2}, \\
& -\epsilon^5 h_1 + \epsilon^5 h_5, -\epsilon^5 h_4 + \epsilon^5 h_5, -2 \epsilon^5 h_1 + 2 \epsilon^5 h_5, -2 \epsilon^5 h_4 + 2 \epsilon^5 h_5, -4 \epsilon^5 h_1 + 4 \epsilon^5 h_5, \\
& \left. -4 \epsilon^5 h_4 + 4 \epsilon^5 h_5, -8 \epsilon^5 h_1 + 8 \epsilon^5 h_5, -8 \epsilon^5 h_4 + 8 \epsilon^5 h_5, -16 \epsilon^5 h_1 + 16 \epsilon^5 h_5, -16 \epsilon^5 h_4 + 16 \epsilon^5 h_5 \right\}
\end{aligned}$$

**LCSCA [5, 5, 31]**

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