Pensieve header: Optimization base for the NOE1 program, with split Nfe and Nlx.

Initialization

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2017-01"];
Once[<< KnotTheory`];
Once[<< "../Projects/Profile/Profile.m"]
```

```
Loading KnotTheory` version of September 6, 2014, 13:37:37.2841.
Read more at http://katlas.org/wiki/KnotTheory.

This is Profile.m, Nov 2016 mods of July 1994 version
```

NOE-1t

```
R+_{i_,j_} := 𝔼[1, Log[t_i] l_j, e_i f_j, e_i l_i f_j + l_i l_j + e_i^2 f_j^2/4];
R-_{i_,j_} := 𝔼[1, -Log[t_i] l_j, -t_i^-1 e_i f_j, -l_i l_j + t_i^-1 e_i l_j f_j - t_i^-2 e_i^2 f_j^2/4];
(ur_{i_} := 𝔼[t_i^-1/2, 0, 0, l_i t_i^-2]; nr_{i_} := 𝔼[t_i^1/2, 0, 0, -l_i t_i^2];)
```

```
DP_{x_→D_α, y_→D_β}[P_][f_] := (* means P[∂_α,∂_β][f] *)
 PP_DP@Total[CoefficientRules[P, {x, y}] /. ({m_, n_} → c_) :> c D[f, {α, m}, {β, n}]]
```

```
CF[𝔼[ω_, L_, Q_, P_]] :=
  PP_CF@𝔼[Expand@Together@ω, Expand@Together@L, Expand@Together@Q, Expand@Together@P];
```

```
𝔼 /: 𝔼[ω1_, L1_, Q1_, P1_] 𝔼[ω2_, L2_, Q2_, P2_] := CF@𝔼[ω1 ω2, L1 + L2, ω2 Q1 + ω1 Q2, ω2^4 P1 + ω1^4 P2];
```

```
Λ[k_] := ((t_k - 1) (2 (α β + δ μ)^2 - α^2 β^2) - 4 e_k l_k f_k δ^2 μ^2 -
      δ (1 + μ) (f_k^2 α^2 + e_k^2 β^2) - e_k^2 f_k^2 δ^3 (1 + 3 μ) -
      2 (α β + 2 δ μ + e_k f_k δ^2 (1 + 2 μ) + 2 l_k δ μ^2) (f_k α + e_k β) - 4 (l_k μ^2 + e_k f_k δ (1 + μ)) (α β + δ μ)) (1 + t_k)/4;
```

```
N_{f_i_ e_j_→k_}[𝔼[ω_, L_, Q_, P_]] := PP_Nfe@With[{q = ((1 - t_k) α β + β e_k + δ e_k f_k + α f_k)/μ}, 𝔼[
      PP_Nfeω@Expand@Together[μ ω /. μ → 1 + (t_k - 1) δ /. δ → ω^-1 ∂_{f_i,e_j} Q],
      L,
      PP_NfeQ@Expand@Together[μ ω q + μ (Q /. f_i | e_j → 0) /. μ → 1 + (t_k - 1) δ /.
          {α → ω^-1 (∂_{f_i} Q /. e_j → 0), β → ω^-1 (∂_{e_j} Q /. f_i → 0), δ → ω^-1 ∂_{f_i,e_j} Q}],
      Plus[
       PP_NfeDP@Expand@Together[μ^4 (DP_{f_i→D_α, e_j→D_β}[P][ℯ^q] /. ℯ- → 1) /. μ → 1 + (t_k - 1) δ /.
           {α → ω^-1 (∂_{f_i} Q /. e_j → 0), β → ω^-1 (∂_{e_j} Q /. f_i → 0), δ → ω^-1 ∂_{f_i,e_j} Q}],
       PP_NfeΛ@Expand@Together[ω^4 Λ[k] /. μ → 1 + (t_k - 1) δ /. {α → ω^-1 (∂_{f_i} Q /. e_j → 0),
           β → ω^-1 (∂_{e_j} Q /. f_i → 0), δ → ω^-1 ∂_{f_i,e_j} Q}]
      ]
     ]];
```

```
N_{l_j_ (x:e|f)_i_→k_}[𝔼[ω_, L_, Q_, P_]] := PP_Nlx@With[{q = ℯ^γ β x_k + γ l_k}, 𝔼[
      ω,
      PP_NlxL@Expand@Together[γ l_k + (L /. l_j → 0) /. γ → ∂_{l_j} L],
      PP_NlxQ@Expand@Together[ω ℯ^γ β x_k + (Q /. x_i → 0) /. {γ → ∂_{l_j} L, β → ω^-1 ∂_{x_i} Q}],
      PP_NlxP@Expand@Together[ℯ^-q DP_{l_j→D_γ, x_i→D_β}[P][ℯ^q] /. {γ → ∂_{l_j} L, β → ω^-1 ∂_{x_i} Q}]
     ]];
```

```
m_i_,j_→k_[Z_𝔼] := PPm@Module[{x, z},
   CF[(Z // Nf_i e_j→x // N1_i e_x→x // Nf_x 1_j→x) /. z_i_|j_|x → z_k]]
```

Rotational Virtual Knots

```
RVK::usage =
  "RVK[xs, rots] represents a Rotational Virtual Knot with a list of n Xp/Xm crossings xs and
    a length 2n list of rotation numbers rots. Crossing sites are indexed 1 through
    2n, and rots[[k]] is the rotation between site k-1 and site k. RVK is also a casting
    operator converting to the RVK presentation from other knot presentations.";
RVK[pd_PD] := Module[{n, xs, x, rots, front, k},
   n = Length[pd];
   xs = List @@ pd /. x_X :> If[PositiveQ[x], Xp[x[[4]], x[[1]]], Xm[x[[2]], x[[1]]]];
   rots = Table[0, {2 n}];
   front = {0};
   For[k = 0, k < 2 n, ++k,
    If[k == 0 ∨ FreeQ[front, -k],
     front = Flatten[front /. k → Catch[xs /. {
             Xp[k + 1, l_] | Xm[l_, k + 1] :> Throw[{l, k + 1, 1 - l}],
             Xp[l_, k + 1] | Xm[k + 1, l_] :> (++rots[[l]]; Throw[{1 - l, k + 1, l}])
            }]],
     If[MatchQ[front, {___, k, ___, -k, ___}], --rots[[k + 1]]
    ]
   ];
   RVK[xs, rots]
  ];
RVK[K_] := RVK[PD[K]];
```

Z

```
ul_ = nl_ = rot[_, 0] = 𝔼[1, 0, 0, 0];
rot[i_, 1] := uri;
rot[i_, n_Integer] /; n > 1 := Module[{y}, rot[i, n - 1] rot[y, 1] // mi,y→i];
rot[i_, -1] := nri;
rot[i_, n_Integer] /; n < -1 := Module[{y}, rot[i, n + 1] rot[y, -1] // mi,y→i];
```

```
t_ = t;
Z[K_] := Z[RVK@K];
Z[rvk_RVK] := PPz@Module[{todo, n, rots, ℨ, done, st, x, ℨ1, i, j, k, k1, k2, k3},
   {todo, rots} = List@@rvk;
   AppendTo[rots, 0];
   n = Length[todo];
   ℨ = 𝔼[1, 0, 0, 0];
   done = {0};
   st = Range[0, 2 n + 1];
   While[todo =!= {},
    {x} = MaximalBy[todo, Length[done ⋂ {#[[1]], #[[2]], #[[1]] - 1, #[[2]] - 1}] &, 1];
    Z$todo = todo; Z$x = x;
    {i, j} = List@@x;
    ℨ1 = Switch[Head[x],
       Xp, mj,k→j[R+i,j (R-k3,k nrk1 ulk2 // mk,k1→k // mk,k2→k // mk,k3→k)],
       Xm, mj,k→j[R-i,j (R+k,k3 nrk1 ulk2 // mk,k1→k // mk,k2→k // mk,k3→k)]
     ];
    ℨ1 = rot[k, rots[[i]]] ℨ1 // mk,i→i; rots[[i]] = 0;
    ℨ1 = ℨ1 rot[k, rots[[i + 1]]] // mi,k→i; rots[[i + 1]] = 0;
    ℨ1 = rot[k, rots[[j]]] ℨ1 // mk,j→j; rots[[j]] = 0;
    ℨ1 = ℨ1 rot[k, rots[[j + 1]]] // mj,k→j; rots[[j + 1]] = 0;
    ℨ *= ℨ1;
    If[MemberQ[done, i], ℨ = ℨ // mi,i+1→i; st = st /. st[[i + 2]] → st[[i + 1]]];
    If[MemberQ[done, i - 1], ℨ = ℨ // mst[[i]],i→st[[i]]; st = st /. st[[i + 1]] → st[[i]]];
    If[MemberQ[done, j], ℨ = ℨ // mj,j+1→j; st = st /. st[[j + 2]] → st[[j + 1]]];
    If[MemberQ[done, j - 1], ℨ = ℨ // mst[[j]],j→st[[j]]; st = st /. st[[j + 1]] → st[[j]]];
    done = done ⋃ {i - 1, i, j - 1, j};
    todo = DeleteCases[todo, x]
   ];
   ℨ /. {e0 → e, l0 → l, f0 → f}
  ]
```

Testing 3₁...

```
BeginProfile[];
Timing[Z[Knot[3, 1]]]
EndProfile[];
```

KnotTheory: Loading precomputed data in PD4Knots`.

$$\left\{0.765625,\ \mathbb{E}\left[-1+\frac{1}{t}+t,\ 0,\ 0,\ -16+2\,e\,f-\frac{2}{t^4}-\frac{2\,e\,f}{t^4}+\frac{2\,l}{t^4}+\frac{7}{t^3}+\frac{4\,e\,f}{t^3}-\frac{6\,l}{t^3}-\frac{14}{t^2}-\frac{6\,e\,f}{t^2}+\right.\right.$$
$$\left.\left.\frac{10\,l}{t^2}+\frac{18}{t}+\frac{2\,e\,f}{t}-\frac{8\,l}{t}+10\,t-6\,e\,f\,t+8\,l\,t-4\,t^2+4\,e\,f\,t^2-10\,l\,t^2+t^3-2\,e\,f\,t^3+6\,l\,t^3-2\,l\,t^4\right]\right\}$$

```
PrintProfile[];
```

```
NlxP: called 60 times, time in 0.251/0.312

  Parents:

    ( 60)  0.251/ 0.312 under Nlx

  Children:

    ( 60)  0.061/ 0.061 above DP


NfeDP: called 30 times, time in 0.173/0.188

  Parents:
```

```
    ( 30)  0.173/ 0.188 under Nfe
  Children:
    ( 30)  0.015/ 0.015 above DP

Nfe△: called 30 times, time in 0.093/0.093
  Parents:
    ( 30)  0.093/ 0.093 under Nfe

DP: called 90 times, time in 0.076/0.076
  Parents:
    ( 30)  0.015/ 0.015 under NfeDP
    ( 60)  0.061/ 0.061 under NlxP

NlxQ: called 60 times, time in 0.032/0.032
  Parents:
    ( 60)  0.032/ 0.032 under Nlx

CF: called 54 times, time in 0.032/0.032
  Parents:
    ( 30)  0.016/ 0.016 under m
    ( 24)  0.016/ 0.016 under z

NlxL: called 60 times, time in 0.016/0.016
  Parents:
    ( 60)  0.016/ 0.016 under Nlx

m: called 30 times, time in 0.016/0.703
  Parents:
    ( 30)  0.016/ 0.703 under z
  Children:
    ( 30)  0.016/ 0.016 above CF
    ( 30)  0.000/ 0.296 above Nfe
    ( 60)  0.015/ 0.375 above Nlx

Nlx: called 60 times, time in 0.015/0.375
  Parents:
    ( 60)  0.015/ 0.375 under m
  Children:
    ( 60)  0.016/ 0.016 above NlxL
    ( 60)  0.251/ 0.312 above NlxP
    ( 60)  0.032/ 0.032 above NlxQ
```

```
NfeQ: called 30 times, time in 0.015/0.015
  Parents:
    ( 30)  0.015/ 0.015 under Nfe

z: called 1 times, time in 0./0.719
  Parents:
    (  1)  0.000/ 0.719 under ProfileRoot
  Children:
    ( 24)  0.016/ 0.016 above CF
    ( 30)  0.016/ 0.703 above m

ProfileRoot: called 0 times, time in 0./0.
  Children:
    (  1)  0.000/ 0.719 above z

Nfeω: called 30 times, time in 0./0.
  Parents:
    ( 30)  0.000/ 0.000 under Nfe

Nfe: called 30 times, time in 0./0.296
  Parents:
    ( 30)  0.000/ 0.296 under m
  Children:
    ( 30)  0.173/ 0.188 above NfeDP
    ( 30)  0.015/ 0.015 above NfeQ
    ( 30)  0.093/ 0.093 above NfeΛ
    ( 30)  0.000/ 0.000 above Nfeω
```

Testing 10_{100}...

```
BeginProfile[];
Timing[Z[Knot[10, 100]]]
EndProfile[];
```

$$\Big\{11.6875,\ \mathbb{E}\Big[13+\frac{1}{t^4}-\frac{4}{t^3}+\frac{9}{t^2}-\frac{12}{t}-12\,t+9\,t^2-4\,t^3+t^4,\ 0,\ 0,$$
$$-2\,563\,146+253\,564\,e\,f-\frac{6}{t^{16}}-\frac{8\,e\,f}{t^{16}}+\frac{8\,l}{t^{16}}+\frac{92}{t^{15}}+\frac{112\,e\,f}{t^{15}}-\frac{120\,l}{t^{15}}-\frac{723}{t^{14}}-\frac{812\,e\,f}{t^{14}}+\frac{924\,l}{t^{14}}+\frac{3818}{t^{13}}+$$
$$\frac{3972\,e\,f}{t^{13}}-\frac{4784\,l}{t^{13}}-\frac{15\,133}{t^{12}}-\frac{14\,616\,e\,f}{t^{12}}+\frac{18\,588\,l}{t^{12}}+\frac{47\,848}{t^{11}}+\frac{42\,936\,e\,f}{t^{11}}-\frac{57\,552\,l}{t^{11}}-\frac{125\,539}{t^{10}}-\frac{104\,604\,e\,f}{t^{10}}+$$
$$\frac{147\,540\,l}{t^{10}}+\frac{281\,054}{t^9}+\frac{216\,948\,e\,f}{t^9}-\frac{321\,552\,l}{t^9}-\frac{548\,129}{t^8}-\frac{390\,040\,e\,f}{t^8}+\frac{606\,988\,l}{t^8}+\frac{945\,756}{t^7}+\frac{614\,936\,e\,f}{t^7}-$$
$$\frac{1\,004\,976\,l}{t^7}-\frac{1\,460\,263}{t^6}-\frac{854\,884\,e\,f}{t^6}+\frac{1\,469\,820\,l}{t^6}+\frac{2\,034\,106}{t^5}+\frac{1\,046\,676\,e\,f}{t^5}-\frac{1\,901\,560\,l}{t^5}-\frac{2\,570\,432}{t^4}-$$
$$\frac{1\,116\,500\,e\,f}{t^4}+\frac{2\,163\,176\,l}{t^4}+\frac{2\,956\,518}{t^3}+\frac{1\,007\,020\,e\,f}{t^3}-\frac{2\,123\,520\,l}{t^3}-\frac{3\,099\,338}{t^2}-\frac{704\,708\,e\,f}{t^2}+\frac{1\,711\,728\,l}{t^2}+$$
$$\frac{2\,958\,726}{t}+\frac{253\,564\,e\,f}{t}-\frac{958\,272\,l}{t}+2\,000\,454\,t-704\,708\,e\,f\,t+958\,272\,l\,t-1\,387\,610\,t^2+1\,007\,020\,e\,f\,t^2-$$
$$1\,711\,728\,l\,t^2+832\,998\,t^3-1\,116\,500\,e\,f\,t^3+2\,123\,520\,l\,t^3-407\,256\,t^4+1\,046\,676\,e\,f\,t^4-2\,163\,176\,l\,t^4+$$
$$132\,546\,t^5-854\,884\,e\,f\,t^5+1\,901\,560\,l\,t^5+9557\,t^6+614\,936\,e\,f\,t^6-1\,469\,820\,l\,t^6-59\,220\,t^7-390\,040\,e\,f\,t^7+$$
$$1\,004\,976\,l\,t^7+58\,859\,t^8+216\,948\,e\,f\,t^8-606\,988\,l\,t^8-40\,498\,t^9-104\,604\,e\,f\,t^9+321\,552\,l\,t^9+22\,001\,t^{10}+$$
$$42\,936\,e\,f\,t^{10}-147\,540\,l\,t^{10}-9704\,t^{11}-14\,616\,e\,f\,t^{11}+57\,552\,l\,t^{11}+3455\,t^{12}+3972\,e\,f\,t^{12}-18\,588\,l\,t^{12}-$$
$$966\,t^{13}-812\,e\,f\,t^{13}+4784\,l\,t^{13}+201\,t^{14}+112\,e\,f\,t^{14}-924\,l\,t^{14}-28\,t^{15}-8\,e\,f\,t^{15}+120\,l\,t^{15}+2\,t^{16}-8\,l\,t^{16}\Big]\Big\}$$

```
PrintProfile[];
```

```
NfeDP: called 100 times, time in 4.531/4.827

  Parents:

    ( 100)  4.531/ 4.827 under Nfe

  Children:

    ( 100)  0.296/ 0.296 above DP


NlxP: called 200 times, time in 4.018/4.815

  Parents:

    ( 200)  4.018/ 4.815 under Nlx

  Children:

    ( 200)  0.797/ 0.797 above DP


DP: called 300 times, time in 1.093/1.093

  Parents:

    ( 100)  0.296/ 0.296 under NfeDP

    ( 200)  0.797/ 0.797 under NlxP


CF: called 180 times, time in 1.077/1.077

  Parents:

    ( 100)  0.766/ 0.766 under m

    (  80)  0.311/ 0.311 under z


Nfe∧: called 100 times, time in 0.501/0.501
```

```
  Parents:
    ( 100)  0.501/ 0.501 under Nfe

NlxQ: called 200 times, time in 0.111/0.111
  Parents:
    ( 200)  0.111/ 0.111 under Nlx

m: called 100 times, time in 0.109/11.361
  Parents:
    ( 100)  0.109/ 11.360 under z
  Children:
    ( 100)  0.766/ 0.766 above CF
    ( 100)  0.061/ 5.497 above Nfe
    ( 200)  0.047/ 4.989 above Nlx

NfeQ: called 100 times, time in 0.108/0.108
  Parents:
    ( 100)  0.108/ 0.108 under Nfe

Nfe: called 100 times, time in 0.061/5.497
  Parents:
    ( 100)  0.061/ 5.497 under m
  Children:
    ( 100)  4.531/ 4.827 above NfeDP
    ( 100)  0.108/ 0.108 above NfeQ
    ( 100)  0.501/ 0.501 above Nfe∧
    ( 100)  0.000/ 0.000 above Nfeω

Nlx: called 200 times, time in 0.047/4.989
  Parents:
    ( 200)  0.047/ 4.989 under m
  Children:
    ( 200)  0.016/ 0.016 above NlxL
    ( 200)  4.018/ 4.815 above NlxP
    ( 200)  0.111/ 0.111 above NlxQ

z: called 1 times, time in 0.016/11.688
  Parents:
    (   1)  0.016/ 11.690 under ProfileRoot
  Children:
    (  80)  0.311/ 0.311 above CF
```

```
    ( 100)  0.109/ 11.360 above m

NlxL: called 200 times, time in 0.016/0.016
  Parents:
    ( 200)  0.016/ 0.016 under Nlx

ProfileRoot: called 0 times, time in 0./0.
  Children:
    (   1)  0.016/ 11.690 above z

Nfeω: called 100 times, time in 0./0.
  Parents:
    ( 100)  0.000/ 0.000 under Nfe
```

Testing $T_{9,5}$...

```
BeginProfile[];
Timing[Z[TorusKnot[9, 5]]]
EndProfile[];
```

$$\Big\{839.469,\ \mathbb{E}\Big[-1+\frac{1}{t^{16}}-\frac{1}{t^{15}}+\frac{1}{t^{11}}-\frac{1}{t^{10}}+\frac{1}{t^{7}}-\frac{1}{t^{5}}+\frac{1}{t^{2}}+t^2-t^5+t^7-t^{10}+t^{11}-t^{15}+t^{16},\ 0,\ 0,$$

$$7580+280\,e\,f-\frac{32\,e\,f}{t^{64}}+\frac{32\,l}{t^{64}}-\frac{1}{t^{63}}+\frac{94\,e\,f}{t^{63}}-\frac{126\,l}{t^{63}}+\frac{3}{t^{62}}-\frac{92\,e\,f}{t^{62}}+\frac{186\,l}{t^{62}}-\frac{3}{t^{61}}+\frac{30\,e\,f}{t^{61}}-\frac{122\,l}{t^{61}}+\frac{1}{t^{60}}+\frac{30\,l}{t^{60}}+$$
$$\frac{1}{t^{59}}-\frac{118\,e\,f}{t^{59}}+\frac{118\,l}{t^{59}}-\frac{8}{t^{58}}+\frac{346\,e\,f}{t^{58}}-\frac{464\,l}{t^{58}}+\frac{18}{t^{57}}-\frac{338\,e\,f}{t^{57}}+\frac{684\,l}{t^{57}}-\frac{16}{t^{56}}+\frac{110\,e\,f}{t^{56}}-\frac{448\,l}{t^{56}}+\frac{6}{t^{55}}-\frac{110\,e\,f}{t^{55}}+$$
$$\frac{220\,l}{t^{55}}-\frac{3}{t^{54}}+\frac{52\,e\,f}{t^{54}}-\frac{162\,l}{t^{54}}-\frac{12}{t^{53}}+\frac{476\,e\,f}{t^{53}}-\frac{424\,l}{t^{53}}+\frac{44}{t^{52}}-\frac{668\,e\,f}{t^{52}}+\frac{1144\,l}{t^{52}}-\frac{45}{t^{51}}+\frac{250\,e\,f}{t^{51}}-\frac{918\,l}{t^{51}}+\frac{25}{t^{50}}-$$
$$\frac{400\,e\,f}{t^{50}}+\frac{650\,l}{t^{50}}-\frac{41}{t^{49}}+\frac{678\,e\,f}{t^{49}}-\frac{1078\,l}{t^{49}}+\frac{28}{t^{48}}+\frac{294\,e\,f}{t^{48}}+\frac{384\,l}{t^{48}}+\frac{62}{t^{47}}-\frac{1022\,e\,f}{t^{47}}+\frac{1316\,l}{t^{47}}-\frac{89}{t^{46}}+\frac{220\,e\,f}{t^{46}}-$$
$$\frac{1242\,l}{t^{46}}+\frac{45}{t^{45}}-\frac{230\,e\,f}{t^{45}}+\frac{450\,l}{t^{45}}-\frac{105}{t^{44}}+\frac{1200\,e\,f}{t^{44}}-\frac{1430\,l}{t^{44}}+\frac{135}{t^{43}}-\frac{90\,e\,f}{t^{43}}+\frac{1290\,l}{t^{43}}+\frac{50}{t^{42}}-\frac{1350\,e\,f}{t^{42}}+\frac{1260\,l}{t^{42}}-$$
$$\frac{125}{t^{41}}-\frac{120\,e\,f}{t^{41}}-\frac{1230\,l}{t^{41}}-\frac{20}{t^{40}}+\frac{780\,e\,f}{t^{40}}-\frac{900\,l}{t^{40}}-\frac{90}{t^{39}}+\frac{1248\,e\,f}{t^{39}}-\frac{468\,l}{t^{39}}+\frac{284}{t^{38}}-\frac{576\,e\,f}{t^{38}}+\frac{1824\,l}{t^{38}}+\frac{3}{t^{37}}-$$
$$\frac{1982\,e\,f}{t^{37}}+\frac{1406\,l}{t^{37}}-\frac{188}{t^{36}}+\frac{250\,e\,f}{t^{36}}-\frac{2232\,l}{t^{36}}-\frac{179}{t^{35}}+\frac{1720\,e\,f}{t^{35}}-\frac{1470\,l}{t^{35}}+\frac{86}{t^{34}}+\frac{972\,e\,f}{t^{34}}+\frac{748\,l}{t^{34}}+\frac{437}{t^{33}}-\frac{1074\,e\,f}{t^{33}}+$$
$$\frac{2046\,l}{t^{33}}-\frac{32}{t^{32}}-\frac{3058\,e\,f}{t^{32}}+\frac{1984\,l}{t^{32}}-\frac{491}{t^{31}}+\frac{1840\,e\,f}{t^{31}}-\frac{4898\,l}{t^{31}}-\frac{214}{t^{30}}+\frac{2020\,e\,f}{t^{30}}-\frac{180\,l}{t^{30}}+\frac{402}{t^{29}}+\frac{512\,e\,f}{t^{29}}+\frac{1508\,l}{t^{29}}+$$
$$\frac{593}{t^{28}}-\frac{1994\,e\,f}{t^{28}}+\frac{2506\,l}{t^{28}}-\frac{131}{t^{27}}-\frac{3128\,e\,f}{t^{27}}+\frac{1134\,l}{t^{27}}-\frac{1110}{t^{26}}+\frac{3320\,e\,f}{t^{26}}-\frac{6448\,l}{t^{26}}+\frac{75}{t^{25}}+\frac{1870\,e\,f}{t^{25}}+\frac{1450\,l}{t^{25}}+$$
$$\frac{804}{t^{24}}-\frac{2\,e\,f}{t^{24}}+\frac{1872\,l}{t^{24}}+\frac{858}{t^{23}}-\frac{3406\,e\,f}{t^{23}}+\frac{3404\,l}{t^{23}}-\frac{738}{t^{22}}-\frac{1602\,e\,f}{t^{22}}-\frac{1804\,l}{t^{22}}-\frac{1669}{t^{21}}+\frac{3900\,e\,f}{t^{21}}-\frac{5502\,l}{t^{21}}+\frac{695}{t^{20}}+$$
$$\frac{1450\,e\,f}{t^{20}}+\frac{2450\,l}{t^{20}}+\frac{1341}{t^{19}}-\frac{944\,e\,f}{t^{19}}+\frac{2394\,l}{t^{19}}+\frac{989}{t^{18}}-\frac{3662\,e\,f}{t^{18}}+\frac{2718\,l}{t^{18}}-\frac{1903}{t^{17}}-\frac{24\,e\,f}{t^{17}}-\frac{3638\,l}{t^{17}}-\frac{1900}{t^{16}}+$$
$$\frac{3720\,e\,f}{t^{16}}-\frac{3744\,l}{t^{16}}+\frac{1613}{t^{15}}+\frac{930\,e\,f}{t^{15}}+\frac{2790\,l}{t^{15}}+\frac{2246}{t^{14}}-\frac{2178\,e\,f}{t^{14}}+\frac{3108\,l}{t^{14}}+\frac{122}{t^{13}}-\frac{2334\,e\,f}{t^{13}}+\frac{156\,l}{t^{13}}-\frac{2952}{t^{12}}+$$
$$\frac{762\,e\,f}{t^{12}}-\frac{3096\,l}{t^{12}}-\frac{1811}{t^{11}}+\frac{2940\,e\,f}{t^{11}}-\frac{2178\,l}{t^{11}}+\frac{3013}{t^{10}}+\frac{130\,e\,f}{t^{10}}+\frac{2810\,l}{t^{10}}+\frac{2956}{t^{9}}-\frac{2174\,e\,f}{t^{9}}+\frac{2304\,l}{t^{9}}-\frac{1656}{t^{8}}-$$
$$\frac{962\,e\,f}{t^{8}}-\frac{1212\,l}{t^{8}}-\frac{3603}{t^{7}}+\frac{816\,e\,f}{t^{7}}-\frac{1778\,l}{t^{7}}-\frac{1390}{t^{6}}+\frac{1740\,e\,f}{t^{6}}-\frac{924\,l}{t^{6}}+\frac{5340}{t^{5}}-\frac{460\,e\,f}{t^{5}}+\frac{2200\,l}{t^{5}}+\frac{2004}{t^{4}}-$$
$$\frac{932\,e\,f}{t^{4}}+\frac{472\,l}{t^{4}}-\frac{3247}{t^{3}}-\frac{266\,e\,f}{t^{3}}-\frac{666\,l}{t^{3}}-\frac{3938}{t^{2}}+\frac{218\,e\,f}{t^{2}}-\frac{484\,l}{t^{2}}-\frac{219}{t}+\frac{280\,e\,f}{t}-\frac{62\,l}{t}-281\,t+218\,e\,f\,t+$$
$$62\,l\,t-4422\,t^2-266\,e\,f\,t^2+484\,l\,t^2-3913\,t^3-932\,e\,f\,t^3+666\,l\,t^3+2476\,t^4-460\,e\,f\,t^4-472\,l\,t^4+7540\,t^5+$$
$$1740\,e\,f\,t^5-2200\,l\,t^5-2314\,t^6+816\,e\,f\,t^6+924\,l\,t^6-5381\,t^7-962\,e\,f\,t^7+1778\,l\,t^7-2868\,t^8-2174\,e\,f\,t^8+$$
$$1212\,l\,t^8+5260\,t^9+130\,e\,f\,t^9-2304\,l\,t^9+5823\,t^{10}+2940\,e\,f\,t^{10}-2810\,l\,t^{10}-3989\,t^{11}+762\,e\,f\,t^{11}+2178\,l\,t^{11}-$$
$$6048\,t^{12}-2334\,e\,f\,t^{12}+3096\,l\,t^{12}+278\,t^{13}-2178\,e\,f\,t^{13}-156\,l\,t^{13}+5354\,t^{14}+930\,e\,f\,t^{14}-3108\,l\,t^{14}+4403\,t^{15}+$$
$$3720\,e\,f\,t^{15}-2790\,l\,t^{15}-5644\,t^{16}-24\,e\,f\,t^{16}+3744\,l\,t^{16}-5541\,t^{17}-3662\,e\,f\,t^{17}+3638\,l\,t^{17}+3707\,t^{18}-944\,e\,f\,t^{18}-$$
$$2718\,l\,t^{18}+3735\,t^{19}+1450\,e\,f\,t^{19}-2394\,l\,t^{19}+3145\,t^{20}+3900\,e\,f\,t^{20}-2450\,l\,t^{20}-7171\,t^{21}-1602\,e\,f\,t^{21}+$$
$$5502\,l\,t^{21}-2542\,t^{22}-3406\,e\,f\,t^{22}+1804\,l\,t^{22}+4262\,t^{23}-2\,e\,f\,t^{23}-3404\,l\,t^{23}+2676\,t^{24}+1870\,e\,f\,t^{24}-1872\,l\,t^{24}+$$
$$1525\,t^{25}+3320\,e\,f\,t^{25}-1450\,l\,t^{25}-7558\,t^{26}-3128\,e\,f\,t^{26}+6448\,l\,t^{26}+1003\,t^{27}-1994\,e\,f\,t^{27}-1134\,l\,t^{27}+$$
$$3099\,t^{28}+512\,e\,f\,t^{28}-2506\,l\,t^{28}+1910\,t^{29}+2020\,e\,f\,t^{29}-1508\,l\,t^{29}-394\,t^{30}+1840\,e\,f\,t^{30}+180\,l\,t^{30}-5389\,t^{31}-$$
$$3058\,e\,f\,t^{31}+4898\,l\,t^{31}+1952\,t^{32}-1074\,e\,f\,t^{32}-1984\,l\,t^{32}+2483\,t^{33}+972\,e\,f\,t^{33}-2046\,l\,t^{33}+834\,t^{34}+$$
$$1720\,e\,f\,t^{34}-748\,l\,t^{34}-1649\,t^{35}+250\,e\,f\,t^{35}+1470\,l\,t^{35}-2420\,t^{36}-1982\,e\,f\,t^{36}+2232\,l\,t^{36}+1409\,t^{37}-$$
$$576\,e\,f\,t^{37}-1406\,l\,t^{37}+2108\,t^{38}+1248\,e\,f\,t^{38}-1824\,l\,t^{38}-558\,t^{39}+780\,e\,f\,t^{39}+468\,l\,t^{39}-920\,t^{40}-120\,e\,f\,t^{40}+$$
$$900\,l\,t^{40}-1355\,t^{41}-1350\,e\,f\,t^{41}+1230\,l\,t^{41}+1310\,t^{42}-90\,e\,f\,t^{42}-1260\,l\,t^{42}+1425\,t^{43}+1200\,e\,f\,t^{43}-$$
$$1290\,l\,t^{43}-1535\,t^{44}-230\,e\,f\,t^{44}+1430\,l\,t^{44}+495\,t^{45}+220\,e\,f\,t^{45}-450\,l\,t^{45}-1331\,t^{46}-1022\,e\,f\,t^{46}+1242\,l\,t^{46}+$$
$$1378\,t^{47}+294\,e\,f\,t^{47}-1316\,l\,t^{47}+412\,t^{48}+678\,e\,f\,t^{48}-384\,l\,t^{48}-1119\,t^{49}-400\,e\,f\,t^{49}+1078\,l\,t^{49}+675\,t^{50}+$$
$$250\,e\,f\,t^{50}-650\,l\,t^{50}-963\,t^{51}-668\,e\,f\,t^{51}+918\,l\,t^{51}+1188\,t^{52}+476\,e\,f\,t^{52}-1144\,l\,t^{52}-436\,t^{53}+52\,e\,f\,t^{53}+$$
$$424\,l\,t^{53}-165\,t^{54}-110\,e\,f\,t^{54}+162\,l\,t^{54}+226\,t^{55}+110\,e\,f\,t^{55}-220\,l\,t^{55}-464\,t^{56}-338\,e\,f\,t^{56}+448\,l\,t^{56}+$$
$$702\,t^{57}+346\,e\,f\,t^{57}-684\,l\,t^{57}-472\,t^{58}-118\,e\,f\,t^{58}+464\,l\,t^{58}+119\,t^{59}-118\,l\,t^{59}+31\,t^{60}+30\,e\,f\,t^{60}-30\,l\,t^{60}-$$
$$125\,t^{61}-92\,e\,f\,t^{61}+122\,l\,t^{61}+189\,t^{62}+94\,e\,f\,t^{62}-186\,l\,t^{62}-127\,t^{63}-32\,e\,f\,t^{63}+126\,l\,t^{63}+32\,t^{64}-32\,l\,t^{64}\Big]\Big\}$$

```
PrintProfile[];
```

```
NfeDP: called 360 times, time in 465.671/472.543

  Parents:

    ( 360)  465.671/ 472.543 under Nfe
```

```
  Children:
    ( 360)    6.872/   6.872 above DP

NlxP: called 720 times, time in 240.702/260.477
  Parents:
    ( 720)  240.702/ 260.477 under Nlx
  Children:
    ( 720)   19.775/  19.775 above DP

CF: called 648 times, time in 79.578/79.578
  Parents:
    ( 360)   61.777/  61.777 under m
    ( 288)   17.801/  17.801 under z

DP: called 1080 times, time in 26.647/26.647
  Parents:
    ( 360)    6.872/   6.872 under NfeDP
    ( 720)   19.775/  19.775 under NlxP

Nfe^: called 360 times, time in 19.127/19.127
  Parents:
    ( 360)   19.127/  19.127 under Nfe

m: called 360 times, time in 3.077/821.384
  Parents:
    ( 360)    3.077/ 821.384 under z
  Children:
    ( 360)   61.777/  61.777 above CF
    ( 360)    0.890/ 494.151 above Nfe
    ( 720)    0.283/ 262.379 above Nlx

NlxQ: called 720 times, time in 1.539/1.539
  Parents:
    ( 720)    1.539/   1.539 under Nlx

NfeQ: called 360 times, time in 1.466/1.466
  Parents:
    ( 360)    1.466/   1.466 under Nfe

Nfe: called 360 times, time in 0.89/494.151
  Parents:
```

```
    ( 360)    0.890/ 494.151 under m
  Children:
    ( 360)  465.671/ 472.543 above NfeDP
    ( 360)    1.466/   1.466 above NfeQ
    ( 360)   19.127/  19.127 above Nfe∧
    ( 360)    0.125/   0.125 above Nfeω

z: called 1 times, time in 0.284/839.469
  Parents:
    (   1)    0.284/ 839.469 under ProfileRoot
  Children:
    ( 288)   17.801/  17.801 above CF
    ( 360)    3.077/ 821.384 above m

Nlx: called 720 times, time in 0.283/262.379
  Parents:
    ( 720)    0.283/ 262.379 under m
  Children:
    ( 720)    0.080/   0.080 above NlxL
    ( 720)  240.702/ 260.477 above NlxP
    ( 720)    1.539/   1.539 above NlxQ

Nfeω: called 360 times, time in 0.125/0.125
  Parents:
    ( 360)    0.125/   0.125 under Nfe

NlxL: called 720 times, time in 0.08/0.08
  Parents:
    ( 720)    0.080/   0.080 under Nlx

ProfileRoot: called 0 times, time in 0./0.
  Children:
    (   1)    0.284/ 839.469 above z
```