

Pensieve header: Optimization base for the NOE1 program, with split Nfe.

## Initialization

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2017-01"];
Once[<< KnotTheory`];
Once[<< "../Projects/Profile/Profile.m"]
```

Loading KnotTheory` version of September 6, 2014, 13:37:37.2841.

Read more at <http://katlas.org/wiki/KnotTheory>.

This is Profile.m, Nov 2016 mods of July 1994 version

## NOE-It

```
R_{i,j}^+ := E[1, Log[t_i] l_j, e_i f_j, e_i l_i f_j + l_i l_j + e_i^2 f_j^2 / 4];
R_{i,j}^- := E[1, -Log[t_i] l_j, -t_i^{-1} e_i f_j, -l_i l_j + t_i^{-1} e_i l_j f_j - t_i^{-2} e_i^2 f_j^2 / 4];
(ur_{i-} := E[t_i^{-1/2}, 0, 0, l_i t_i^2]; nr_{i-} := E[t_i^{1/2}, 0, 0, -l_i t_i^2];)
```

```
DP_{x \to D_\alpha, y \to D_\beta}[P_][f_] := (* means P[\partial_\alpha, \partial_\beta][f] *)
PP_{DP}@Total[CoefficientRules[P, {x, y}] /. ({m_, n_} \to c_) \to c D[f, {\alpha, m}, {\beta, n}]]
```

```
CF[E[\omega_, L_, Q_, P_]] :=
  PP_{CF}@E[Expand@Together@\omega, Expand@Together@L, Expand@Together@Q, Expand@PPTogether4P@Together@P];
```

```
E /: E[\omega1_, L1_, Q1_, P1_] E[\omega2_, L2_, Q2_, P2_] := CF@E[\omega1 \omega2, L1 + L2, \omega2 Q1 + \omega1 Q2, \omega2^4 P1 + \omega1^4 P2];
```

$$\Delta[k_] := \left( (t_k - 1) (2(\alpha\beta + \delta\mu)^2 - \alpha^2\beta^2) - 4e_k l_k f_k \delta^2 \mu^2 - \delta(1 + \mu)(f_k^2 \alpha^2 + e_k^2 \beta^2) - e_k^2 f_k^2 \delta^3 (1 + 3\mu) - 2(\alpha\beta + 2\delta\mu + e_k f_k \delta^2 (1 + 2\mu) + 2l_k \delta \mu^2)(f_k \alpha + e_k \beta) - 4(l_k \mu^2 + e_k f_k \delta(1 + \mu))(\alpha\beta + \delta\mu)(1 + t_k) \right) / 4;$$

```
N_{f_i e_j \to k}[E[\omega_, L_, Q_, P_]] := PP_{Nfe}@With[{q = ((1 - t_k) \alpha \beta + \beta e_k + \delta e_k f_k + \alpha f_k) / \mu}, E[
  PP_{Nfe\omega}@Expand@Together[\mu \omega /. \mu \to 1 + (t_k - 1) \delta /. \delta \to \omega^{-1} \partial_{f_i, e_j} Q],
  L,
  PP_{NfeQ}@Expand@Together[\mu \omega q + \mu (Q /. f_i | e_j \to \theta) /. \mu \to 1 + (t_k - 1) \delta /.
    {\alpha \to \omega^{-1} (\partial_{f_i} Q /. e_j \to \theta), \beta \to \omega^{-1} (\partial_{e_j} Q /. f_i \to \theta), \delta \to \omega^{-1} \partial_{f_i, e_j} Q}],
  Plus[
    PP_{NfeDP}@Expand@Together[\mu^4 (DP_{f_i \to D_\alpha, e_j \to D_\beta}[P][e^q] /. e \to 1) /. \mu \to 1 + (t_k - 1) \delta /.
      {\alpha \to \omega^{-1} (\partial_{f_i} Q /. e_j \to \theta), \beta \to \omega^{-1} (\partial_{e_j} Q /. f_i \to \theta), \delta \to \omega^{-1} \partial_{f_i, e_j} Q}],
    PP_{Nfe\Delta}@Expand@Together[\omega^4 \Delta[k] /. \mu \to 1 + (t_k - 1) \delta /. {\alpha \to \omega^{-1} (\partial_{f_i} Q /. e_j \to \theta),
      \beta \to \omega^{-1} (\partial_{e_j} Q /. f_i \to \theta), \delta \to \omega^{-1} \partial_{f_i, e_j} Q}]]
  ]];
```

```
N_{l_j (x:e|f)_i \to k}[E[\omega_, L_, Q_, P_]] := PP_{Nlx}@With[{q = e^y \beta x_k + \gamma l_k}, CF[
  E[\omega, \gamma l_k + (L /. l_j \to \theta), \omega e^y \beta x_k + (Q /. x_i \to \theta), e^{-q} DP_{l_j \to D_\gamma, x_i \to D_\beta}[P][e^q]] /. {\gamma \to \partial_{l_j} L, \beta \to \omega^{-1} \partial_{x_i} Q}]]];
```

```
m_{i,j \to k}[Z_E] := PP_m@Module[{x, z},
  CF[(Z // N_{f_i e_j \to x} // N_{l_i e_x \to x} // N_{f_x l_j \to x}) /. Z_{-i|j|x} \to z_k]]
```

## Rotational Virtual Knots

```

RVK::usage =
  "RVK[xs, rots] represents a Rotational Virtual Knot with a list of n Xp/Xm crossings xs and
  a length 2n list of rotation numbers rots. Crossing sites are indexed 1 through
  2n, and rots[[k]] is the rotation between site k-1 and site k. RVK is also a casting
  operator converting to the RVK presentation from other knot presentations.";
RVK[pd_PD] := Module[{n, xs, x, rots, front, k},
  n = Length[pd];
  xs = List@@pd /. x_X => If[PositiveQ[x], Xp[x[[4]], x[[1]], Xm[x[[2]], x[[1]]];
  rots = Table[0, {2 n});
  front = {0};
  For[k = 0, k < 2 n, ++k,
  If[k == 0 ∨ FreeQ[front, -k],
  front = Flatten[front /. k → Catch[xs /. {
    Xp[k + 1, L_] | Xm[L_, k + 1] => Throw[{L, k + 1, 1 - L}},
    Xp[L_, k + 1] | Xm[k + 1, L_] => (++)rots[[L]]; Throw[{1 - L, k + 1, L}}]
  ]],
  If[MatchQ[front, {___, k, ___, -k, ___}], --rots[[k + 1]]
  ];
  RVK[xs, rots]
  ];
RVK[K_] := RVK[PD[K]];

```

## Z

```

ul_ = n1_ = rot[_, 0] = E[1, 0, 0, 0];
rot[i_, 1] := uri;
rot[i_, n_Integer] /; n > 1 := Module[{y}, rot[i, n - 1] rot[y, 1] // mi,y→i;
rot[i_, -1] := nri;
rot[i_, n_Integer] /; n < -1 := Module[{y}, rot[i, n + 1] rot[y, -1] // mi,y→i;

```

```

t_ = t;
Z[K_] := Z[RVK@K];
Z[rvk_RVK] := PPz@Module[{todo, n, rots, ζ, done, st, x, ζ1, i, j, k, k1, k2, k3},
  {todo, rots} = List@@rvk;
AppendTo[rots, 0];
n = Length[todo];
ζ = E[1, 0, 0, 0];
done = {0};
st = Range[0, 2 n + 1];
While[todo != {},
  {x} = MaximalBy[todo, Length[done ∩ {#[[1]], #[[2]], #[[1]] - 1, #[[2]] - 1}] &, 1];
Z$todo = todo; Z$x = x;
{i, j} = List@@x;
ζ1 = Switch[Head[x],
  Xp, mj,k→j [R+i,j (R-k3,k nrk1 ulk2 // mk,k1→k // mk,k2→k // mk,k3→k) ],
  Xm, mj,k→j [R-i,j (R+k,k3 nrk1 ulk2 // mk,k1→k // mk,k2→k // mk,k3→k) ]
];
ζ1 = rot[k, rots[[i]] ζ1 // mk,i→i; rots[[i]] = 0;
ζ1 = ζ1 rot[k, rots[[i + 1]] // mi,k→i; rots[[i + 1]] = 0;
ζ1 = rot[k, rots[[j]] ζ1 // mk,j→j; rots[[j]] = 0;
ζ1 = ζ1 rot[k, rots[[j + 1]] // mj,k→j; rots[[j + 1]] = 0;
ζ *= ζ1;
If[MemberQ[done, i], ζ = ζ // mi,i+1→i; st = st /. st[[i + 2]] → st[[i + 1]];
If[MemberQ[done, i - 1], ζ = ζ // mst[[i],i→st[[i]]; st = st /. st[[i + 1]] → st[[i]];
If[MemberQ[done, j], ζ = ζ // mj,j+1→j; st = st /. st[[j + 2]] → st[[j + 1]];
If[MemberQ[done, j - 1], ζ = ζ // mst[[j],j→st[[j]]; st = st /. st[[j + 1]] → st[[j]];
done = done ∪ {i - 1, i, j - 1, j};
todo = DeleteCases[todo, x]
];
ζ /. {e0 → e, l0 → l, f0 → f}
]

```

## Testing 3<sub>1</sub>...

```

BeginProfile[];
Timing[Z[Knot[3, 1]]]
EndProfile[];

```

 KnotTheory: Loading precomputed data in PD4Knots` 

$$\left\{ 0.75, \mathbb{E} \left[ -1 + \frac{1}{t} + t, 0, 0, -16 + 2ef - \frac{2}{t^4} - \frac{2ef}{t^4} + \frac{2l}{t^4} + \frac{7}{t^3} + \frac{4ef}{t^3} - \frac{6l}{t^3} - \frac{14}{t^2} - \frac{6ef}{t^2} + \frac{10l}{t^2} + \frac{18}{t} + \frac{2ef}{t} - \frac{8l}{t} + 10t - 6eft + 8lt - 4t^2 + 4eft^2 - 10lt^2 + t^3 - 2eft^3 + 6lt^3 - 2lt^4 \right] \right\}$$

```
PrintProfile[];
```

Together4P: called 114 times, time in 0.218/0.218

Parents:

( 114) 0.218/ 0.218 under CF

NfeDP: called 30 times, time in 0.187/0.203

Parents:

( 30) 0.187/ 0.203 under Nfe

Children:

( 30) 0.016/ 0.016 above DP

CF: called 114 times, time in 0.094/0.312

Parents:

( 30) 0.016/ 0.016 under m

( 60) 0.062/ 0.265 under Nlx

( 24) 0.016/ 0.031 under z

Children:

( 114) 0.218/ 0.218 above Together4P

Nfe $\Delta$ : called 30 times, time in 0.064/0.064

Parents:

( 30) 0.064/ 0.064 under Nfe

DP: called 90 times, time in 0.061/0.061

Parents:

( 30) 0.016/ 0.016 under NfeDP

( 60) 0.045/ 0.045 under Nlx

NfeQ: called 30 times, time in 0.032/0.032

Parents:

( 30) 0.032/ 0.032 under Nfe

m: called 30 times, time in 0.031/0.672

Parents:

( 30) 0.031/ 0.672 under z

Children:

( 30) 0.016/ 0.016 above CF

( 30) 0.000/ 0.299 above Nfe

( 60) 0.016/ 0.326 above Nlx

z: called 1 times, time in 0.016/0.719

Parents:

( 1) 0.016/ 0.719 under ProfileRoot

Children:

( 24) 0.016/ 0.031 above CF

( 30) 0.031/ 0.672 above m

Nlx: called 60 times, time in 0.016/0.326

Parents:

( 60) 0.016/ 0.326 under m

Children:

( 60) 0.062/ 0.265 above CF

( 60) 0.045/ 0.045 above DP

ProfileRoot: called 0 times, time in 0./0.

Children:

( 1) 0.016/ 0.719 above z

Nfe $\omega$ : called 30 times, time in 0./0.

Parents:

( 30) 0.000/ 0.000 under Nfe

Nfe: called 30 times, time in 0./0.299

Parents:

( 30) 0.000/ 0.299 under m

Children:

( 30) 0.187/ 0.203 above NfeDP

( 30) 0.032/ 0.032 above NfeQ

( 30) 0.064/ 0.064 above Nfe $\Delta$ ( 30) 0.000/ 0.000 above Nfe $\omega$ 

## Testing $10_{100}$ ...

**BeginProfile** [];**Timing**[Z[Knot[10, 100]]]**EndProfile** [];
$$\left\{ 12.0469, \mathbb{E} \left[ 13 + \frac{1}{t^4} - \frac{4}{t^3} + \frac{9}{t^2} - \frac{12}{t} - 12t + 9t^2 - 4t^3 + t^4, 0, 0, \right. \right.$$

$$- 2563146 + 253564ef - \frac{6}{t^{16}} - \frac{8ef}{t^{16}} + \frac{81}{t^{16}} + \frac{92}{t^{15}} + \frac{112ef}{t^{15}} - \frac{1201}{t^{15}} - \frac{723}{t^{14}} - \frac{812ef}{t^{14}} + \frac{9241}{t^{14}} + \frac{3818}{t^{13}} +$$

$$\frac{3972ef}{t^{13}} - \frac{47841}{t^{13}} - \frac{15133}{t^{12}} - \frac{14616ef}{t^{12}} + \frac{185881}{t^{12}} + \frac{47848}{t^{11}} + \frac{42936ef}{t^{11}} - \frac{575521}{t^{11}} - \frac{125539}{t^{10}} - \frac{104604ef}{t^{10}} +$$

$$\frac{1475401}{t^{10}} + \frac{281054}{t^9} + \frac{216948ef}{t^9} - \frac{3215521}{t^9} - \frac{548129}{t^8} - \frac{390040ef}{t^8} + \frac{6069881}{t^8} + \frac{945756}{t^7} + \frac{614936ef}{t^7} -$$

$$\frac{10049761}{t^7} - \frac{1460263}{t^6} - \frac{854884ef}{t^6} + \frac{14698201}{t^6} + \frac{2034106}{t^5} + \frac{1046676ef}{t^5} - \frac{19015601}{t^5} - \frac{2570432}{t^4} -$$

$$\frac{1116500ef}{t^4} + \frac{21631761}{t^4} + \frac{2956518}{t^3} + \frac{1007020ef}{t^3} - \frac{21235201}{t^3} - \frac{3099338}{t^2} - \frac{704708ef}{t^2} + \frac{17117281}{t^2} +$$

$$\frac{2958726}{t} + \frac{253564ef}{t} - \frac{9582721}{t} + 2000454t - 704708eft + 9582721t - 1387610t^2 + 1007020eft^2 -$$

$$17117281t^2 + 832998t^3 - 1116500eft^3 + 21235201t^3 - 407256t^4 + 1046676eft^4 - 21631761t^4 +$$

$$132546t^5 - 854884eft^5 + 19015601t^5 + 9557t^6 + 614936eft^6 - 14698201t^6 - 59220t^7 - 390040eft^7 +$$

$$10049761t^7 + 58859t^8 + 216948eft^8 - 6069881t^8 - 40498t^9 - 104604eft^9 + 3215521t^9 + 22001t^{10} +$$

$$42936eft^{10} - 1475401t^{10} - 9704t^{11} - 14616eft^{11} + 575521t^{11} + 3455t^{12} + 3972eft^{12} - 185881t^{12} -$$

$$966t^{13} - 812eft^{13} + 47841t^{13} + 201t^{14} + 112eft^{14} - 9241t^{14} - 28t^{15} - 8eft^{15} + 1201t^{15} + 2t^{16} - 81t^{16} \left. \right\}$$
**PrintProfile** [];

Together4P: called 380 times, time in 4.812/4.812

Parents:

( 380) 4.812/ 4.812 under CF

NfeDP: called 100 times, time in 4.5/4.752

Parents:

( 100) 4.500/ 4.752 under Nfe

Children:

( 100) 0.252/ 0.252 above DP

DP: called 300 times, time in 1.046/1.046

Parents:

( 100) 0.252/ 0.252 under NfeDP

( 200) 0.794/ 0.794 under Nlx

CF: called 380 times, time in 0.675/5.487

Parents:

( 100) 0.204/ 0.860 under m

( 200) 0.392/ 4.190 under Nlx

( 80) 0.079/ 0.437 under z

Children:

( 380) 4.812/ 4.812 above Together4P

Nfe $\Delta$ : called 100 times, time in 0.486/0.486

Parents:

( 100) 0.486/ 0.486 under Nfe

m: called 100 times, time in 0.189/11.61

Parents:

( 100) 0.189/ 11.610 under z

Children:

( 100) 0.204/ 0.860 above CF

( 100) 0.015/ 5.391 above Nfe

( 200) 0.186/ 5.170 above Nlx

Nlx: called 200 times, time in 0.186/5.17

Parents:

( 200) 0.186/ 5.170 under m

Children:

( 200) 0.392/ 4.190 above CF

( 200) 0.794/ 0.794 above DP

NfeQ: called 100 times, time in 0.108/0.108

Parents:

( 100) 0.108 / 0.108 under Nfe

Nfe $\omega$ : called 100 times, time in 0.03/0.03

Parents:

( 100) 0.030 / 0.030 under Nfe

Nfe: called 100 times, time in 0.015/5.391

Parents:

( 100) 0.015 / 5.391 under m

Children:

( 100) 4.500 / 4.752 above NfeDP

( 100) 0.108 / 0.108 above NfeQ

( 100) 0.486 / 0.486 above Nfe $\Delta$

( 100) 0.030 / 0.030 above Nfe $\omega$

z: called 1 times, time in 0./12.047

Parents:

( 1) 0.000 / 12.050 under ProfileRoot

Children:

( 80) 0.079 / 0.437 above CF

( 100) 0.189 / 11.610 above m

ProfileRoot: called 0 times, time in 0./0.

Children:

( 1) 0.000 / 12.050 above z

Testing  $T_{9,5}$ ...

```
BeginProfile[];
Timing[Z[TorusKnot[9, 5]]]
EndProfile[];
```

$$\left\{ 792.469, \mathbb{E} \left[ -1 + \frac{1}{t^{16}} - \frac{1}{t^{15}} + \frac{1}{t^{11}} - \frac{1}{t^{10}} + \frac{1}{t^7} - \frac{1}{t^5} + \frac{1}{t^2} + t^2 - t^5 + t^7 - t^{10} + t^{11} - t^{15} + t^{16}, 0, 0, \right. \right.$$

$$7580 + 280 e f - \frac{32 e f}{t^{64}} + \frac{32 l}{t^{64}} - \frac{1}{t^{63}} + \frac{94 e f}{t^{63}} - \frac{126 l}{t^{63}} + \frac{3}{t^{62}} - \frac{92 e f}{t^{62}} + \frac{186 l}{t^{62}} - \frac{3}{t^{61}} + \frac{30 e f}{t^{61}} - \frac{122 l}{t^{61}} + \frac{1}{t^{60}} + \frac{30 l}{t^{60}} +$$

$$\frac{1}{t^{59}} - \frac{118 e f}{t^{59}} + \frac{118 l}{t^{59}} - \frac{8}{t^{58}} + \frac{346 e f}{t^{58}} - \frac{464 l}{t^{58}} + \frac{18}{t^{57}} - \frac{338 e f}{t^{57}} + \frac{684 l}{t^{57}} - \frac{16}{t^{56}} + \frac{110 e f}{t^{56}} - \frac{448 l}{t^{56}} + \frac{6}{t^{55}} - \frac{110 e f}{t^{55}} +$$

$$\frac{220 l}{t^{55}} - \frac{3}{t^{54}} + \frac{52 e f}{t^{54}} - \frac{162 l}{t^{54}} - \frac{12}{t^{53}} + \frac{476 e f}{t^{53}} - \frac{424 l}{t^{53}} + \frac{44}{t^{52}} - \frac{668 e f}{t^{52}} + \frac{1144 l}{t^{52}} - \frac{45}{t^{51}} + \frac{250 e f}{t^{51}} - \frac{918 l}{t^{51}} + \frac{25}{t^{50}} -$$

$$\frac{400 e f}{t^{50}} + \frac{650 l}{t^{50}} - \frac{41}{t^{49}} + \frac{678 e f}{t^{49}} - \frac{1078 l}{t^{49}} + \frac{28}{t^{48}} + \frac{294 e f}{t^{48}} + \frac{384 l}{t^{48}} + \frac{62}{t^{47}} - \frac{1022 e f}{t^{47}} + \frac{1316 l}{t^{47}} - \frac{89}{t^{46}} + \frac{220 e f}{t^{46}} -$$

$$\frac{1242 l}{t^{46}} + \frac{45}{t^{45}} - \frac{230 e f}{t^{45}} + \frac{450 l}{t^{45}} - \frac{105}{t^{44}} + \frac{1200 e f}{t^{44}} - \frac{1430 l}{t^{44}} + \frac{135}{t^{43}} - \frac{90 e f}{t^{43}} + \frac{1290 l}{t^{43}} + \frac{50}{t^{42}} - \frac{1350 e f}{t^{42}} + \frac{1260 l}{t^{42}} -$$

$$\frac{125}{t^{41}} - \frac{120 e f}{t^{41}} - \frac{1230 l}{t^{41}} - \frac{20}{t^{40}} + \frac{780 e f}{t^{40}} - \frac{900 l}{t^{40}} - \frac{90}{t^{39}} + \frac{1248 e f}{t^{39}} - \frac{468 l}{t^{39}} + \frac{284}{t^{38}} - \frac{576 e f}{t^{38}} + \frac{1824 l}{t^{38}} + \frac{3}{t^{37}} -$$

$$\frac{1982 e f}{t^{37}} + \frac{1406 l}{t^{37}} - \frac{188}{t^{36}} + \frac{250 e f}{t^{36}} - \frac{2232 l}{t^{36}} - \frac{179}{t^{35}} + \frac{1720 e f}{t^{35}} - \frac{1470 l}{t^{35}} + \frac{86}{t^{34}} + \frac{972 e f}{t^{34}} + \frac{748 l}{t^{34}} + \frac{437}{t^{33}} - \frac{1074 e f}{t^{33}} +$$

$$\frac{2046 l}{t^{33}} - \frac{32}{t^{32}} - \frac{3058 e f}{t^{32}} + \frac{1984 l}{t^{32}} - \frac{491}{t^{31}} + \frac{1840 e f}{t^{31}} - \frac{4898 l}{t^{31}} - \frac{214}{t^{30}} + \frac{2020 e f}{t^{30}} - \frac{180 l}{t^{30}} + \frac{402}{t^{29}} + \frac{512 e f}{t^{29}} + \frac{1508 l}{t^{29}} -$$

$$\frac{593}{t^{28}} - \frac{1994 e f}{t^{28}} + \frac{2506 l}{t^{28}} - \frac{131}{t^{27}} - \frac{3128 e f}{t^{27}} + \frac{1134 l}{t^{27}} - \frac{1110}{t^{26}} + \frac{3320 e f}{t^{26}} - \frac{6448 l}{t^{26}} + \frac{75}{t^{25}} + \frac{1870 e f}{t^{25}} + \frac{1450 l}{t^{25}} +$$

$$\frac{804}{t^{24}} - \frac{2 e f}{t^{24}} + \frac{1872 l}{t^{24}} + \frac{858}{t^{23}} - \frac{3406 e f}{t^{23}} + \frac{3404 l}{t^{23}} - \frac{738}{t^{22}} - \frac{1602 e f}{t^{22}} - \frac{1804 l}{t^{22}} - \frac{1669}{t^{21}} + \frac{3900 e f}{t^{21}} - \frac{5502 l}{t^{21}} + \frac{695}{t^{20}} -$$

$$\frac{1450 e f}{t^{20}} + \frac{2450 l}{t^{20}} + \frac{1341}{t^{19}} - \frac{944 e f}{t^{19}} + \frac{2394 l}{t^{19}} + \frac{989}{t^{18}} - \frac{3662 e f}{t^{18}} + \frac{2718 l}{t^{18}} - \frac{1903}{t^{17}} - \frac{24 e f}{t^{17}} - \frac{3638 l}{t^{17}} - \frac{1900}{t^{16}} +$$

$$\frac{3720 e f}{t^{16}} - \frac{3744 l}{t^{16}} + \frac{1613}{t^{15}} + \frac{930 e f}{t^{15}} + \frac{2790 l}{t^{15}} + \frac{2246}{t^{14}} - \frac{2178 e f}{t^{14}} + \frac{3108 l}{t^{14}} + \frac{122}{t^{13}} - \frac{2334 e f}{t^{13}} + \frac{156 l}{t^{13}} - \frac{2952}{t^{12}} +$$

$$\frac{762 e f}{t^{12}} - \frac{3096 l}{t^{12}} - \frac{1811}{t^{11}} + \frac{2940 e f}{t^{11}} - \frac{2178 l}{t^{11}} + \frac{3013}{t^{10}} + \frac{130 e f}{t^{10}} + \frac{2810 l}{t^{10}} + \frac{2956}{t^9} - \frac{2174 e f}{t^9} + \frac{2304 l}{t^9} - \frac{1656}{t^8} -$$

$$\frac{962 e f}{t^8} - \frac{1212 l}{t^8} - \frac{3603}{t^7} + \frac{816 e f}{t^7} - \frac{1778 l}{t^7} - \frac{1390}{t^6} + \frac{1740 e f}{t^6} - \frac{924 l}{t^6} + \frac{5340}{t^5} - \frac{460 e f}{t^5} + \frac{2200 l}{t^5} + \frac{2004}{t^4} -$$

$$\frac{932 e f}{t^4} + \frac{472 l}{t^4} - \frac{3247}{t^3} - \frac{266 e f}{t^3} - \frac{666 l}{t^3} - \frac{3938}{t^2} + \frac{218 e f}{t^2} - \frac{484 l}{t^2} - \frac{219}{t} + \frac{280 e f}{t} - \frac{62 l}{t} - 281 t + 218 e f t +$$

$$62 l t - 4422 t^2 - 266 e f t^2 + 484 l t^2 - 3913 t^3 - 932 e f t^3 + 666 l t^3 + 2476 t^4 - 460 e f t^4 - 472 l t^4 + 7540 t^5 +$$

$$1740 e f t^5 - 2200 l t^5 - 2314 t^6 + 816 e f t^6 + 924 l t^6 - 5381 t^7 - 962 e f t^7 + 1778 l t^7 - 2868 t^8 - 2174 e f t^8 +$$

$$1212 l t^8 + 5260 t^9 + 130 e f t^9 - 2304 l t^9 + 5823 t^{10} + 2940 e f t^{10} - 2810 l t^{10} - 3989 t^{11} + 762 e f t^{11} + 2178 l t^{11} -$$

$$6048 t^{12} - 2334 e f t^{12} + 3096 l t^{12} + 278 t^{13} - 2178 e f t^{13} - 156 l t^{13} + 5354 t^{14} + 930 e f t^{14} - 3108 l t^{14} + 4403 t^{15} +$$

$$3720 e f t^{15} - 2790 l t^{15} - 5644 t^{16} - 24 e f t^{16} + 3744 l t^{16} - 5541 t^{17} - 3662 e f t^{17} + 3638 l t^{17} + 3707 t^{18} - 944 e f t^{18} -$$

$$2718 l t^{18} + 3735 t^{19} + 1450 e f t^{19} - 2394 l t^{19} + 3145 t^{20} + 3900 e f t^{20} - 2450 l t^{20} - 7171 t^{21} - 1602 e f t^{21} +$$

$$5502 l t^{21} - 2542 t^{22} - 3406 e f t^{22} + 1804 l t^{22} + 4262 t^{23} - 2 e f t^{23} - 3404 l t^{23} + 2676 t^{24} + 1870 e f t^{24} - 1872 l t^{24} +$$

$$1525 t^{25} + 3320 e f t^{25} - 1450 l t^{25} - 7558 t^{26} - 3128 e f t^{26} + 6448 l t^{26} + 1003 t^{27} - 1994 e f t^{27} - 1134 l t^{27} +$$

$$3099 t^{28} + 512 e f t^{28} - 2506 l t^{28} + 1910 t^{29} + 2020 e f t^{29} - 1508 l t^{29} - 394 t^{30} + 1840 e f t^{30} + 180 l t^{30} - 5389 t^{31} -$$

$$3058 e f t^{31} + 4898 l t^{31} + 1952 t^{32} - 1074 e f t^{32} - 1984 l t^{32} + 2483 t^{33} + 972 e f t^{33} - 2046 l t^{33} + 834 t^{34} +$$

$$1720 e f t^{34} - 748 l t^{34} - 1649 t^{35} + 250 e f t^{35} + 1470 l t^{35} - 2420 t^{36} - 1982 e f t^{36} + 2232 l t^{36} + 1409 t^{37} -$$

$$576 e f t^{37} - 1406 l t^{37} + 2108 t^{38} + 1248 e f t^{38} - 1824 l t^{38} - 558 t^{39} + 780 e f t^{39} + 468 l t^{39} - 920 t^{40} - 120 e f t^{40} +$$

$$900 l t^{40} - 1355 t^{41} - 1350 e f t^{41} + 1230 l t^{41} + 1310 t^{42} - 90 e f t^{42} - 1260 l t^{42} + 1425 t^{43} + 1200 e f t^{43} -$$

$$1290 l t^{43} - 1535 t^{44} - 230 e f t^{44} + 1430 l t^{44} + 495 t^{45} + 220 e f t^{45} - 450 l t^{45} - 1331 t^{46} - 1022 e f t^{46} + 1242 l t^{46} +$$

$$1378 t^{47} + 294 e f t^{47} - 1316 l t^{47} + 412 t^{48} + 678 e f t^{48} - 384 l t^{48} - 1119 t^{49} - 400 e f t^{49} + 1078 l t^{49} + 675 t^{50} +$$

$$250 e f t^{50} - 650 l t^{50} - 963 t^{51} - 668 e f t^{51} + 918 l t^{51} + 1188 t^{52} + 476 e f t^{52} - 1144 l t^{52} - 436 t^{53} + 52 e f t^{53} +$$

$$424 l t^{53} - 165 t^{54} - 110 e f t^{54} + 162 l t^{54} + 226 t^{55} + 110 e f t^{55} - 220 l t^{55} - 464 t^{56} - 338 e f t^{56} + 448 l t^{56} +$$

$$702 t^{57} + 346 e f t^{57} - 684 l t^{57} - 472 t^{58} - 118 e f t^{58} + 464 l t^{58} + 119 t^{59} - 118 l t^{59} + 31 t^{60} + 30 e f t^{60} - 30 l t^{60} -$$

$$125 t^{61} - 92 e f t^{61} + 122 l t^{61} + 189 t^{62} + 94 e f t^{62} - 186 l t^{62} - 127 t^{63} - 32 e f t^{63} + 126 l t^{63} + 32 t^{64} - 32 l t^{64} \left. \right\}$$

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PrintProfile[];
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NfeDP: called 360 times, time in 442.421/449.

Parents:

( 360) 442.421/ 449.000 under Nfe



Children:

( 360) 6.579/ 6.579 above DP

Together4P: called 1368 times, time in 288.141/288.141

Parents:

( 1368) 288.141/ 288.141 under CF

DP: called 1080 times, time in 25.821/25.821

Parents:

( 360) 6.579/ 6.579 under NfeDP

( 720) 19.242/ 19.242 under Nlx

Nfe $\Delta$ : called 360 times, time in 17.376/17.376

Parents:

( 360) 17.376/ 17.376 under Nfe

CF: called 1368 times, time in 10.38/298.521

Parents:

( 360) 2.439/ 56.548 under m

( 720) 7.127/ 225.224 under Nlx

( 288) 0.814/ 16.749 under z

Children:

( 1368) 288.141/ 288.141 above Together4P

Nlx: called 720 times, time in 2.791/247.257

Parents:

( 720) 2.791/ 247.257 under m

Children:

( 720) 7.127/ 225.224 above CF

( 720) 19.242/ 19.242 above DP

m: called 360 times, time in 2.776/775.47

Parents:

( 360) 2.776/ 775.470 under z

Children:

( 360) 2.439/ 56.548 above CF

( 360) 0.969/ 468.889 above Nfe

( 720) 2.791/ 247.257 above Nlx

NfeQ: called 360 times, time in 1.388/1.388

Parents:

( 360) 1.388/ 1.388 under Nfe

Nfe: called 360 times, time in 0.969/468.889

Parents:

( 360) 0.969/ 468.889 under m

Children:

( 360) 442.421/ 449.000 above NfeDP

( 360) 1.388/ 1.388 above NfeQ

( 360) 17.376/ 17.376 above Nfe $\Delta$

( 360) 0.156/ 0.156 above Nfe $\omega$

z: called 1 times, time in 0.25/792.469

Parents:

( 1) 0.250/ 792.469 under ProfileRoot

Children:

( 288) 0.814/ 16.749 above CF

( 360) 2.776/ 775.470 above m

Nfe $\omega$ : called 360 times, time in 0.156/0.156

Parents:

( 360) 0.156/ 0.156 under Nfe

ProfileRoot: called 0 times, time in 0./0.

Children:

( 1) 0.250/ 792.469 above z