

Pensieve header: Optimization for the NOE1 program base D with  $\mu$  consolidated at NfeDP, with some tracing.

## Initialization

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2017-01"];
<< "NOE-Utilities.m"
```

Loading KnotTheory` version of September 6, 2014, 13:37:37.2841.

Read more at <http://katlas.org/wiki/KnotTheory>.

This is Profile.m. This version: January 2017. Original version: July 1994.

## NOE-It

```
R_{i,j}^+ := E[1, Log[t_i] l_j, e_i f_j, e_i l_i f_j + l_i l_j + e_i^2 f_j^2 / 4];
R_{i,j}^- := E[1, -Log[t_i] l_j, -t_i^{-1} e_i f_j, -l_i l_j + t_i^{-1} e_i l_j f_j - t_i^{-2} e_i^2 f_j^2 / 4];
(ur_{i-} := E[t_i^{-1/2}, 0, 0, l_i t_i^{-2}]; nr_{i-} := E[t_i^{1/2}, 0, 0, -l_i t_i^2];)
```

```
DP_{x \to D_\alpha, y \to D_\beta}[P_][f_] := (* means P[\partial_\alpha, \partial_\beta][f] *)
PP_DP@Total[CoefficientRules[P, {x, y}] /. ({m_, n_} \to c_) \to c D[f, {\alpha, m}, {\beta, n}]]
```

```
CF[E[\omega_, L_, Q_, P_]] := PP_Cf@E[Expand@omega, Expand@L, Expand@Q, Expand@P];
```

```
E /: E[\omega1_, L1_, Q1_, P1_] E[\omega2_, L2_, Q2_, P2_] := CF@E[\omega1 \omega2, L1 + L2, \omega2 Q1 + \omega1 Q2, \omega2^4 P1 + \omega1^4 P2];
```

```
\Delta[k_] := ((t_k - 1) (2 (\alpha \beta + \delta \mu)^2 - \alpha^2 \beta^2) - 4 e_k l_k f_k \delta^2 \mu^2 -
\delta (1 + \mu) (f_k^2 \alpha^2 + e_k^2 \beta^2) - e_k^2 f_k^2 \delta^3 (1 + 3 \mu) -
2 (\alpha \beta + 2 \delta \mu + e_k f_k \delta^2 (1 + 2 \mu) + 2 l_k \delta \mu^2) (f_k \alpha + e_k \beta) - 4 (l_k \mu^2 + e_k f_k \delta (1 + \mu)) (\alpha \beta + \delta \mu)) (1 + t_k) / 4;
```

```
N_{f_i, e_j \to k}[E[\omega_, L_, Q_, P_]] := PP_Nfe@With[{q = ((1 - t_k) \alpha \beta + \beta e_k + \delta e_k f_k + \alpha f_k) / \mu}, E[
PP_Nfep@Expand@Together[\mu \omega / . \mu \to 1 + (t_k - 1) \delta / . \delta \to \omega^{-1} \partial_{f_i, e_j} Q],
L,
PP_Nfeq@Expand@Together[\mu \omega q + \mu (Q / . f_i | e_j \to \theta) / . \mu \to 1 + (t_k - 1) \delta / .
{\alpha \to \omega^{-1} (\partial_{f_i} Q / . e_j \to \theta), \beta \to \omega^{-1} (\partial_{e_j} Q / . f_i \to \theta), \delta \to \omega^{-1} \partial_{f_i, e_j} Q}],
Plus[
PP_Nfedp@Expand@Together[Echo[\mu^4 (DP_{f_i \to D_\alpha, e_j \to D_\beta}[P][e^q] / . e \to 1)] / . Echo@{
\alpha \to \omega^{-1} (\partial_{f_i} Q / . e_j \to \theta), \beta \to \omega^{-1} (\partial_{e_j} Q / . f_i \to \theta), \delta \to \omega^{-1} \partial_{f_i, e_j} Q, \mu \to \omega^{-1} Expand[\omega + (t_k - 1) \partial_{f_i, e_j} Q]
}],
PP_Nfe\Delta@Expand@Together[
\omega^4 \Delta[k] / . \mu \to 1 + (t_k - 1) \delta / . {\alpha \to \omega^{-1} (\partial_{f_i} Q / . e_j \to \theta), \beta \to \omega^{-1} (\partial_{e_j} Q / . f_i \to \theta), \delta \to \omega^{-1} \partial_{f_i, e_j} Q}
]
];
```

```
N_{l_j (x:e|f) \to k}[E[\omega_, L_, Q_, P_]] := PP_Nlx@With[{q = e^x \beta x_k + \gamma l_k}, E[
\omega,
PP_NlxL@Expand@Together[\gamma l_k + (L / . l_j \to \theta) / . \gamma \to \partial_{l_j} L],
PP_NlxQ@Expand@Together[\omega e^x \beta x_k + (Q / . x_i \to \theta) / . {\gamma \to \partial_{l_j} L, \beta \to \omega^{-1} \partial_{x_i} Q}],
PP_NlxP@Expand@Together[e^{-q} DP_{l_j \to D_\gamma, x_i \to D_\beta}[P][e^q] / . {\gamma \to \partial_{l_j} L, \beta \to \omega^{-1} \partial_{x_i} Q}
]
];
```

```

mi,j→k[Z_E] := PP@Module[{x, z},
  CF[(Z // Nfiej→x // Nliex→x // Nfxlj→x) / . Z-i|j|x → Zk]]

```

## Testing 3<sub>1</sub>...

```

BeginProfile[];
Timing[Z[Knot[3, 1]]]
EndProfile[];
PrintProfile[]

```

**KnotTheory:** Loading precomputed data in PD4Knots`.

- »  $\mu^4 \left( \frac{1}{4} t^2 e_{k\$571}^2 f_{k\$571}^2 - t^2 l_{k\$571} + t^2 e_{k\$571} f_{k3\$571} l_{k\$571} + t^2 l_{k3\$571} l_{k\$571} \right)$
- »  $\{\alpha \rightarrow 0, \beta \rightarrow 0, \delta \rightarrow 0, \mu \rightarrow 1\}$
- »  $\mu^4 \left( \frac{1}{4} t^2 e_{k\$571}^2 f_{k3\$571}^2 - t^2 l_{k\$571} + t^2 e_{k\$571} f_{k3\$571} l_{k\$571} + t^2 l_{k3\$571} l_{k\$571} \right)$
- »  $\{\alpha \rightarrow 0, \beta \rightarrow 0, \delta \rightarrow 0, \mu \rightarrow 1\}$
- »  $\mu^4 \left( \frac{1}{4} t^2 e_{k\$571}^2 f_{k3\$571}^2 - t^2 l_{k\$571} + t^2 e_{k\$571} f_{k3\$571} l_{k\$571} + t^2 l_{k3\$571} l_{k\$571} \right)$
- »  $\{\alpha \rightarrow 0, \beta \rightarrow 0, \delta \rightarrow 0, \mu \rightarrow 1\}$
- »  $\mu^4 \left( \frac{t^2 ((1-t)\alpha + e_{x\$599})^2 f_{k\$571}^2}{4\mu^2} - \frac{e_4^2 ((1-t)\beta + f_{x\$599})^2}{4\mu^2} + \frac{t e_4 ((1-t)\beta + f_{x\$599}) l_1}{\mu} - t^2 l_1 l_4 - t^2 l_{k\$571} + \frac{t^2 ((1-t)\alpha + e_{x\$599}) f_{k\$571} l_{k\$571}}{\mu} + t^2 l_{k\$571}^2 \right)$
- »  $\left\{ \alpha \rightarrow -\frac{e_4}{t}, \beta \rightarrow f_{k\$571}, \delta \rightarrow 0, \mu \rightarrow 1 \right\}$
- »  $\mu^4 \left( \frac{1}{4} t^2 e_1^2 f_1^2 - \frac{t^2 ((1-t)\alpha + e_{x\$606})^2 f_1^2}{4\mu^2} - t^4 l_1 + t^3 e_1 f_1 l_1 + t^4 l_1^2 + \frac{((1-t)\alpha + e_{x\$606})(t^2 e_1 f_1^2 + t^3 f_1 l_1)}{\mu} - t^3 e_1 f_1 l_4 - t^4 l_1 l_4 - t^4 l_{k\$571} \right)$
- »  $\left\{ \alpha \rightarrow 0, \beta \rightarrow -\frac{f_1}{t}, \delta \rightarrow 0, \mu \rightarrow 1 \right\}$
- »  $\mu^4 \left( t^3 e_4 f_1 + \frac{1}{4} t^2 e_1^2 f_1^2 + t^2 e_1 e_4 f_1^2 - \frac{1}{4} t^2 e_4^2 f_1^2 - t^4 l_1 + t^3 e_1 f_1 l_1 + t^3 e_4 f_1 l_1 + t^4 l_1^2 - t^4 l_4 - t^3 e_1 f_1 l_4 - t^4 l_1 l_4 \right)$
- »  $\{\alpha \rightarrow 0, \beta \rightarrow 0, \delta \rightarrow 0, \mu \rightarrow 1\}$
- »  $\mu^4 \left( t^3 e_4 f_1 - \frac{1}{4} t^2 e_4^2 f_1^2 + \frac{t^2 ((1-t)\alpha + e_{x\$620})^2 f_1^2}{4\mu^2} - t^4 l_1 + t^3 e_4 f_1 l_1 + t^4 l_1^2 - t^4 l_4 - t^4 l_1 l_4 + \frac{((1-t)\alpha + e_{x\$620})(t^2 e_4 f_1^2 + t^3 f_1 l_1 - t^3 f_1 l_4)}{\mu} \right)$
- »  $\left\{ \alpha \rightarrow 0, \beta \rightarrow \frac{f_1}{t}, \delta \rightarrow 0, \mu \rightarrow 1 \right\}$
- »  $\mu^4 \left( \frac{(\frac{1}{4} t^2 e_1^2 + t^2 e_1 e_4 - \frac{1}{4} t^2 e_4^2) ((1-t)\beta + f_{x\$627})^2}{\mu^2} - t^4 l_1 + t^4 l_1^2 - t^4 l_4 - t^4 l_1 l_4 + \frac{((1-t)\beta + f_{x\$627})(t^3 e_4 + t^3 e_1 l_1 + t^3 e_4 l_1 - t^3 e_1 l_4)}{\mu} \right)$
- »  $\left\{ \alpha \rightarrow \frac{e_1 - e_4}{t}, \beta \rightarrow 0, \delta \rightarrow 0, \mu \rightarrow 1 \right\}$
- »  $\mu^4 \left( t^3 e_4 f_1 - \frac{1}{4} t^2 e_4^2 f_1^2 + \frac{t^2 ((1-t)\alpha + e_{x\$634})^2 f_1^2}{4\mu^2} - t^4 l_1 + t^3 e_4 f_1 l_1 + t^4 l_1^2 - t^4 l_4 - t^4 l_1 l_4 + \frac{((1-t)\alpha + e_{x\$634})(t^2 e_4 f_1^2 + t^3 f_1 l_1 - t^3 f_1 l_4)}{\mu} \right)$

- »  $\left\{ \alpha \rightarrow 0, \beta \rightarrow \frac{f_1}{t}, \delta \rightarrow 0, \mu \rightarrow 1 \right\}$
- »  $\mu^4 \left( \frac{1}{4} t^2 e_{k\$571}^2 f_{k3\$571}^2 - t^2 l_{k1\$571} + t^2 e_{k\$571} f_{k3\$571} l_{k\$571} + t^2 l_{k3\$571} l_{k\$571} \right)$
- »  $\left\{ \alpha \rightarrow 0, \beta \rightarrow 0, \delta \rightarrow 0, \mu \rightarrow 1 \right\}$
- »  $\mu^4 \left( \frac{1}{4} t^2 e_{k\$571}^2 f_{k3\$571}^2 - t^2 l_{k\$571} + t^2 e_{k\$571} f_{k3\$571} l_{k\$571} + t^2 l_{k3\$571} l_{k\$571} \right)$
- »  $\left\{ \alpha \rightarrow 0, \beta \rightarrow 0, \delta \rightarrow 0, \mu \rightarrow 1 \right\}$
- »  $\mu^4 \left( \frac{1}{4} t^2 e_{k\$571}^2 f_{k3\$571}^2 - t^2 l_{k\$571} + t^2 e_{k\$571} f_{k3\$571} l_{k\$571} + t^2 l_{k3\$571} l_{k\$571} \right)$
- »  $\left\{ \alpha \rightarrow 0, \beta \rightarrow 0, \delta \rightarrow 0, \mu \rightarrow 1 \right\}$
- »  $\mu^4 \left( \frac{t^2 \left( (1-t) \alpha + e_{x\$666} \right)^2 f_{k3\$571}^2 - e_2^2 \left( (1-t) \beta + f_{x\$666} \right)^2}{4 \mu^2} + \frac{t e_2 \left( (1-t) \beta + f_{x\$666} \right) l_5}{\mu} - t^2 l_2 l_5 - t^2 l_{k\$571} + \frac{t^2 \left( (1-t) \alpha + e_{x\$666} \right) f_{k\$571} l_{k\$571} + t^2 l_{k\$571}^2}{\mu} \right)$
- »  $\left\{ \alpha \rightarrow -\frac{e_2}{t}, \beta \rightarrow f_{k\$571}, \delta \rightarrow 0, \mu \rightarrow 1 \right\}$
- »  $\mu^4 \left( \frac{1}{4} e_5^2 f_5^2 - \frac{\left( (1-t) \alpha + e_{x\$673} \right)^2 f_5^2}{4 \mu^2} - t e_5 f_5 l_2 - t^2 l_5 + t e_5 f_5 l_5 - t^2 l_2 l_5 + t^2 l_5^2 + \frac{\left( (1-t) \alpha + e_{x\$673} \right) \left( e_5 f_5^2 + t f_5 l_5 \right)}{\mu} \right)$
- »  $\left\{ \alpha \rightarrow 0, \beta \rightarrow -\frac{f_5}{t}, \delta \rightarrow 0, \mu \rightarrow 1 \right\}$
- »  $\mu^4 \left( -\frac{1}{4} e_2^2 f_5^2 + e_2 e_5 f_5^2 + \frac{1}{4} e_5^2 f_5^2 - t e_5 f_5 l_2 - t^2 l_5 + t e_2 f_5 l_5 + t e_5 f_5 l_5 - t^2 l_2 l_5 + t^2 l_5^2 \right)$
- »  $\left\{ \alpha \rightarrow 0, \beta \rightarrow 0, \delta \rightarrow 0, \mu \rightarrow 1 \right\}$
- »  $\mu^4 \left( -\frac{1}{4} e_2^2 f_5^2 + \frac{\left( (1-t) \alpha + e_{x\$687} \right)^2 f_5^2}{4 \mu^2} - t^2 l_5 + t e_2 f_5 l_5 - t^2 l_2 l_5 + t^2 l_5^2 + \frac{\left( (1-t) \alpha + e_{x\$687} \right) \left( e_2 f_5^2 - t f_5 l_2 + t f_5 l_5 \right)}{\mu} \right)$
- »  $\left\{ \alpha \rightarrow 0, \beta \rightarrow \frac{f_5}{t}, \delta \rightarrow 0, \mu \rightarrow 1 \right\}$
- »  $\mu^4 \left( \frac{\left( -\frac{e_2}{4} + e_2 e_5 + \frac{e_5}{4} \right) \left( (1-t) \beta + f_{x\$694} \right)^2}{\mu^2} - t^2 l_5 - t^2 l_2 l_5 + t^2 l_5^2 + \frac{\left( (1-t) \beta + f_{x\$694} \right) \left( -t e_5 l_2 + t e_2 l_5 + t e_5 l_5 \right)}{\mu} \right)$
- »  $\left\{ \alpha \rightarrow \frac{-\frac{e_2}{\sqrt{t}} + \frac{e_5}{\sqrt{t}}}{\sqrt{t}}, \beta \rightarrow 0, \delta \rightarrow 0, \mu \rightarrow 1 \right\}$
- »  $\mu^4 \left( \frac{1}{4} t^4 e_5^2 f_5^2 - \frac{t^4 \left( (1-t) \alpha + e_{x\$701} \right)^2 f_5^2}{4 \mu^2} + \frac{\left( \frac{1}{4} t^4 e_0^2 + t^4 e_0 e_4 - \frac{1}{4} t^4 e_4^2 \right) \left( (1-t) \beta + f_{x\$701} \right)^2}{\mu^2} - \frac{t^6 l_0 + t^6 l_0^2 - t^5 e_5 f_5 l_2 - t^6 l_4 - t^6 l_0 l_4 + \frac{\left( (1-t) \beta + f_{x\$701} \right) \left( t^5 e_4 + t^5 e_0 l_0 + t^5 e_4 l_0 - t^5 e_0 l_4 \right)}{\mu}}{t^6 l_5 + t^5 e_5 f_5 l_5 - t^6 l_2 l_5 + t^6 l_5^2 + \frac{\left( (1-t) \alpha + e_{x\$701} \right) \left( t^4 e_5 f_5^2 + t^5 f_5 l_5 \right)}{\mu}} \right)$
- »  $\left\{ \alpha \rightarrow \frac{\sqrt{t} e_0 - \sqrt{t} e_4}{t^{3/2}}, \beta \rightarrow -\frac{f_5}{t}, \delta \rightarrow 0, \mu \rightarrow 1 \right\}$
- »  $\mu^4 \left( t^5 e_4 f_0 + \frac{1}{4} t^4 e_0^2 f_0^2 + t^4 e_0 e_4 f_0^2 - \frac{1}{4} t^4 e_4^2 f_0^2 - t^4 e_4 f_5 + t^5 e_4 f_5 - 3 t^3 e_0 e_4 f_0 f_5 + t^3 e_4^2 f_0 f_5 - \frac{1}{4} t^2 e_0^2 f_5^2 + 2 t^2 e_0 e_4 f_5^2 - t^3 e_0 e_4 f_5^2 - \frac{3}{4} t^2 e_4^2 f_5^2 + t^3 e_4^2 f_5^2 - \frac{1}{4} t^4 e_4^2 f_5^2 + \frac{t^4 \left( (1-t) \alpha + e_{x\$708} \right)^2 f_5^2}{4 \mu^2} - t^6 l_0 + t^5 e_0 f_0 l_0 + t^5 e_4 f_0 l_0 - 2 t^4 e_4 f_5 l_0 + t^6 l_0^2 - t^6 l_4 - t^5 e_0 f_0 l_4 + t^4 e_0 f_5 l_4 - t^6 l_0 l_4 - t^6 l_5 - t^5 e_0 f_0 l_5 + t^5 e_4 f_0 l_5 + t^4 e_0 f_5 l_5 - t^4 e_4 f_5 l_5 + t^5 e_4 f_5 l_5 - t^6 l_0 l_5 + t^6 l_5^2 + \frac{1}{\mu} \left( (1-t) \alpha + e_{x\$708} \right) \left( -t^4 e_0 f_0 f_5 + t^4 e_4 f_0 f_5 + t^3 e_0 f_5^2 - t^3 e_4 f_5^2 + t^4 e_4 f_5^2 - t^5 f_5 l_0 + t^5 f_5 l_5 \right) \right)$
- »  $\left\{ \alpha \rightarrow 0, \beta \rightarrow \frac{f_5}{t}, \delta \rightarrow 0, \mu \rightarrow 1 \right\}$
- »  $\mu^4 \left( \frac{1}{4} t^2 e_{k\$571}^2 f_{k3\$571}^2 - t^2 l_{k1\$571} + t^2 e_{k\$571} f_{k3\$571} l_{k\$571} + t^2 l_{k3\$571} l_{k\$571} \right)$
- »  $\left\{ \alpha \rightarrow 0, \beta \rightarrow 0, \delta \rightarrow 0, \mu \rightarrow 1 \right\}$

- »  $\mu^4 \left( \frac{1}{4} t^2 e_{k\$571}^2 f_{k3\$571}^2 - t^2 l_{k\$571} + t^2 e_{k\$571} f_{k3\$571} l_{k\$571} + t^2 l_{k3\$571} l_{k\$571} \right)$
- »  $\{\alpha \rightarrow 0, \beta \rightarrow 0, \delta \rightarrow 0, \mu \rightarrow 1\}$
- »  $\mu^4 \left( \frac{1}{4} t^2 e_{k\$571}^2 f_{k3\$571}^2 - t^2 l_{k\$571} + t^2 e_{k\$571} f_{k3\$571} l_{k\$571} + t^2 l_{k3\$571} l_{k\$571} \right)$
- »  $\{\alpha \rightarrow 0, \beta \rightarrow 0, \delta \rightarrow 0, \mu \rightarrow 1\}$
- »  $\mu^4 \left( \frac{t^2 ((1-t) \alpha + e_{x\$737})^2 f_{k\$571}^2}{4 \mu^2} - \frac{e_6^2 ((1-t) \beta + f_{x\$737})^2}{4 \mu^2} + \frac{t e_6 ((1-t) \beta + f_{x\$737}) l_3}{\mu} - t^2 l_3 l_6 - t^2 l_{k\$571} + \frac{t^2 ((1-t) \alpha + e_{x\$737}) f_{k\$571} l_{k\$571}}{\mu} + t^2 l_{k\$571}^2 \right)$
- »  $\left\{ \alpha \rightarrow -\frac{e_6}{t}, \beta \rightarrow f_{k\$571}, \delta \rightarrow 0, \mu \rightarrow 1 \right\}$
- »  $\mu^4 \left( \frac{1}{4} e_3^2 f_3^2 - \frac{((1-t) \alpha + e_{x\$744})^2 f_3^2}{4 \mu^2} - t^2 l_3 + t e_3 f_3 l_3 + t^2 l_3^2 + \frac{((1-t) \alpha + e_{x\$744}) (e_3 f_3^2 + t f_3 l_3)}{\mu} - t e_3 f_3 l_6 - t^2 l_3 l_6 \right)$
- »  $\left\{ \alpha \rightarrow 0, \beta \rightarrow -\frac{f_3}{t}, \delta \rightarrow 0, \mu \rightarrow 1 \right\}$
- »  $\mu^4 \left( \frac{1}{4} e_3^2 f_3^2 + e_3 e_6 f_3^2 - \frac{1}{4} e_6^2 f_3^2 - t^2 l_3 + t e_3 f_3 l_3 + t e_6 f_3 l_3 + t^2 l_3^2 - t e_3 f_3 l_6 - t^2 l_3 l_6 \right)$
- »  $\{\alpha \rightarrow 0, \beta \rightarrow 0, \delta \rightarrow 0, \mu \rightarrow 1\}$
- »  $\mu^4 \left( -\frac{1}{4} e_6^2 f_3^2 + \frac{((1-t) \alpha + e_{x\$758})^2 f_3^2}{4 \mu^2} - t^2 l_3 + t e_6 f_3 l_3 + t^2 l_3^2 - t^2 l_3 l_6 + \frac{((1-t) \alpha + e_{x\$758}) (e_6 f_3^2 + t f_3 l_3 - t f_3 l_6)}{\mu} \right)$
- »  $\left\{ \alpha \rightarrow 0, \beta \rightarrow \frac{f_3}{t}, \delta \rightarrow 0, \mu \rightarrow 1 \right\}$
- »  $\mu^4 \left( \frac{(e_3^2 + e_3 e_6 - \frac{e_6^2}{4}) ((1-t) \beta + f_{x\$765})^2}{\mu^2} - t^2 l_3 + t^2 l_3^2 - t^2 l_3 l_6 + \frac{((1-t) \beta + f_{x\$765}) (t e_3 l_3 + t e_6 l_3 - t e_3 l_6)}{\mu} \right)$
- »  $\left\{ \alpha \rightarrow \frac{\frac{e_3}{\sqrt{t}} - \frac{e_6}{\sqrt{t}}}{\sqrt{t}}, \beta \rightarrow 0, \delta \rightarrow 0, \mu \rightarrow 1 \right\}$
- »  $\mu^4 \left( t^7 e_4 f_0 + \frac{1}{4} t^6 e_0^2 f_0^2 + t^6 e_0 e_4 f_0^2 - \frac{1}{4} t^6 e_4^2 f_0^2 + \frac{1}{4} t^6 e_3^2 f_3^2 - \frac{t^6 ((1-t) \alpha + e_{x\$772})^2 f_3^2}{4 \mu^2} + \frac{(-\frac{1}{4} t^4 e_0^2 + 2 t^4 e_0 e_4 + t^5 e_0 e_4 - \frac{3}{4} t^4 e_4^2 + t^6 e_4^2) ((1-t) \beta + f_{x\$772})^2}{\mu^2} - t^8 l_0 + t^7 e_0 f_0 l_0 + t^7 e_4 f_0 l_0 + t^8 l_0^2 - t^8 l_3 + t^7 e_3 f_3 l_3 + t^8 l_3^2 + \frac{((1-t) \alpha + e_{x\$772}) (t^6 e_3 f_3^2 + t^7 f_3 l_3)}{\mu} - 2 t^8 l_4 - 2 t^7 e_0 f_0 l_4 + t^7 e_4 f_0 l_4 - 2 t^8 l_0 l_4 + t^8 l_4^2 + \frac{1}{\mu} ((1-t) \beta + f_{x\$772}) (-t^6 e_4 - 3 t^5 e_0 e_4 f_0 - 2 t^6 e_0 e_4 f_0 + t^5 e_4^2 f_0 + t^6 e_4^2 f_0 - 2 t^6 e_4 l_0 - 2 t^7 e_4 l_0 + 2 t^6 e_0 l_4 - t^6 e_4 l_4 + 2 t^7 e_4 l_4) - t^7 e_3 f_3 l_6 - t^8 l_3 l_6 \right)$
- »  $\left\{ \alpha \rightarrow \frac{-e_0 + e_4}{t^2}, \beta \rightarrow -\frac{f_3}{t}, \delta \rightarrow 0, \mu \rightarrow 1 \right\}$

$$\begin{aligned}
& \gg \mu^4 \left( \frac{1}{4} t^6 e_0^2 f_0^2 - \frac{1}{4} t^4 e_0^2 f_4^2 + \frac{((1-t)\alpha + e_{x\$779})^2 \left(-\frac{1}{4} t^6 f_0^2 + t^5 f_0 f_4 + t^6 f_0 f_4 - \frac{3}{4} t^4 f_4^2 + t^6 f_4^2\right)}{\mu^2} + \right. \\
& \quad \left. \frac{\left(-\frac{3}{4} t^2 e_0^2 + t^3 e_0^2 - \frac{1}{4} t^4 e_0^2 - t^4 e_0 e_3 + t^5 e_0 e_3 + \frac{1}{4} t^6 e_3^2\right) \left((1-t)\beta + f_{x\$779}\right)^2}{\mu^2} + \right. \\
& \quad \left. \left(-t^4 f_0 + 2 t^3 f_4 - t^4 f_4 - t^5 f_4\right) \left( \frac{2(1-t) \left((1-t)\alpha + e_{x\$779}\right)}{\mu^2} + \frac{\left((1-t)\alpha + e_{x\$779}\right)^2 \left((1-t)\beta + f_{x\$779}\right)}{\mu^3} \right) + \right. \\
& \quad \left. \left(3 t^2 e_0 - 4 t^3 e_0 + 3 t^4 e_0 - t^5 e_0 + t^4 e_3 - t^5 e_3 + t^6 e_3\right) \right. \\
& \quad \left. \left( \frac{2(1-t) \left((1-t)\beta + f_{x\$779}\right)}{\mu^2} + \frac{\left((1-t)\alpha + e_{x\$779}\right) \left((1-t)\beta + f_{x\$779}\right)^2}{\mu^3} \right) + \left( -\frac{5 t^2}{4} + 2 t^3 - \frac{7 t^4}{4} + t^5 - \frac{t^6}{4} \right) \right. \\
& \quad \left. \left( \frac{2(1-t)^2}{\mu^2} + \frac{4(1-t) \left((1-t)\alpha + e_{x\$779}\right) \left((1-t)\beta + f_{x\$779}\right)}{\mu^3} + \frac{\left((1-t)\alpha + e_{x\$779}\right)^2 \left((1-t)\beta + f_{x\$779}\right)^2}{\mu^4} \right) - \right. \\
& \quad t^8 l_0 + t^7 e_0 f_0 l_0 + t^8 l_0^2 - t^8 l_3 + t^6 e_0 f_4 l_3 + t^8 l_3^2 - 2 t^8 l_4 - 2 t^7 e_0 f_0 l_4 + 2 t^6 e_0 f_4 l_4 - \\
& \quad \left. 2 t^8 l_0 l_4 - t^8 l_3 l_4 + t^8 l_4^2 + \left( \frac{1-t}{\mu} + \frac{\left((1-t)\alpha + e_{x\$779}\right) \left((1-t)\beta + f_{x\$779}\right)}{\mu^2} \right) \right. \\
& \quad \left. \left( t^5 - t^6 + t^7 + 3 t^4 e_0 f_0 - t^5 e_0 f_0 - 5 t^3 e_0 f_4 + t^4 e_0 f_4 - t^5 e_3 f_4 + 2 t^5 l_0 + t^5 l_3 - t^6 l_3 + t^7 l_3 + 2 t^5 l_4 - 2 t^6 l_4 \right) + \right. \\
& \quad \frac{1}{\mu} \left( (1-t)\beta + f_{x\$779} \right) \left( t^3 e_0^2 f_4 + t^5 e_0 e_3 f_4 - t^5 e_0 l_3 + t^6 e_0 l_3 + t^7 e_3 l_3 - 3 t^5 e_0 l_4 + t^6 e_0 l_4 - t^7 e_3 l_4 \right) + \\
& \quad \frac{1}{\mu} \left( (1-t)\alpha + e_{x\$779} \right) \left( t^7 f_0 + t^6 e_0 f_0^2 - t^6 f_4 - 3 t^5 e_0 f_0 f_4 - 2 t^6 e_0 f_0 f_4 + 2 t^4 e_0 f_4^2 + \right. \\
& \quad \left. t^5 e_0 f_4^2 + t^7 f_0 l_0 - 2 t^6 f_4 l_0 - 2 t^7 f_4 l_0 - t^6 f_4 l_3 + t^7 f_0 l_4 - t^6 f_4 l_4 + 2 t^7 f_4 l_4 \right) \left. \right) \\
& \gg \left\{ \alpha \rightarrow \frac{-e_0 + \frac{e_a}{t} + t e_3}{t^2}, \beta \rightarrow \frac{-t f_0 + f_4}{t^2}, \delta \rightarrow \frac{1 - \frac{1}{t} - t}{t^2}, \mu \rightarrow \frac{-2 + \frac{1}{t} + 2 t}{t^2} \right\}
\end{aligned}$$

$$\begin{aligned}
& \mu^4 \left( -129 - \frac{2}{t^4} + \frac{14}{t^3} - \frac{47}{t^2} + \frac{96}{t} + 116t - 68t^2 + 24t^3 - 4t^4 + 12e_0 f_3 - \frac{4e_0 f_3}{t} - 18te_0 f_3 + 12t^2 e_0 f_3 - 4t^3 e_0 f_3 + \frac{7}{2} e_0^2 f_3^2 + \right. \\
& \frac{3e_0^2 f_3^2}{4t^2} - \frac{2e_0^2 f_3^2}{t} - 2te_0^2 f_3^2 + \frac{1}{2} t^2 e_0^2 f_3^2 + \frac{((1-t)\alpha + e_{x\$786})^2 \left( \frac{119f_3^2}{2} + \frac{f_3^2}{t^4} - \frac{8f_3^2}{t^3} + \frac{109f_3^2}{4t^2} - \frac{52f_3^2}{t} - 38te_0 f_3^2 + \frac{21}{2} t^2 f_3^2 \right)}{\mu^2} + \\
& \frac{\left( \frac{119e_0^2}{2} + \frac{e_0^2}{t^4} - \frac{8e_0^2}{t^3} + \frac{109e_0^2}{4t^2} - \frac{52e_0^2}{t} - 38te_0^2 + \frac{21}{2} t^2 e_0^2 \right) \left( (1-t)\beta + f_{x\$786} \right)^2}{\mu^2} + \\
& \left. \left( -11f_3 + \frac{f_3}{t^3} - \frac{5f_3}{t^2} + \frac{10f_3}{t} + 4te_0 f_3 + t^2 f_3 \right) \left( \frac{2(1-t)\left( (1-t)\alpha + e_{x\$786} \right)}{\mu^2} + \frac{\left( (1-t)\alpha + e_{x\$786} \right)^2 \left( (1-t)\beta + f_{x\$786} \right)}{\mu^3} \right) + \right. \\
& \left. \left( -3e_0 + \frac{e_0}{t^3} - \frac{4e_0}{t^2} + \frac{6e_0}{t} - 4te_0 + 5t^2 e_0 \right) \left( \frac{2(1-t)\left( (1-t)\beta + f_{x\$786} \right)}{\mu^2} + \frac{\left( (1-t)\alpha + e_{x\$786} \right) \left( (1-t)\beta + f_{x\$786} \right)^2}{\mu^3} \right) + \right. \\
& \left. \left( \frac{7}{2} + \frac{3}{4t^2} - \frac{2}{t} - 2t + \frac{t^2}{2} \right) \right. \\
& \left. \left( \frac{2(1-t)^2}{\mu^2} + \frac{4(1-t)\left( (1-t)\alpha + e_{x\$786} \right) \left( (1-t)\beta + f_{x\$786} \right)}{\mu^3} + \frac{\left( (1-t)\alpha + e_{x\$786} \right)^2 \left( (1-t)\beta + f_{x\$786} \right)^2}{\mu^4} \right) + \frac{l_0}{t^4} - \right. \\
& \frac{6l_0}{t^3} + \frac{16l_0}{t^2} - \frac{20l_0}{t} + 40tl_0 - 64t^2 l_0 + 48t^3 l_0 - 16t^4 l_0 + 32e_0 f_3 l_0 - \frac{2e_0 f_3 l_0}{t^3} + \frac{10e_0 f_3 l_0}{t^2} - \frac{24e_0 f_3 l_0}{t} - \\
& 24te_0 f_3 l_0 + 8t^2 e_0 f_3 l_0 + 136l_0^2 + \frac{l_0^2}{t^4} - \frac{8l_0^2}{t^3} + \frac{32l_0^2}{t^2} - \frac{80l_0^2}{t} - 160tl_0^2 + 128t^2 l_0^2 - 64t^3 l_0^2 + 16t^4 l_0^2 + \frac{l_3}{t^4} - \frac{6l_3}{t^3} + \frac{16l_3}{t^2} - \\
& \frac{20l_3}{t} + 40tl_3 - 64t^2 l_3 + 48t^3 l_3 - 16t^4 l_3 - \frac{e_0 f_3 l_3}{t^3} + \frac{4e_0 f_3 l_3}{t^2} - \frac{6e_0 f_3 l_3}{t} + 12te_0 f_3 l_3 - 16t^2 e_0 f_3 l_3 + 8t^3 e_0 f_3 l_3 - \\
& 272l_0 l_3 - \frac{2l_0 l_3}{t^4} + \frac{16l_0 l_3}{t^3} - \frac{64l_0 l_3}{t^2} + \frac{160l_0 l_3}{t} + 320tl_0 l_3 - 256t^2 l_0 l_3 + 128t^3 l_0 l_3 - 32t^4 l_0 l_3 + 136l_3^2 + \frac{l_3^2}{t^4} - \\
& \frac{8l_3^2}{t^3} + \frac{32l_3^2}{t^2} - \frac{80l_3^2}{t} - 160tl_3^2 + 128t^2 l_3^2 - 64t^3 l_3^2 + 16t^4 l_3^2 + \left( \frac{1-t}{\mu} + \frac{\left( (1-t)\alpha + e_{x\$786} \right) \left( (1-t)\beta + f_{x\$786} \right)}{\mu^2} \right) \\
& \left. \left( -20 + \frac{1}{t^3} - \frac{6}{t^2} + \frac{14}{t} + 18t - 12t^2 + 4t^3 - 98e_0 f_3 - \frac{2e_0 f_3}{t^4} + \frac{12e_0 f_3}{t^3} - \frac{38e_0 f_3}{t^2} + \frac{76e_0 f_3}{t} + 80te_0 f_3 - \right. \right. \\
& \left. \left. 34t^2 e_0 f_3 - \frac{l_0}{t^3} + \frac{4l_0}{t^2} - \frac{6l_0}{t} + 12tl_0 - 16t^2 l_0 + 8t^3 l_0 + 32l_3 - \frac{2l_3}{t^3} + \frac{10l_3}{t^2} - \frac{24l_3}{t} - 24tl_3 + 8t^2 l_3 \right) + \right. \\
& \frac{1}{\mu} \left( (1-t)\beta + f_{x\$786} \right) \left( -84e_0 - \frac{2e_0}{t^4} + \frac{12e_0}{t^3} - \frac{36e_0}{t^2} + \frac{68e_0}{t} + 66te_0 - 28t^2 e_0 + 4t^3 e_0 - 11e_0^2 f_3 + \frac{e_0^2 f_3}{t^3} - \frac{5e_0^2 f_3}{t^2} + \right. \\
& \frac{10e_0^2 f_3}{t} + 4te_0^2 f_3 + t^2 e_0^2 f_3 - 200e_0 l_0 - \frac{2e_0 l_0}{t^4} + \frac{17e_0 l_0}{t^3} - \frac{64e_0 l_0}{t^2} + \frac{142e_0 l_0}{t} + 180te_0 l_0 - 96t^2 e_0 l_0 + \\
& 24t^3 e_0 l_0 + 168e_0 l_3 + \frac{2e_0 l_3}{t^4} - \frac{14e_0 l_3}{t^3} + \frac{50e_0 l_3}{t^2} - \frac{112e_0 l_3}{t} - 168te_0 l_3 + 104t^2 e_0 l_3 - 32t^3 e_0 l_3 \left. \right) + \\
& \frac{1}{\mu} \left( (1-t)\alpha + e_{x\$786} \right) \left( -84f_3 - \frac{2f_3}{t^4} + \frac{12f_3}{t^3} - \frac{36f_3}{t^2} + \frac{68f_3}{t} + 66tf_3 - 28t^2 f_3 + 4t^3 f_3 - 3e_0 f_3^2 + \frac{e_0 f_3^2}{t^3} - \frac{4e_0 f_3^2}{t^2} + \right. \\
& \frac{6e_0 f_3^2}{t} - 4te_0 f_3^2 + 5t^2 e_0 f_3^2 + 168f_3 l_0 + \frac{2f_3 l_0}{t^4} - \frac{14f_3 l_0}{t^3} + \frac{50f_3 l_0}{t^2} - \frac{112f_3 l_0}{t} - 168tf_3 l_0 + 104t^2 f_3 l_0 - \\
& \left. \left. 32t^3 f_3 l_0 - 200f_3 l_3 - \frac{2f_3 l_3}{t^4} + \frac{17f_3 l_3}{t^3} - \frac{64f_3 l_3}{t^2} + \frac{142f_3 l_3}{t} + 180tf_3 l_3 - 96t^2 f_3 l_3 + 24t^3 f_3 l_3 \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left\{ \alpha \rightarrow \frac{e_0}{-2 + \frac{1}{t} + 2t}, \beta \rightarrow \frac{f_3}{-2 + \frac{1}{t} + 2t}, \delta \rightarrow -\frac{1}{-2 + \frac{1}{t} + 2t}, \mu \rightarrow \frac{-1 + \frac{1}{t} + t}{-2 + \frac{1}{t} + 2t} \right\} \\
& \left\{ 0.765625, \mathbb{E} \left[ -1 + \frac{1}{t} + t, 0, 0, -16 + 2ef - \frac{2}{t^4} - \frac{2ef}{t^4} + \frac{2l}{t^4} + \frac{7}{t^3} + \frac{4ef}{t^3} - \frac{6l}{t^3} - \frac{14}{t^2} - \frac{6ef}{t^2} + \right. \right. \\
& \left. \left. \frac{10l}{t^2} + \frac{18}{t} + \frac{2ef}{t} - \frac{8l}{t} + 10t - 6eft + 8lt - 4t^2 + 4eft^2 - 10lt^2 + t^3 - 2eft^3 + 6lt^3 - 2lt^4 \right] \right\}
\end{aligned}$$

NfeDP: called 30 times, time in 0.298/0.313

Parents:

( 30 ) 0.298/ 0.313 under Nfe

Children:

( 30 ) 0.015/ 0.015 above DP

NlxP: called 60 times, time in 0.248/0.326

Parents:

( 60 ) 0.248/ 0.326 under Nlx

Children:

( 60 ) 0.078/ 0.078 above DP

DP: called 90 times, time in 0.093/0.093

Parents:

```

( 30) 0.015/ 0.015 under NfeDP
( 60) 0.078/ 0.078 under NlxP
Nfe $\Delta$ : called 30 times, time in 0.031/0.031
Parents:
( 30) 0.031/ 0.031 under Nfe
z: called 1 times, time in 0.016/0.734
Parents:
( 1) 0.016/ 0.734 under ProfileRoot
Children:
( 24) 0.000/ 0.000 above CF
( 30) 0.000/ 0.718 above m
NlxL: called 60 times, time in 0.016/0.016
Parents:
( 60) 0.016/ 0.016 under Nlx
Nfe $\omega$ : called 30 times, time in 0.016/0.016
Parents:
( 30) 0.016/ 0.016 under Nfe
NfeQ: called 30 times, time in 0.016/0.016
Parents:
( 30) 0.016/ 0.016 under Nfe
ProfileRoot: called 0 times, time in 0./0.
Children:
( 1) 0.016/ 0.734 above z
NlxQ: called 60 times, time in 0./0.
Parents:
( 60) 0.000/ 0.000 under Nlx
Nlx: called 60 times, time in 0./0.342
Parents:
( 60) 0.000/ 0.342 under m
Children:
( 60) 0.016/ 0.016 above NlxL
( 60) 0.248/ 0.326 above NlxP
( 60) 0.000/ 0.000 above NlxQ
Nfe: called 30 times, time in 0./0.376
Parents:
( 30) 0.000/ 0.376 under m
Children:
( 30) 0.298/ 0.313 above NfeDP
( 30) 0.016/ 0.016 above NfeQ
( 30) 0.031/ 0.031 above Nfe $\Delta$ 
( 30) 0.016/ 0.016 above Nfe $\omega$ 
m: called 30 times, time in 0./0.718
Parents:
( 30) 0.000/ 0.718 under z
Children:
( 30) 0.000/ 0.000 above CF
( 30) 0.000/ 0.376 above Nfe
( 60) 0.000/ 0.342 above Nlx
CF: called 54 times, time in 0./0.
Parents:
( 30) 0.000/ 0.000 under m
( 24) 0.000/ 0.000 under z

```

Testing IO<sub>100</sub>...

```
BeginProfile [];  
Timing[Z[Knot[10, 100]]]  
EndProfile [];  
PrintProfile []
```

## Testing $T_{9,5}$ ...

```
BeginProfile [];  
Timing[Z[TorusKnot[9, 5]]]  
EndProfile [];  
PrintProfile []
```