

Pensieve header: Figuring out the commutation relations of $sl(3)$.

$$sl3rule = \left\{ h1 \rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, h2 \rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, e1 \rightarrow \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \right.$$

$$\left. e2 \rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, e3 \rightarrow \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, f1 \rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, f2 \rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, f3 \rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \right\}$$

$h1 \rightarrow \{\{1, 0, 0\}, \{0, -1, 0\}, \{0, 0, 0\}\}$, $h2 \rightarrow \{\{0, 0, 0\}, \{0, 1, 0\}, \{0, 0, -1\}\}$,
 $e1 \rightarrow \{\{0, 1, 0\}, \{0, 0, 0\}, \{0, 0, 0\}\}$, $e2 \rightarrow \{\{0, 0, 0\}, \{0, 0, 1\}, \{0, 0, 0\}\}$,
 $e3 \rightarrow \{\{0, 0, 1\}, \{0, 0, 0\}, \{0, 0, 0\}\}$, $f1 \rightarrow \{\{0, 0, 0\}, \{1, 0, 0\}, \{0, 0, 0\}\}$,
 $f2 \rightarrow \{\{0, 0, 0\}, \{0, 0, 0\}, \{0, 1, 0\}\}$, $f3 \rightarrow \{\{0, 0, 0\}, \{0, 0, 0\}, \{1, 0, 0\}\}$

```
InverseRule[rules_List][m_] := Module[{k, s, a, g},
  k = 0; s = Sum[a[+k] g, {g, First/@rules}];
  s /. First@Solve[
    Thread[Flatten[s /. rules] == Flatten[m]]
  ]
];
```

```
B[x_, y_] := InverseRule[sl3rule][x.y - y.x /. sl3rule]
```

```
{B[h1, e1], B[h1, f1], B[e1, f1]}
```

```
{2 e1, -2 f1, h1}
```

```
{B[h2, e2], B[h2, f2], B[e2, f2]}
```

```
{2 e2, -2 f2, h2}
```

```
{B[h1, e3], B[h1, f3], B[e3, f3]}
```

```
{e3, -f3, h1 + h2}
```

```
{B[h1 + h2, e3], B[h1 + h2, f3], B[e3, f3]}
```

```
{2 e3, -2 f3, h1 + h2}
```

```
{B[e1, e2], B[e1, e3], B[e2, e3]}
```

```
{e3, 0, 0}
```

```
{B[f1, f2], B[f1, f3], B[f2, f3]}
```

```
{-f3, 0, 0}
```

```
{B[e1, f2], B[e1, f3], B[e2, f3]}
```

```
{0, -f2, f1}
```