

Pensieve header: Figuring out polynomiality for the NOE1 program.

Initialization

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2016-12"];
Once[<< KnotTheory`];
Once[<< "../Projects/Profile/Profile.m"]
```

Loading KnotTheory` version of September 6, 2014, 13:37:37.2841.

Read more at <http://katlas.org/wiki/KnotTheory>.

This is Profile.m, Nov 2016 mods of July 1994 version

Rotational Virtual Knots

```
RVK::usage =
  "RVK[xs, rots] represents a Rotational Virtual Knot with a list of n Xp/Xm crossings xs and
  a length 2n list of rotation numbers rots. Crossing sites are indexed 1 through
  2n, and rots[[k]] is the rotation between site k-1 and site k. RVK is also a casting
  operator converting to the RVK presentation from other knot presentations.";
RVK[pd_PD] := Module[{n, xs, x, rots, front, k},
  n = Length[pd];
  xs = List@@pd /. x_X => If[PositiveQ[x], Xp[x[[4]], x[[1]], Xm[x[[2]], x[[1]]];
  rots = Table[0, {2 n}];
  front = {0};
  For[k = 0, k < 2 n, ++k,
  If[k == 0 || FreeQ[front, -k],
  front = Flatten[front /. k -> Catch[xs /. {
    Xp[k + 1, L_] | Xm[L_, k + 1] => Throw[{L, k + 1, 1 - L}],
    Xp[L_, k + 1] | Xm[k + 1, L_] => (++rots[[L]]; Throw[{1 - L, k + 1, L})
  }]],
  If[MatchQ[front, {___, k, ___, -k, ___}], --rots[[k + 1]]
  ]
  ];
  RVK[xs, rots]
  ];
RVK[K_] := RVK[PD[K]];
```

NOE-It

Warning: Logos may be disabled.

$$\Lambda[k_] := \theta \left((t_k - 1) \left(2 (\alpha \beta + \delta \mu)^2 - \alpha^2 \beta^2 \right) - 4 v_k c_k w_k \delta^2 \mu^2 - \right. \\ \left. \delta (1 + \mu) \left(w_k^2 \alpha^2 + v_k^2 \beta^2 \right) - v_k^2 w_k^2 \delta^3 (1 + 3 \mu) - \right. \\ \left. 2 (\alpha \beta + 2 \delta \mu + v_k w_k \delta^2 (1 + 2 \mu) + 2 c_k \delta \mu^2) (w_k \alpha + v_k \beta) - 4 (c_k \mu^2 + v_k w_k \delta (1 + \mu)) (\alpha \beta + \delta \mu) (1 + t_k) \right) / 4;$$

$$R_{i,j}^+ := \mathbb{E} \left[1, \text{Log}[t_i] c_j, v_i w_j, v_i c_i w_j + c_i c_j + v_i^2 w_j^2 / 4 \right]; \\ R_{i,j}^- := \mathbb{E} \left[1, -\text{Log}[t_i] c_j, -t_i^{-1} v_i w_j, -c_i c_j + t_i^{-1} v_i c_j w_j - t_i^{-2} v_i^2 w_j^2 / 4 \right]; \\ (ur_{i-} := \mathbb{E} [t_i^{-1/2}, \theta, \theta, c_i t_i^2]; nr_{i-} := \mathbb{E} [t_i^{1/2}, \theta, \theta, -c_i t_i^2];)$$

1DP

```

DPx→Dα,y→Dβ[P-][f-] := (* means P[∂α,∂β][f] *)
PPDp@Total[CoefficientRules[P,{x,y}]/.({m-,n-}→c-)⇒cD[f,{α,m},{β,n}]]

```

1Util

```

E /: E[ω1-,L1-,Q1-,P1-] E[ω2-,L2-,Q2-,P2-] := CF@E[ω1 ω2, L1 + L2, ω2 Q1 + ω1 Q2, ω24 P1 + ω14 P2];

```

1NOuw

```

Nwi-vj→k-[E[ω-,L-,Q-,P-]] := PPNwv@With[{q = ((1 - tk) α β + β vk + δ vk wk + α wk) / μ}, CF[
E[μ ω, L, μ ω q + μ (Q / . wi | vj → 0), μ4 (DPwi→Dα,vj→Dβ[P][eq] / . e- → 1) + ω4 Δ[k]] / . μ → 1 + (tk - 1) δ / .
{α → ω-1 (∂wi Q / . vj → 0), β → ω-1 (∂vj Q / . wi → 0), δ → ω-1 ∂wi,vj Q}]]];

```

1NOc

```

Ncj-(x:v|w)i→k-[E[ω-,L-,Q-,P-]] := PPNcx@With[{q = eγ β xk + γ ck}, CF[
E[ω, γ ck + (L / . cj → 0), ω eγ β xk + (Q / . xi → 0), e-q DPcj→Dγ,xi→Dβ[P][eq]] / . {γ → ∂cj L, β → ω-1 ∂xi Q}]]];

```

1m

```

mi-,j-→k-[Z-E] := PPm@Module[{x,z},
CF[(Z // Nwi vj→x // Nci vx→x // Nwx cj→x) / . z-i|j|x → zk]]]

```

Z

```

ul- = nl- = rot[_ , 0] = E[1, 0, 0, 0];
rot[i-, 1] := uri;
rot[i-, n_Integer] /; n > 1 := Module[{y}, rot[i, n - 1] rot[y, 1] // mi,y→i];
rot[i-, -1] := nri;
rot[i-, n_Integer] /; n < -1 := Module[{y}, rot[i, n + 1] rot[y, -1] // mi,y→i];

```

```

t_ = t;
Z[K_] := Z[RVK@K];
Z[rvk_RVK] := PPz@Module[{todo, n, rots, g, done, st, x, g1, i, j, k, k1, k2, k3},
  {todo, rots} = List@@rvk;
AppendTo[rots, 0];
n = Length[todo];
g = E[1, 0, 0, 0];
done = {0};
st = Range[0, 2 n + 1];
While[todo != {},
  {x} = MaximalBy[todo, Length[done ∩ {#[[1]], #[[2]], #[[1]] - 1, #[[2]] - 1}] &, 1];
Z$todo = todo; Z$x = x;
{i, j} = List@@x;
g1 = Switch[Head[x],
  Xp, m_{j,k→j} [R_{i,j}^+ (R_{k3,k}^- nr_{k1} ul_{k2} // m_{k,k1→k} // m_{k,k2→k} // m_{k,k3→k})],
  Xm, m_{j,k→j} [R_{i,j}^- (R_{k,k3}^+ nr_{k1} ul_{k2} // m_{k,k1→k} // m_{k,k2→k} // m_{k,k3→k})]
];
g1 = rot[k, rots[[i]] g1 // m_{k,i→i}; rots[[i]] = 0;
g1 = g1 rot[k, rots[[i + 1]] // m_{i,k→i}; rots[[i + 1]] = 0;
g1 = rot[k, rots[[j]] g1 // m_{k,j→j}; rots[[j]] = 0;
g1 = g1 rot[k, rots[[j + 1]] // m_{j,k→j}; rots[[j + 1]] = 0;
g *= g1;
If[MemberQ[done, i], g = g // m_{i,i+1→i}; st = st /. st[[i + 2]] → st[[i + 1]];
If[MemberQ[done, i - 1], g = g // m_{st[[i],i→st[[i]]}; st = st /. st[[i + 1]] → st[[i]];
If[MemberQ[done, j], g = g // m_{j,j+1→j}; st = st /. st[[j + 2]] → st[[j + 1]];
If[MemberQ[done, j - 1], g = g // m_{st[[j],j→st[[j]]}; st = st /. st[[j + 1]] → st[[j]];
done = done ∪ {i - 1, i, j - 1, j};
todo = DeleteCases[todo, x]
];
g /. {V0 → v, C0 → c, W0 → w}
]

```

1Utl

```

CF1[E[ω, L, Q, P]] :=
  PPCF1@E[Expand@Together@ω, Expand@Together@L, Expand@Together@Q, Expand@PPTogether4P@Together@P];
CF[E[ω, L, Q, P]] := PPCF@E[Factor@ω, Factor@L,
  PPCF4Q@Module[{vars},
    vars = Union@Cases[Q, (a | c | v | w)_, ∞];
    Total[CoefficientRules[Q, vars] /. (p_ → cc_) ⇒ (PPFactor@Factor@cc) (Times@@(vars^p))]],
  PPCF4P@Module[{vars},
    vars = Union@Cases[P, (a | c | v | w)_, ∞];
    Total[CoefficientRules[P, vars] /. (p_ → cc_) ⇒ (PPFactor@Factor@cc) (Times@@(vars^p))]]
];

```

Timing[Z[Knot[3, 1]]]

 KnotTheory: Loading precomputed data in PD4Knots` 

{1.64063,

$$E\left[\frac{1-t+t^2}{t}, 0, 0, -\frac{2c(-1+t)(1+t)(1-t+t^2)^3}{t^4} + \frac{(-1+t)(1-t+t^2)^2(2-t+t^2)}{t^4} - \frac{2(1+t)(1-t+t^2)^3vw}{t^4}\right]}$$

Timing[Z[Knot[4, 1]]]

$$\left\{ 0.734375, \mathbb{E} \left[-\frac{1-3t+t^2}{t}, \theta, \theta, \right. \right. \\ \left. \left. \frac{(-1+t)(1+t)(1-3t+t^2)^3}{t^4} - \frac{2c(-1+t)(1+t)(1-3t+t^2)^3}{t^4} - \frac{2(1+t)(1-3t+t^2)^3 v w}{t^4} \right] \right\}$$

Quuww = $\mathbb{E}[\omega, \text{Sum}[l_{i,j} \text{Log}[t_i] c_j, \{i, 3\}, \{j, 3\}], \text{Sum}[a_{i,j} v_i w_j, \{i, 3\}, \{j, 3\}],$

$\text{Sum}[a_{1000 i+100 j+10 k+k} v_i v_j w_k w_l, \{i, 3\}, \{j, i, 3\}, \{k, 3\}, \{l, k, 3\}]];$

Quuww // N_{w1 v2→4}

$$\mathbb{E} \left[\omega - a_{2,1} + t a_{2,1}, \text{Log}[t] (c_1 l_{1,1} + c_2 l_{1,2} + c_3 l_{1,3} + c_1 l_{2,1} + c_2 l_{2,2} + c_3 l_{2,3} + c_1 l_{3,1} + c_2 l_{3,2} + c_3 l_{3,3}), \right. \\ v_1 w_4 a_{1,1} + v_4 w_4 a_{2,1} + v_4 w_2 a_{2,2} + \frac{v_1 w_2 (\omega a_{1,2} - a_{1,2} a_{2,1} + t a_{1,2} a_{2,1} + a_{1,1} a_{2,2} - t a_{1,1} a_{2,2})}{\omega} + \\ v_4 w_3 a_{2,3} + \frac{v_1 w_3 (\omega a_{1,3} - a_{1,3} a_{2,1} + t a_{1,3} a_{2,1} + a_{1,1} a_{2,3} - t a_{1,1} a_{2,3})}{\omega} + v_3 w_4 a_{3,1} + \\ \left. \frac{v_3 w_2 (a_{2,2} a_{3,1} - t a_{2,2} a_{3,1} + \omega a_{3,2} - a_{2,1} a_{3,2} + t a_{2,1} a_{3,2})}{\omega} + \frac{v_3 w_3 (a_{2,3} a_{3,1} - t a_{2,3} a_{3,1} + \omega a_{3,3} - a_{2,1} a_{3,3} + t a_{2,1} a_{3,3})}{\omega} \right], \\ a_{2211} v_4^2 w_4^2 - \frac{2(-1+t) a_{2211} v_1 v_4 w_4^2 a_{1,1}}{\omega} + \frac{(-1+t)^2 a_{2211} v_1^2 w_4^2 a_{1,1}^2}{\omega^2} - \frac{4(-1+t) a_{2211} v_4 w_4 (\omega - a_{2,1} + t a_{2,1})}{\omega} + \\ \frac{a_{1211} v_1 v_4 w_4^2 (\omega - a_{2,1} + t a_{2,1})}{\omega} + \frac{a_{2311} v_3 v_4 w_4^2 (\omega - a_{2,1} + t a_{2,1})}{\omega} + \frac{4(-1+t)^2 a_{2211} v_1 v_4 a_{1,1} (\omega - a_{2,1} + t a_{2,1})}{\omega^2} - \\ \frac{(-1+t) a_{1211} v_1^2 w_4^2 a_{1,1} (\omega - a_{2,1} + t a_{2,1})}{\omega^2} - \frac{(-1+t) a_{2311} v_1 v_3 w_4^2 a_{1,1} (\omega - a_{2,1} + t a_{2,1})}{\omega^2} + \\ \frac{2(-1+t)^2 a_{2211} (\omega - a_{2,1} + t a_{2,1})^2}{\omega^2} + \frac{a_{2222} v_4^2 w_2^2 (\omega - a_{2,1} + t a_{2,1})^2}{\omega^2} + \frac{a_{2222} v_4^2 w_2 w_3 (\omega - a_{2,1} + t a_{2,1})^2}{\omega^2} + \\ \frac{a_{2233} v_4^2 w_3^2 (\omega - a_{2,1} + t a_{2,1})^2}{\omega^2} - \frac{2(-1+t) a_{1211} v_1 v_4 (\omega - a_{2,1} + t a_{2,1})^2}{\omega^2} - \frac{2(-1+t) a_{2311} v_3 w_4 (\omega - a_{2,1} + t a_{2,1})^2}{\omega^2} + \\ \frac{a_{1111} v_1^2 w_4^2 (\omega - a_{2,1} + t a_{2,1})^2}{\omega^2} + \frac{a_{1311} v_1 v_3 w_4^2 (\omega - a_{2,1} + t a_{2,1})^2}{\omega^2} + \frac{a_{3311} v_3^2 w_4^2 (\omega - a_{2,1} + t a_{2,1})^2}{\omega^2} - \\ \frac{2(-1+t) a_{2222} v_1 v_4 w_2^2 a_{1,1} (\omega - a_{2,1} + t a_{2,1})^2}{\omega^3} - \frac{2(-1+t) a_{2222} v_1 v_4 w_2 w_3 a_{1,1} (\omega - a_{2,1} + t a_{2,1})^2}{\omega^3} - \\ \frac{2(-1+t) a_{2233} v_1 v_4 w_3^2 a_{1,1} (\omega - a_{2,1} + t a_{2,1})^2}{\omega^3} + \frac{(-1+t)^2 a_{2222} v_1^2 w_2^2 a_{1,1}^2 (\omega - a_{2,1} + t a_{2,1})^2}{\omega^4} + \\ \frac{(-1+t)^2 a_{2233} v_1^2 w_2 w_3 a_{1,1}^2 (\omega - a_{2,1} + t a_{2,1})^2}{\omega^4} + \frac{(-1+t)^2 a_{2233} v_1^2 w_3^2 a_{1,1}^2 (\omega - a_{2,1} + t a_{2,1})^2}{\omega^4} + \\ \frac{a_{1222} v_1 v_4 w_2^2 (\omega - a_{2,1} + t a_{2,1})^3}{\omega^3} + \frac{a_{2322} v_3 v_4 w_2^2 (\omega - a_{2,1} + t a_{2,1})^3}{\omega^3} + \frac{a_{1222} v_1 v_4 w_2 w_3 (\omega - a_{2,1} + t a_{2,1})^3}{\omega^3} + \\ \frac{a_{2322} v_3 v_4 w_2 w_3 (\omega - a_{2,1} + t a_{2,1})^3}{\omega^3} + \frac{a_{1233} v_1 v_4 w_3^2 (\omega - a_{2,1} + t a_{2,1})^3}{\omega^3} + \frac{a_{2333} v_3 v_4 w_3^2 (\omega - a_{2,1} + t a_{2,1})^3}{\omega^3} - \\ \frac{(-1+t) a_{1222} v_1^2 w_2^2 a_{1,1} (\omega - a_{2,1} + t a_{2,1})^3}{\omega^4} - \frac{(-1+t) a_{2322} v_1 v_3 w_2^2 a_{1,1} (\omega - a_{2,1} + t a_{2,1})^3}{\omega^4} - \\ \frac{(-1+t) a_{1222} v_1^2 w_2 w_3 a_{1,1} (\omega - a_{2,1} + t a_{2,1})^3}{\omega^4} - \frac{(-1+t) a_{2322} v_1 v_3 w_2 w_3 a_{1,1} (\omega - a_{2,1} + t a_{2,1})^3}{\omega^4} - \\ \frac{(-1+t) a_{1233} v_1^2 w_3^2 a_{1,1} (\omega - a_{2,1} + t a_{2,1})^3}{\omega^4} - \frac{(-1+t) a_{2333} v_1 v_3 w_3^2 a_{1,1} (\omega - a_{2,1} + t a_{2,1})^3}{\omega^4} + \\ \frac{a_{1122} v_1^2 w_2^2 (\omega - a_{2,1} + t a_{2,1})^4}{\omega^4} + \frac{a_{1322} v_1 v_3 w_2^2 (\omega - a_{2,1} + t a_{2,1})^4}{\omega^4} + \frac{a_{3322} v_3^2 w_2^2 (\omega - a_{2,1} + t a_{2,1})^4}{\omega^4} + \\ \frac{a_{1122} v_1^2 w_2 w_3 (\omega - a_{2,1} + t a_{2,1})^4}{\omega^4} + \frac{a_{1322} v_1 v_3 w_2 w_3 (\omega - a_{2,1} + t a_{2,1})^4}{\omega^4} + \frac{a_{3322} v_3^2 w_2 w_3 (\omega - a_{2,1} + t a_{2,1})^4}{\omega^4} + \\ \frac{a_{1133} v_1^2 w_3^2 (\omega - a_{2,1} + t a_{2,1})^4}{\omega^4} + \frac{a_{1333} v_1 v_3 w_3^2 (\omega - a_{2,1} + t a_{2,1})^4}{\omega^4} + \frac{a_{3333} v_3^2 w_3^2 (\omega - a_{2,1} + t a_{2,1})^4}{\omega^4} + \\ \frac{a_{2211} v_4^2 w_2 w_4 (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,2} - 2 t a_{2,2})}{\omega} - \frac{2(-1+t) a_{2211} v_1 v_4 w_2 w_4 a_{1,1} (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,2} - 2 t a_{2,2})}{\omega^2} + \\ \frac{(-1+t)^2 a_{2211} v_1^2 w_2 w_4 a_{1,1}^2 (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,2} - 2 t a_{2,2})}{\omega^3} - \frac{1}{\omega^2} \\ 2(-1+t) a_{2211} v_4 w_2 (\omega - a_{2,1} + t a_{2,1}) (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,2} - 2 t a_{2,2}) + \\ \frac{a_{1211} v_1 v_4 w_2 w_4 (\omega - a_{2,1} + t a_{2,1}) (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,2} - 2 t a_{2,2})}{\omega^2} +$$

$$\begin{aligned}
& \frac{a_{2311} v_3 v_4 w_2 w_4 (\omega - a_{2,1} + t a_{2,1}) (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,2} - 2 t a_{2,2})}{\omega^2} + \frac{1}{\omega^3} \\
& 2 (-1 + t)^2 a_{2211} v_1 w_2 a_{1,1} (\omega - a_{2,1} + t a_{2,1}) (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,2} - 2 t a_{2,2}) - \frac{1}{\omega^3} \\
& (-1 + t) a_{1211} v_1^2 w_2 w_4 a_{1,1} (\omega - a_{2,1} + t a_{2,1}) (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,2} - 2 t a_{2,2}) - \frac{1}{\omega^3} \\
& (-1 + t) a_{2311} v_1 v_3 w_2 w_4 a_{1,1} (\omega - a_{2,1} + t a_{2,1}) (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,2} - 2 t a_{2,2}) - \\
& \frac{1}{\omega^3} (-1 + t) a_{1211} v_1 w_2 (\omega - a_{2,1} + t a_{2,1})^2 (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,2} - 2 t a_{2,2}) - \frac{1}{\omega^3} \\
& (-1 + t) a_{2311} v_3 w_2 (\omega - a_{2,1} + t a_{2,1})^2 (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,2} - 2 t a_{2,2}) + \\
& \frac{a_{1111} v_1^2 w_2 w_4 (\omega - a_{2,1} + t a_{2,1})^2 (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,2} - 2 t a_{2,2})}{\omega^3} + \\
& \frac{a_{1311} v_1 v_3 w_2 w_4 (\omega - a_{2,1} + t a_{2,1})^2 (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,2} - 2 t a_{2,2})}{\omega^3} + \\
& \frac{a_{3311} v_3^2 w_2 w_4 (\omega - a_{2,1} + t a_{2,1})^2 (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,2} - 2 t a_{2,2})}{\omega^3} - \\
& \frac{(-1 + t) a_{2211} v_4^2 w_2^2 a_{2,2} (\omega - a_{2,1} + t a_{2,1} + a_{2,2} - t a_{2,2})}{\omega^2} + \\
& 2 (-1 + t)^2 a_{2211} v_1 v_4 w_2^2 a_{1,1} a_{2,2} (\omega - a_{2,1} + t a_{2,1} + a_{2,2} - t a_{2,2}) - \\
& \frac{(-1 + t)^3 a_{2211} v_1^2 w_2^2 a_{1,1}^2 a_{2,2} (\omega - a_{2,1} + t a_{2,1} + a_{2,2} - t a_{2,2})}{\omega^4} - \frac{1}{\omega^3} \\
& (-1 + t) a_{1211} v_1 v_4 w_2^2 (\omega - a_{2,1} + t a_{2,1}) a_{2,2} (\omega - a_{2,1} + t a_{2,1} + a_{2,2} - t a_{2,2}) - \frac{1}{\omega^3} \\
& (-1 + t) a_{2311} v_3 v_4 w_2^2 (\omega - a_{2,1} + t a_{2,1}) a_{2,2} (\omega - a_{2,1} + t a_{2,1} + a_{2,2} - t a_{2,2}) + \frac{1}{\omega^4} \\
& (-1 + t)^2 a_{1211} v_1^2 w_2^2 a_{1,1} (\omega - a_{2,1} + t a_{2,1}) a_{2,2} (\omega - a_{2,1} + t a_{2,1} + a_{2,2} - t a_{2,2}) + \frac{1}{\omega^4} \\
& (-1 + t)^2 a_{2311} v_1 v_3 w_2^2 a_{1,1} (\omega - a_{2,1} + t a_{2,1}) a_{2,2} (\omega - a_{2,1} + t a_{2,1} + a_{2,2} - t a_{2,2}) - \\
& \frac{1}{\omega^4} (-1 + t) a_{1111} v_1^2 w_2^2 (\omega - a_{2,1} + t a_{2,1})^2 a_{2,2} (\omega - a_{2,1} + t a_{2,1} + a_{2,2} - t a_{2,2}) - \frac{1}{\omega^4} \\
& (-1 + t) a_{1311} v_1 v_3 w_2^2 (\omega - a_{2,1} + t a_{2,1})^2 a_{2,2} (\omega - a_{2,1} + t a_{2,1} + a_{2,2} - t a_{2,2}) - \frac{1}{\omega^4} \\
& (-1 + t) a_{3311} v_3^2 w_2^2 (\omega - a_{2,1} + t a_{2,1})^2 a_{2,2} (\omega - a_{2,1} + t a_{2,1} + a_{2,2} - t a_{2,2}) + \\
& \frac{a_{2211} v_4^2 w_3 w_4 (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,3} - 2 t a_{2,3})}{\omega} - \frac{2 (-1 + t) a_{2211} v_1 v_4 w_3 w_4 a_{1,1} (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,3} - 2 t a_{2,3})}{\omega^2} + \\
& \frac{(-1 + t)^2 a_{2211} v_1^2 w_3 w_4 a_{1,1}^2 (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,3} - 2 t a_{2,3})}{\omega^3} - \frac{1}{\omega^2} \\
& 2 (-1 + t) a_{2211} v_4 w_3 (\omega - a_{2,1} + t a_{2,1}) (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,3} - 2 t a_{2,3}) + \\
& \frac{a_{1211} v_1 v_4 w_3 w_4 (\omega - a_{2,1} + t a_{2,1}) (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,3} - 2 t a_{2,3})}{\omega^2} + \\
& \frac{a_{2311} v_3 v_4 w_3 w_4 (\omega - a_{2,1} + t a_{2,1}) (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,3} - 2 t a_{2,3})}{\omega^2} + \frac{1}{\omega^3} \\
& 2 (-1 + t)^2 a_{2211} v_1 w_3 a_{1,1} (\omega - a_{2,1} + t a_{2,1}) (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,3} - 2 t a_{2,3}) - \frac{1}{\omega^3} \\
& (-1 + t) a_{1211} v_1^2 w_3 w_4 a_{1,1} (\omega - a_{2,1} + t a_{2,1}) (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,3} - 2 t a_{2,3}) - \frac{1}{\omega^3} \\
& (-1 + t) a_{2311} v_1 v_3 w_3 w_4 a_{1,1} (\omega - a_{2,1} + t a_{2,1}) (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,3} - 2 t a_{2,3}) - \\
& \frac{1}{\omega^3} (-1 + t) a_{1211} v_1 w_3 (\omega - a_{2,1} + t a_{2,1})^2 (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,3} - 2 t a_{2,3}) - \frac{1}{\omega^3} \\
& (-1 + t) a_{2311} v_3 w_3 (\omega - a_{2,1} + t a_{2,1})^2 (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,3} - 2 t a_{2,3}) + \\
& \frac{a_{1111} v_1^2 w_3 w_4 (\omega - a_{2,1} + t a_{2,1})^2 (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,3} - 2 t a_{2,3})}{\omega^3} + \\
& \frac{a_{1311} v_1 v_3 w_3 w_4 (\omega - a_{2,1} + t a_{2,1})^2 (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,3} - 2 t a_{2,3})}{\omega^3} + \\
& \frac{a_{3311} v_3^2 w_3 w_4 (\omega - a_{2,1} + t a_{2,1})^2 (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,3} - 2 t a_{2,3})}{\omega^3} - \\
& \frac{(-1 + t) a_{2211} v_4^2 w_3^2 a_{2,3} (\omega - a_{2,1} + t a_{2,1} + a_{2,3} - t a_{2,3})}{\omega^2} + \\
& 2 (-1 + t)^2 a_{2211} v_1 v_4 w_3^2 a_{1,1} a_{2,3} (\omega - a_{2,1} + t a_{2,1} + a_{2,3} - t a_{2,3}) -
\end{aligned}$$

$$\begin{aligned}
& \frac{(-1+t)^3 a_{2211} v_1^2 w_3^2 a_{1,1}^2 a_{2,3} (\omega - a_{2,1} + t a_{2,1} + a_{2,3} - t a_{2,3})}{\omega^4} - \frac{1}{\omega^3} \\
& (-1+t) a_{1211} v_1 v_4 w_3^2 (\omega - a_{2,1} + t a_{2,1}) a_{2,3} (\omega - a_{2,1} + t a_{2,1} + a_{2,3} - t a_{2,3}) - \frac{1}{\omega^3} \\
& (-1+t) a_{2311} v_3 v_4 w_3^2 (\omega - a_{2,1} + t a_{2,1}) a_{2,3} (\omega - a_{2,1} + t a_{2,1} + a_{2,3} - t a_{2,3}) + \frac{1}{\omega^4} \\
& (-1+t)^2 a_{1211} v_1^2 w_3^2 a_{1,1} (\omega - a_{2,1} + t a_{2,1}) a_{2,3} (\omega - a_{2,1} + t a_{2,1} + a_{2,3} - t a_{2,3}) + \frac{1}{\omega^4} \\
& (-1+t)^2 a_{2311} v_1 v_3 w_3^2 a_{1,1} (\omega - a_{2,1} + t a_{2,1}) a_{2,3} (\omega - a_{2,1} + t a_{2,1} + a_{2,3} - t a_{2,3}) - \\
& \frac{1}{\omega^4} (-1+t) a_{1111} v_1^2 w_3^2 (\omega - a_{2,1} + t a_{2,1})^2 a_{2,3} (\omega - a_{2,1} + t a_{2,1} + a_{2,3} - t a_{2,3}) - \frac{1}{\omega^4} \\
& (-1+t) a_{1311} v_1 v_3 w_3^2 (\omega - a_{2,1} + t a_{2,1})^2 a_{2,3} (\omega - a_{2,1} + t a_{2,1} + a_{2,3} - t a_{2,3}) - \frac{1}{\omega^4} \\
& (-1+t) a_{3311} v_3^2 w_3^2 (\omega - a_{2,1} + t a_{2,1})^2 a_{2,3} (\omega - a_{2,1} + t a_{2,1} + a_{2,3} - t a_{2,3}) - \frac{1}{\omega^2} \\
& (-1+t) a_{2211} v_4^2 w_2 w_3 (\omega a_{2,2} - a_{2,1} a_{2,2} + t a_{2,1} a_{2,2} + \omega a_{2,3} - a_{2,1} a_{2,3} + t a_{2,1} a_{2,3} + 2 a_{2,2} a_{2,3} - 2 t a_{2,2} a_{2,3}) + \frac{1}{\omega^3} \\
& 2 (-1+t)^2 a_{2211} v_1 v_4 w_2 w_3 a_{1,1} (\omega a_{2,2} - a_{2,1} a_{2,2} + t a_{2,1} a_{2,2} + \omega a_{2,3} - a_{2,1} a_{2,3} + t a_{2,1} a_{2,3} + 2 a_{2,2} a_{2,3} - 2 t a_{2,2} a_{2,3}) - \\
& \frac{1}{\omega^4} (-1+t)^3 a_{2211} v_1^2 w_2 w_3 a_{1,1}^2 (\omega a_{2,2} - a_{2,1} a_{2,2} + t a_{2,1} a_{2,2} + \omega a_{2,3} - a_{2,1} a_{2,3} + t a_{2,1} a_{2,3} + 2 a_{2,2} a_{2,3} - 2 t a_{2,2} a_{2,3}) - \\
& \frac{1}{\omega^3} (-1+t) a_{1211} v_1 v_4 w_2 w_3 (\omega - a_{2,1} + t a_{2,1}) \\
& (\omega a_{2,2} - a_{2,1} a_{2,2} + t a_{2,1} a_{2,2} + \omega a_{2,3} - a_{2,1} a_{2,3} + t a_{2,1} a_{2,3} + 2 a_{2,2} a_{2,3} - 2 t a_{2,2} a_{2,3}) - \frac{1}{\omega^3} (-1+t) a_{2311} v_3 v_4 \\
& w_2 w_3 (\omega - a_{2,1} + t a_{2,1}) (\omega a_{2,2} - a_{2,1} a_{2,2} + t a_{2,1} a_{2,2} + \omega a_{2,3} - a_{2,1} a_{2,3} + t a_{2,1} a_{2,3} + 2 a_{2,2} a_{2,3} - 2 t a_{2,2} a_{2,3}) + \\
& \frac{1}{\omega^4} (-1+t)^2 a_{1211} v_1^2 w_2 w_3 a_{1,1} (\omega - a_{2,1} + t a_{2,1}) \\
& (\omega a_{2,2} - a_{2,1} a_{2,2} + t a_{2,1} a_{2,2} + \omega a_{2,3} - a_{2,1} a_{2,3} + t a_{2,1} a_{2,3} + 2 a_{2,2} a_{2,3} - 2 t a_{2,2} a_{2,3}) + \frac{1}{\omega^4} (-1+t)^2 a_{2311} v_1 v_3 w_2 w_3 a_{1,1} \\
& (\omega - a_{2,1} + t a_{2,1}) (\omega a_{2,2} - a_{2,1} a_{2,2} + t a_{2,1} a_{2,2} + \omega a_{2,3} - a_{2,1} a_{2,3} + t a_{2,1} a_{2,3} + 2 a_{2,2} a_{2,3} - 2 t a_{2,2} a_{2,3}) - \frac{1}{\omega^4} (-1+t) \\
& a_{1111} v_1^2 w_2 w_3 (\omega - a_{2,1} + t a_{2,1})^2 (\omega a_{2,2} - a_{2,1} a_{2,2} + t a_{2,1} a_{2,2} + \omega a_{2,3} - a_{2,1} a_{2,3} + t a_{2,1} a_{2,3} + 2 a_{2,2} a_{2,3} - 2 t a_{2,2} a_{2,3}) - \\
& \frac{1}{\omega^4} (-1+t) a_{1311} v_1 v_3 w_2 w_3 (\omega - a_{2,1} + t a_{2,1})^2 \\
& (\omega a_{2,2} - a_{2,1} a_{2,2} + t a_{2,1} a_{2,2} + \omega a_{2,3} - a_{2,1} a_{2,3} + t a_{2,1} a_{2,3} + 2 a_{2,2} a_{2,3} - 2 t a_{2,2} a_{2,3}) - \frac{1}{\omega^4} (-1+t) a_{3311} v_3^2 w_2 \\
& w_3 (\omega - a_{2,1} + t a_{2,1})^2 (\omega a_{2,2} - a_{2,1} a_{2,2} + t a_{2,1} a_{2,2} + \omega a_{2,3} - a_{2,1} a_{2,3} + t a_{2,1} a_{2,3} + 2 a_{2,2} a_{2,3} - 2 t a_{2,2} a_{2,3}) - \\
& \frac{2 (-1+t) a_{2211} v_3 v_4 w_4^2 a_{3,1}}{\omega} + \frac{2 (-1+t)^2 a_{2211} v_1 v_3 w_4^2 a_{1,1} a_{3,1}}{\omega^2} + \frac{4 (-1+t)^2 a_{2211} v_3 w_4 (\omega - a_{2,1} + t a_{2,1}) a_{3,1}}{\omega^2} - \\
& \frac{(-1+t) a_{1211} v_1 v_3 w_4^2 (\omega - a_{2,1} + t a_{2,1}) a_{3,1}}{\omega^2} - \frac{(-1+t) a_{2311} v_3^2 w_4^2 (\omega - a_{2,1} + t a_{2,1}) a_{3,1}}{\omega^2} - \\
& \frac{2 (-1+t) a_{2222} v_3 v_4 w_2^2 (\omega - a_{2,1} + t a_{2,1})^2 a_{3,1}}{\omega^3} - \frac{2 (-1+t) a_{2222} v_3 v_4 w_2 w_3 (\omega - a_{2,1} + t a_{2,1})^2 a_{3,1}}{\omega^3} - \\
& \frac{2 (-1+t) a_{2233} v_3 v_4 w_2^2 (\omega - a_{2,1} + t a_{2,1})^2 a_{3,1}}{\omega^3} + \frac{2 (-1+t)^2 a_{2222} v_1 v_3 w_2^2 a_{1,1} (\omega - a_{2,1} + t a_{2,1})^2 a_{3,1}}{\omega^4} + \\
& \frac{2 (-1+t)^2 a_{2222} v_1 v_3 w_2 w_3 a_{1,1} (\omega - a_{2,1} + t a_{2,1})^2 a_{3,1}}{\omega^4} + \frac{2 (-1+t)^2 a_{2233} v_1 v_3 w_3^2 a_{1,1} (\omega - a_{2,1} + t a_{2,1})^2 a_{3,1}}{\omega^4} - \\
& \frac{(-1+t) a_{1222} v_1 v_3 w_2^2 (\omega - a_{2,1} + t a_{2,1})^3 a_{3,1}}{\omega^4} - \frac{(-1+t) a_{2322} v_3^2 w_2^2 (\omega - a_{2,1} + t a_{2,1})^3 a_{3,1}}{\omega^4} - \\
& \frac{(-1+t) a_{1222} v_1 v_3 w_2 w_3 (\omega - a_{2,1} + t a_{2,1})^3 a_{3,1}}{\omega^4} - \frac{(-1+t) a_{2322} v_3^2 w_2 w_3 (\omega - a_{2,1} + t a_{2,1})^3 a_{3,1}}{\omega^4} - \\
& \frac{(-1+t) a_{1233} v_1 v_3 w_2^2 (\omega - a_{2,1} + t a_{2,1})^3 a_{3,1}}{\omega^4} - \frac{(-1+t) a_{2333} v_3^2 w_2^2 (\omega - a_{2,1} + t a_{2,1})^3 a_{3,1}}{\omega^4} - \\
& \frac{2 (-1+t) a_{2211} v_3 v_4 w_2 w_4 (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,2} - 2 t a_{2,2}) a_{3,1}}{\omega^2} + \\
& \frac{2 (-1+t)^2 a_{2211} v_1 v_3 w_2 w_4 a_{1,1} (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,2} - 2 t a_{2,2}) a_{3,1}}{\omega^3} + \frac{1}{\omega^3} \\
& 2 (-1+t)^2 a_{2211} v_3 w_2 (\omega - a_{2,1} + t a_{2,1}) (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,2} - 2 t a_{2,2}) a_{3,1} - \frac{1}{\omega^3} \\
& (-1+t) a_{1211} v_1 v_3 w_2 w_4 (\omega - a_{2,1} + t a_{2,1}) (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,2} - 2 t a_{2,2}) a_{3,1} - \\
& \frac{1}{\omega^3} (-1+t) a_{2311} v_3^2 w_2 w_4 (\omega - a_{2,1} + t a_{2,1}) (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,2} - 2 t a_{2,2}) a_{3,1} +
\end{aligned}$$

$$\begin{aligned}
& \frac{2(-1+t)^2 a_{2211} v_3 v_4 w_2^2 a_{2,2} (\omega - a_{2,1} + t a_{2,1} + a_{2,2} - t a_{2,2}) a_{3,1}}{\omega^3} - \\
& \frac{2(-1+t)^3 a_{2211} v_1 v_3 w_2^2 a_{1,1} a_{2,2} (\omega - a_{2,1} + t a_{2,1} + a_{2,2} - t a_{2,2}) a_{3,1}}{\omega^4} + \frac{1}{\omega^4} \\
& (-1+t)^2 a_{1211} v_1 v_3 w_2^2 (\omega - a_{2,1} + t a_{2,1}) a_{2,2} (\omega - a_{2,1} + t a_{2,1} + a_{2,2} - t a_{2,2}) a_{3,1} + \frac{1}{\omega^4} \\
& (-1+t)^2 a_{2311} v_3^2 w_2^2 (\omega - a_{2,1} + t a_{2,1}) a_{2,2} (\omega - a_{2,1} + t a_{2,1} + a_{2,2} - t a_{2,2}) a_{3,1} - \\
& \frac{2(-1+t) a_{2211} v_3 v_4 w_3 w_4 (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,3} - 2 t a_{2,3}) a_{3,1}}{\omega^2} + \\
& \frac{2(-1+t)^2 a_{2211} v_1 v_3 w_3 w_4 a_{1,1} (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,3} - 2 t a_{2,3}) a_{3,1}}{\omega^3} + \frac{1}{\omega^3} \\
& 2(-1+t)^2 a_{2211} v_3 w_3 (\omega - a_{2,1} + t a_{2,1}) (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,3} - 2 t a_{2,3}) a_{3,1} - \frac{1}{\omega^3} \\
& (-1+t) a_{1211} v_1 v_3 w_3 w_4 (\omega - a_{2,1} + t a_{2,1}) (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,3} - 2 t a_{2,3}) a_{3,1} - \\
& \frac{1}{\omega^3} (-1+t) a_{2311} v_3^2 w_3 w_4 (\omega - a_{2,1} + t a_{2,1}) (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,3} - 2 t a_{2,3}) a_{3,1} + \\
& \frac{2(-1+t)^2 a_{2211} v_3 v_4 w_3^2 a_{2,3} (\omega - a_{2,1} + t a_{2,1} + a_{2,3} - t a_{2,3}) a_{3,1}}{\omega^3} - \\
& \frac{2(-1+t)^3 a_{2211} v_1 v_3 w_3^2 a_{1,1} a_{2,3} (\omega - a_{2,1} + t a_{2,1} + a_{2,3} - t a_{2,3}) a_{3,1}}{\omega^4} + \frac{1}{\omega^4} \\
& (-1+t)^2 a_{1211} v_1 v_3 w_3^2 (\omega - a_{2,1} + t a_{2,1}) a_{2,3} (\omega - a_{2,1} + t a_{2,1} + a_{2,3} - t a_{2,3}) a_{3,1} + \frac{1}{\omega^4} \\
& (-1+t)^2 a_{2311} v_3^2 w_3^2 (\omega - a_{2,1} + t a_{2,1}) a_{2,3} (\omega - a_{2,1} + t a_{2,1} + a_{2,3} - t a_{2,3}) a_{3,1} + \frac{1}{\omega^3} \\
& 2(-1+t)^2 a_{2211} v_3 v_4 w_2 w_3 (\omega a_{2,2} - a_{2,1} a_{2,2} + t a_{2,1} a_{2,2} + \omega a_{2,3} - a_{2,1} a_{2,3} + t a_{2,1} a_{2,3} + 2 a_{2,2} a_{2,3} - 2 t a_{2,2} a_{2,3}) a_{3,1} - \\
& \frac{1}{\omega^4} 2(-1+t)^3 a_{2211} v_1 v_3 w_2 w_3 a_{1,1} (\omega a_{2,2} - a_{2,1} a_{2,2} + t a_{2,1} a_{2,2} + \omega a_{2,3} - a_{2,1} a_{2,3} + t a_{2,1} a_{2,3} + 2 a_{2,2} a_{2,3} - 2 t a_{2,2} a_{2,3}) \\
& a_{3,1} + \frac{1}{\omega^4} (-1+t)^2 a_{1211} v_1 v_3 w_2 w_3 (\omega - a_{2,1} + t a_{2,1}) \\
& (\omega a_{2,2} - a_{2,1} a_{2,2} + t a_{2,1} a_{2,2} + \omega a_{2,3} - a_{2,1} a_{2,3} + t a_{2,1} a_{2,3} + 2 a_{2,2} a_{2,3} - 2 t a_{2,2} a_{2,3}) a_{3,1} + \frac{1}{\omega^4} (-1+t)^2 a_{2311} v_3^2 \\
& w_2 w_3 (\omega - a_{2,1} + t a_{2,1}) (\omega a_{2,2} - a_{2,1} a_{2,2} + t a_{2,1} a_{2,2} + \omega a_{2,3} - a_{2,1} a_{2,3} + t a_{2,1} a_{2,3} + 2 a_{2,2} a_{2,3} - 2 t a_{2,2} a_{2,3}) a_{3,1} + \\
& \frac{(-1+t)^2 a_{2211} v_3^2 w_4^2 a_{3,1}^2}{\omega^2} + \frac{(-1+t)^2 a_{2222} v_3^2 w_2^2 (\omega - a_{2,1} + t a_{2,1})^2 a_{3,1}^2}{\omega^4} + \frac{(-1+t)^2 a_{2222} v_3^2 w_2 w_3 (\omega - a_{2,1} + t a_{2,1})^2 a_{3,1}^2}{\omega^4} + \\
& \frac{(-1+t)^2 a_{2233} v_3^2 w_3^2 (\omega - a_{2,1} + t a_{2,1})^2 a_{3,1}^2}{\omega^4} + \frac{(-1+t)^2 a_{2211} v_3^2 w_2 w_4 (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,2} - 2 t a_{2,2}) a_{3,1}^2}{\omega^3} - \\
& \frac{(-1+t)^3 a_{2211} v_3^2 w_2^2 a_{2,2} (\omega - a_{2,1} + t a_{2,1} + a_{2,2} - t a_{2,2}) a_{3,1}^2}{\omega^4} + \\
& \frac{(-1+t)^2 a_{2211} v_3^2 w_3 w_4 (\omega - a_{2,1} + t a_{2,1} + 2 a_{2,3} - 2 t a_{2,3}) a_{3,1}^2}{\omega^3} - \\
& \frac{(-1+t)^3 a_{2211} v_3^2 w_3^2 a_{2,3} (\omega - a_{2,1} + t a_{2,1} + a_{2,3} - t a_{2,3}) a_{3,1}^2}{\omega^4} - \frac{1}{\omega^4} \\
& (-1+t)^3 a_{2211} v_3^2 w_2 w_3 (\omega a_{2,2} - a_{2,1} a_{2,2} + t a_{2,1} a_{2,2} + \omega a_{2,3} - a_{2,1} a_{2,3} + t a_{2,1} a_{2,3} + 2 a_{2,2} a_{2,3} - 2 t a_{2,2} a_{2,3}) a_{3,1}^2]
\end{aligned}$$

Quuww = $\mathbb{E}[\omega, \text{Sum}[\mathbf{l}_{i,j} \text{Log}[\mathbf{t}_i] \mathbf{c}_j, \{\mathbf{i}, 3\}, \{\mathbf{j}, 3\}], \text{Sum}[\mathbf{a}_{i,j} \mathbf{v}_i \mathbf{w}_j, \{\mathbf{i}, 3\}, \{\mathbf{j}, 3\}],$

$\text{Sum}[\mathbf{a}_{1000 \mathbf{i}+100 \mathbf{j}+10 \mathbf{k}+\mathbf{k}} \mathbf{v}_i \mathbf{v}_j \mathbf{w}_k \mathbf{w}_l, \{\mathbf{i}, 3\}, \{\mathbf{j}, 3\}, \{\mathbf{k}, 3\}, \{\mathbf{l}, 3\}]];$

Quuww // m_{1,2-4}