Pensieve header: Cheap CF optimization for the NOE1 program (V6.1).

Initialization

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2016-12"];
Once[<< KnotTheory`];
Once[<< "../Projects/Profile/Profile.m"]
```

```
Loading KnotTheory` version of September 6, 2014, 13:37:37.2841.
Read more at http://katlas.org/wiki/KnotTheory.

This is Profile.m, Nov 2016 mods of July 1994 version
```

Rotational Virtual Knots

```
RVK::usage =
  "RVK[xs, rots] represents a Rotational Virtual Knot with a list of n Xp/Xm crossings xs and
    a length 2n list of rotation numbers rots. Crossing sites are indexed 1 through
    2n, and rots[[k]] is the rotation between site k-1 and site k. RVK is also a casting
    operator converting to the RVK presentation from other knot presentations.";
RVK[pd_PD] := Module[{n, xs, x, rots, front, k},
   n = Length[pd];
   xs = List @@ pd /. x_X :> If[PositiveQ[x], Xp[x[[4]], x[[1]]], Xm[x[[2]], x[[1]]]];
   rots = Table[0, {2 n}];
   front = {0};
   For[k = 0, k < 2 n, ++k,
    If[k == 0 \[Or] FreeQ[front, -k],
     front = Flatten[front /. k -> Catch[xs /. {
             Xp[k + 1, l_] | Xm[l_, k + 1] :> Throw[{l, k + 1, 1 - l}],
             Xp[l_, k + 1] | Xm[k + 1, l_] :> (++rots[[l]]; Throw[{1 - l, k + 1, l}])
            }]],
     If[MatchQ[front, {___, k, ___, -k, ___}], --rots[[k + 1]]]
    ]
   ];
   RVK[xs, rots]
  ];
RVK[K_] := RVK[PD[K]];
```

NOE-1t

1Gens

```
R^+_{i_,j_} := 𝔼[1, Log[t_i] c_j, v_i w_j, v_i c_i w_j + c_i c_j + v_i^2 w_j^2/4];
R^-_{i_,j_} := 𝔼[1, -Log[t_i] c_j, -t_i^{-1} v_i w_j, -c_i c_j + t_i^{-1} v_i c_j w_j - t_i^{-2} v_i^2 w_j^2/4];
(ur_{i_} := 𝔼[t_i^{-1/2}, 0, 0, c_i t_i^{-2}]; nr_{i_} := 𝔼[t_i^{1/2}, 0, 0, -c_i t_i^2];)
```

1DP

```
DP_{x_->D_α, y_->D_β}[P_][f_] := (* means P[∂_α,∂_β][f] *)
 PP_DP@Total[CoefficientRules[P, {x, y}] /. ({m_, n_} -> c_) :> c D[f, {α, m}, {β, n}]]
```

1Util

```
CF[𝔼[ω_, L_, Q_, P_]] :=
  PP_CF@𝔼[Expand@Together@ω, Expand@Together@L, Expand@Together@Q, Expand@PP_Together4P@Together@P];
```

1Util

```
𝔼 /: 𝔼[ω1_, L1_, Q1_, P1_] 𝔼[ω2_, L2_, Q2_, P2_] := CF@𝔼[ω1 ω2, L1 + L2, ω2 Q1 + ω1 Q2, ω2^4 P1 + ω1^4 P2];
```

Logos

```
Λ[k_] := ((t_k - 1) (2 (α β + δ μ)^2 - α^2 β^2) - 4 v_k c_k w_k δ^2 μ^2 -
      δ (1 + μ) (w_k^2 α^2 + v_k^2 β^2) - v_k^2 w_k^2 δ^3 (1 + 3 μ) -
      2 (α β + 2 δ μ + v_k w_k δ^2 (1 + 2 μ) + 2 c_k δ μ^2) (w_k α + v_k β) - 4 (c_k μ^2 + v_k w_k δ (1 + μ)) (α β + δ μ)) (1 + t_k) / 4;
```

1NOuw

```
N_{w_i_ v_j_→k_}[𝔼[ω_, L_, Q_, P_]] := PP_Nwv@With[{q = ((1 - t_k) α β + β v_k + δ v_k w_k + α w_k) / μ},
    𝔼[μ ω, L, μ ω q + μ (Q /. w_i | v_j → 0), μ^4 (DP_{w_i→D_α, v_j→D_β}[P][e^q] /. e^- → 1) + ω^4 Λ[k]] // CF //
      ReplaceAll[μ → Expand[ω + (t_k - 1) ∂_{w_i, v_j} Q] / ω] // CF //
     ReplaceAll[{α → ω^-1 (∂_{w_i} Q /. v_j → 0), β → ω^-1 (∂_{v_j} Q /. w_i → 0), δ → ω^-1 ∂_{w_i, v_j} Q}] // CF
   ];
```

1NOc

```
N_{c_j_ (x:v|w)_i_→k_}[𝔼[ω_, L_, Q_, P_]] := PP_Ncx@With[{q = e^γ β x_k + γ c_k}, CF[
      𝔼[ω, γ c_k + (L /. c_j → 0), ω e^γ β x_k + (Q /. x_i → 0), e^-q DP_{c_j→D_γ, x_i→D_β}[P][e^q]]] /. {γ → ∂_{c_j} L, β → ω^-1 ∂_{x_i} Q}]];
```

1m

```
m_{i_,j_→k_}[Z_𝔼] := PP_m@Module[{x, z},
   CF[(Z // N_{w_i v_j→x} // N_{c_i v_x→x} // N_{w_x c_j→x}) /. z_{i_|j_|x} → z_k]]
```

Z

```
ul_ = nl_ = rot[_, 0] = 𝔼[1, 0, 0, 0];
rot[i_, 1] := ur_i;
rot[i_, n_Integer] /; n > 1 := Module[{y}, rot[i, n - 1] rot[y, 1] // m_{i,y→i}];
rot[i_, -1] := nr_i;
rot[i_, n_Integer] /; n < -1 := Module[{y}, rot[i, n + 1] rot[y, -1] // m_{i,y→i}];
```

```
t_ = t;
Z[K_] := Z[RVK@K];
Z[rvk_RVK] := PPz@Module[{todo, n, rots, ℰ, done, st, x, ℰ1, i, j, k, k1, k2, k3},
   {todo, rots} = List@@rvk;
   AppendTo[rots, 0];
   n = Length[todo];
   ℰ = 𝔼[1, 0, 0, 0];
   done = {0};
   st = Range[0, 2 n + 1];
   While[todo =!= {},
    {x} = MaximalBy[todo, Length[done ⋂ {#[[1]], #[[2]], #[[1]] - 1, #[[2]] - 1}] &, 1];
    Z$todo = todo; Z$x = x;
    {i, j} = List@@x;
    ℰ1 = Switch[Head[x],
       Xp, mj,k→j[R+i,j (R-k3,k nrk1 ulk2 // mk,k1→k // mk,k2→k // mk,k3→k)],
       Xm, mj,k→j[R-i,j (R+k,k3 nrk1 ulk2 // mk,k1→k // mk,k2→k // mk,k3→k)]
     ];
    ℰ1 = rot[k, rots[[i]]] ℰ1 // mk,i→i; rots[[i]] = 0;
    ℰ1 = ℰ1 rot[k, rots[[i + 1]]] // mi,k→i; rots[[i + 1]] = 0;
    ℰ1 = rot[k, rots[[j]]] ℰ1 // mk,j→j; rots[[j]] = 0;
    ℰ1 = ℰ1 rot[k, rots[[j + 1]]] // mj,k→j; rots[[j + 1]] = 0;
    ℰ *= ℰ1;
    If[MemberQ[done, i], ℰ = ℰ // mi,i+1→i; st = st /. st[[i + 2]] → st[[i + 1]]];
    If[MemberQ[done, i - 1], ℰ = ℰ // mst[[i]],i→st[[i]]; st = st /. st[[i + 1]] → st[[i]]];
    If[MemberQ[done, j], ℰ = ℰ // mj,j+1→j; st = st /. st[[j + 2]] → st[[j + 1]]];
    If[MemberQ[done, j - 1], ℰ = ℰ // mst[[j]],j→st[[j]]; st = st /. st[[j + 1]] → st[[j]]];
    done = done ⋃ {i - 1, i, j - 1, j};
    todo = DeleteCases[todo, x]
   ];
   ℰ /. {v0 → v, c0 → c, w0 → w}
  ]
```

```
Timing[Z[Knot[3, 1]]]
```

KnotTheory: Loading precomputed data in PD4Knots`.

$$\{4.51563,\ \mathbb{E}\left[-1+\frac{1}{t}+t,\ 0,\ 0,\ -16-\frac{2}{t^4}+\frac{2\,c}{t^4}+\frac{7}{t^3}-\frac{6\,c}{t^3}-\frac{14}{t^2}+\frac{10\,c}{t^2}+\frac{18}{t}-\frac{8\,c}{t}+10\,t+8\,c\,t-\right.$$
$$\left.4\,t^2-10\,c\,t^2+t^3+6\,c\,t^3-2\,c\,t^4+2\,v\,w-\frac{2\,v\,w}{t^4}+\frac{4\,v\,w}{t^3}-\frac{6\,v\,w}{t^2}+\frac{2\,v\,w}{t}-6\,t\,v\,w+4\,t^2\,v\,w-2\,t^3\,v\,w\right]\}$$

Testing 10_{100}...

```
Timing[Z[Knot[10, 100]]]
```

```
BeginProfile[];
Timing[Z[Knot[10, 100]]]
EndProfile[];
```

Testing $T_{9,5}$...

```
BeginProfile[];
Timing[Z[TorusKnot[9, 5]]]
EndProfile[];
```

```
PrintProfile[];
```