

Pensieve header: Cheap CF optimization for the NOE1 program (V5).

## Initialization

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2016-12"];
Once[<< KnotTheory`];
Once[<< "../Projects/Profile/Profile.m"]
```

Loading KnotTheory` version of September 6, 2014, 13:37:37.2841.

Read more at <http://katlas.org/wiki/KnotTheory>.

This is Profile.m, Nov 2016 mods of July 1994 version

## Rotational Virtual Knots

```
RVK::usage =
"RVK[xs, rots] represents a Rotational Virtual Knot with a list of n Xp/Xm crossings xs and
a length 2n list of rotation numbers rots. Crossing sites are indexed 1 through
2n, and rots[[k]] is the rotation between site k-1 and site k. RVK is also a casting
operator converting to the RVK presentation from other knot presentations.";
RVK[pd_PD] := Module[{n, xs, x, rots, front, k},
  n = Length[pd];
  xs = List@@pd /. x_X => If[PositiveQ[x], Xp[x[[4]], x[[1]], Xm[x[[2]], x[[1]]];
  rots = Table[0, {2 n});
  front = {0};
  For[k = 0, k < 2 n, ++k,
    If[k == 0 ∨ FreeQ[front, -k],
      front = Flatten[front /. k → Catch[xs /. {
        Xp[k + 1, L_] | Xm[L_, k + 1] => Throw[{L, k + 1, 1 - L]},
        Xp[L_, k + 1] | Xm[k + 1, L_] => ({++rots[[L]]; Throw[{1 - L, k + 1, L]})
      }]],
      If[MatchQ[front, {___, k, ___, -k, ___}], --rots[[k + 1]]
    ]
  ];
  RVK[xs, rots]
];
RVK[K_] := RVK[PD[K]];
```

## NOE-It

1Gens

```
Ri_,j_+ := E[1, Log[ti cj, vi wj, vi ci wj + ci cj + vi2 wj2 / 4];
Ri_,j_- := E[1, -Log[ti cj, -ti-1 vi wj, -ci cj + ti-1 vi cj wj - ti-2 vi2 wj2 / 4];
(uri_ := E[ti-1/2, 0, 0, ci ti-2]; nri_ := E[ti1/2, 0, 0, -ci ti2];)
```

1DP

```
DPx_→D_α,y_→D_β[P_][f_] := (* means P[∂α, ∂β][f] *)
PPDP@Total[CoefficientRules[P, {x, y}] /. ({m_, n_} → c_) => c D[f, {α, m}, {β, n}]]
```

1Util

```
CF[E[ω_, L_, Q_, P_]] :=
PPCF@E[Expand@Together@ω, Expand@Together@L, Expand@Together@Q, Expand@PPTogether4P@Together@P];
```

1Util

```
 $\mathbb{E} /: \mathbb{E}[\omega 1_, L1_, Q1_, P1_] \mathbb{E}[\omega 2_, L2_, Q2_, P2_] := \text{CF} @ \mathbb{E}[\omega 1 \omega 2, L1 + L2, \omega 2 Q1 + \omega 1 Q2, \omega 2^4 P1 + \omega 1^4 P2];$ 
```

Logos

```
 $\Delta[k_] := ((t_k - 1) (2 (\alpha \beta + \delta \mu)^2 - \alpha^2 \beta^2) - 4 v_k c_k w_k \delta^2 \mu^2 - \delta (1 + \mu) (w_k^2 \alpha^2 + v_k^2 \beta^2) - v_k^2 w_k^2 \delta^3 (1 + 3 \mu) - 2 (\alpha \beta + 2 \delta \mu + v_k w_k \delta^2 (1 + 2 \mu) + 2 c_k \delta \mu^2) (w_k \alpha + v_k \beta) - 4 (c_k \mu^2 + v_k w_k \delta (1 + \mu)) (\alpha \beta + \delta \mu)) (1 + t_k) / 4;$ 
```

1NOuw

```
 $\text{N}_{w_i, v_j \rightarrow k}[\mathbb{E}[\eta_, L_, Q_, P_]] := \text{PP}_{\text{Nwv}} @ \text{With}[\{q = ((1 - t_k) \alpha \beta + \beta v_k + \delta v_k w_k + \alpha w_k) \omega / n\mu\}, \mathbb{E}[n\mu, L, \mu \omega q + \mu (Q /. w_i | v_j \rightarrow \theta), \mu^4 (\text{DP}_{w_i \rightarrow D_\alpha, v_j \rightarrow D_\beta}[P][e^q] /. e \rightarrow 1) + \omega^4 \Delta[k]] /. n\mu \rightarrow \omega + (t_k - 1) n\delta /. \{n\alpha \rightarrow \partial_{w_i} Q /. v_j \rightarrow \theta, n\beta \rightarrow \partial_{v_j} Q /. w_i \rightarrow \theta, n\delta \rightarrow \partial_{w_i, v_j} Q\}];$ 
```

1NOc

```
 $\text{N}_{c_j} (x: v | w)_{i \rightarrow k}[\mathbb{E}[\omega_, L_, Q_, P_]] := \text{PP}_{\text{Ncx}} @ \text{With}[\{q = e^\gamma \beta x_k + \gamma c_k\}, \text{CF}[\mathbb{E}[\omega, \gamma c_k + (L /. c_j \rightarrow \theta), \omega e^\gamma \beta x_k + (Q /. x_i \rightarrow \theta), e^{-q} \text{DP}_{c_j \rightarrow D_\gamma, x_i \rightarrow D_\beta}[P][e^q]] /. \{\gamma \rightarrow \partial_{c_j} L, \beta \rightarrow \omega^{-1} \partial_{x_i} Q\}]]];$ 
```

1m

```
 $m_{i, j \rightarrow k}[\mathbb{Z}_{\mathbb{E}}] := \text{PP}_m @ \text{Module}[\{x, z\}, \text{CF}[(Z // \text{N}_{w_i, v_j \rightarrow x} // \text{N}_{c_i, v_x \rightarrow x} // \text{N}_{w_x, c_j \rightarrow x}) /. \mathbb{Z}_{-i | j | x} \rightarrow z_k]]]$ 
```

## Z

```
 $ul_ = nl_ = \text{rot}[_ , \theta] = \mathbb{E}[1, \theta, \theta, \theta];$   

 $\text{rot}[i_, 1] := \text{ur}_i;$   

 $\text{rot}[i_, n\_Integer] /; n > 1 := \text{Module}[\{y\}, \text{rot}[i, n - 1] \text{rot}[y, 1] // m_{i, y \rightarrow i}];$   

 $\text{rot}[i_, -1] := \text{nr}_i;$   

 $\text{rot}[i_, n\_Integer] /; n < -1 := \text{Module}[\{y\}, \text{rot}[i, n + 1] \text{rot}[y, -1] // m_{i, y \rightarrow i}];$ 
```

```

t_ = t;
Z[K_] := Z[RVK@K];
Z[rvk_RVK] := PPz@Module[{todo, n, rots, ζ, done, st, x, ζ1, i, j, k, k1, k2, k3},
  {todo, rots} = List@@rvk;
  AppendTo[rots, 0];
  n = Length[todo];
  ζ = E[1, 0, 0, 0];
  done = {0};
  st = Range[0, 2 n + 1];
  While[todo != {},
    {x} = MaximalBy[todo, Length[done ∩ {#[[1]], #[[2]], #[[1]] - 1, #[[2]] - 1}] &, 1];
    Z$todo = todo; Z$x = x;
    {i, j} = List@@x;
    ζ1 = Switch[Head[x],
      Xp, m_{j,k→j} [R_{i,j}^+ (R_{k3,k}^- nr_{k1} ul_{k2} // m_{k,k1→k} // m_{k,k2→k} // m_{k,k3→k})],
      Xm, m_{j,k→j} [R_{i,j}^- (R_{k,k3}^+ nr_{k1} ul_{k2} // m_{k,k1→k} // m_{k,k2→k} // m_{k,k3→k})]
    ];
    ζ1 = rot[k, rots[[i]] ζ1 // m_{k,i→i}; rots[[i]] = 0;
    ζ1 = ζ1 rot[k, rots[[i + 1]] // m_{i,k→i}; rots[[i + 1]] = 0;
    ζ1 = rot[k, rots[[j]] ζ1 // m_{k,j→j}; rots[[j]] = 0;
    ζ1 = ζ1 rot[k, rots[[j + 1]] // m_{j,k→j}; rots[[j + 1]] = 0;
    ζ *= ζ1;
    If[MemberQ[done, i], ζ = ζ // m_{i,i+1→i}; st = st /. st[[i + 2]] → st[[i + 1]];
    If[MemberQ[done, i - 1], ζ = ζ // m_{st[[i],i→st[[i]]}; st = st /. st[[i + 1]] → st[[i]];
    If[MemberQ[done, j], ζ = ζ // m_{j,j+1→j}; st = st /. st[[j + 2]] → st[[j + 1]];
    If[MemberQ[done, j - 1], ζ = ζ // m_{st[[j],j→st[[j]]}; st = st /. st[[j + 1]] → st[[j]];
    done = done ∪ {i - 1, i, j - 1, j};
    todo = DeleteCases[todo, x]
  ];
  ζ /. {V_0 → V, C_0 → C, W_0 → W}
]

```

Timing[Z[Knot[3, 1]]]

KnotTheory: Loading precomputed data in PD4Knots`.

$$\left\{ 2.39063, \mathbb{E} \left[ -1 + \frac{1}{t} + t, 0, 0, -16 - \frac{2}{t^4} + \frac{2c}{t^4} + \frac{7}{t^3} - \frac{6c}{t^3} - \frac{14}{t^2} + \frac{10c}{t^2} + \frac{18}{t} - \frac{8c}{t} + 10t + 8ct - 4t^2 - 10ct^2 + t^3 + 6ct^3 - 2ct^4 + 2vw - \frac{2vw}{t^4} + \frac{4vw}{t^3} - \frac{6vw}{t^2} + \frac{2vw}{t} - 6tvw + 4t^2vw - 2t^3vw \right] \right\}$$

Testing  $10_{100}$ ...

Timing[Z[Knot[10, 100]]]

$$\left\{ 118.266, \mathbb{E} \left[ 13 + \frac{1}{t^4} - \frac{4}{t^3} + \frac{9}{t^2} - \frac{12}{t} - 12t + 9t^2 - 4t^3 + t^4, 0, 0, \right. \right. \\
- 2563146 - \frac{6}{t^{16}} + \frac{8c}{t^{16}} + \frac{92}{t^{15}} - \frac{120c}{t^{15}} - \frac{723}{t^{14}} + \frac{924c}{t^{14}} + \frac{3818}{t^{13}} - \frac{4784c}{t^{13}} - \frac{15133}{t^{12}} + \frac{18588c}{t^{12}} + \frac{47848}{t^{11}} - \frac{57552c}{t^{11}} - \\
\frac{125539}{t^{10}} + \frac{147540c}{t^{10}} + \frac{281054}{t^9} - \frac{321552c}{t^9} - \frac{548129}{t^8} + \frac{606988c}{t^8} + \frac{945756}{t^7} - \frac{1004976c}{t^7} - \frac{1460263}{t^6} + \\
\frac{1469820c}{t^6} + \frac{2034106}{t^5} - \frac{1901560c}{t^5} - \frac{2570432}{t^4} + \frac{2163176c}{t^4} + \frac{2956518}{t^3} - \frac{2123520c}{t^3} - \frac{3099338}{t^2} + \\
\frac{1711728c}{t^2} + \frac{2958726}{t} - \frac{958272c}{t} + 2000454t + 958272ct - 1387610t^2 - 1711728ct^2 + 832998t^3 + \\
2123520ct^3 - 407256t^4 - 2163176ct^4 + 132546t^5 + 1901560ct^5 + 9557t^6 - 1469820ct^6 - 59220t^7 + \\
1004976ct^7 + 58859t^8 - 606988ct^8 - 40498t^9 + 321552ct^9 + 22001t^{10} - 147540ct^{10} - 9704t^{11} + \\
57552ct^{11} + 3455t^{12} - 18588ct^{12} - 966t^{13} + 4784ct^{13} + 201t^{14} - 924ct^{14} - 28t^{15} + 120ct^{15} + 2t^{16} - \\
8ct^{16} + 253564vw - \frac{8vw}{t^{16}} + \frac{112vw}{t^{15}} - \frac{812vw}{t^{14}} + \frac{3972vw}{t^{13}} - \frac{14616vw}{t^{12}} + \frac{42936vw}{t^{11}} - \frac{104604vw}{t^{10}} + \frac{216948vw}{t^9} - \\
\frac{390040vw}{t^8} + \frac{614936vw}{t^7} - \frac{854884vw}{t^6} + \frac{1046676vw}{t^5} - \frac{1116500vw}{t^4} + \frac{1007020vw}{t^3} - \frac{704708vw}{t^2} + \frac{253564vw}{t} - \\
704708tvw + 1007020t^2vw - 1116500t^3vw + 1046676t^4vw - 854884t^5vw + 614936t^6vw - 390040t^7vw + \\
216948t^8vw - 104604t^9vw + 42936t^{10}vw - 14616t^{11}vw + 3972t^{12}vw - 812t^{13}vw + 112t^{14}vw - 8t^{15}vw \left. \right\}$$

BeginProfile[];

Timing[Z[Knot[10, 100]]]

EndProfile[];

$$\left\{ 119.047, \mathbb{E} \left[ 13 + \frac{1}{t^4} - \frac{4}{t^3} + \frac{9}{t^2} - \frac{12}{t} - 12t + 9t^2 - 4t^3 + t^4, 0, 0, \right. \right. \\
- 2563146 - \frac{6}{t^{16}} + \frac{8c}{t^{16}} + \frac{92}{t^{15}} - \frac{120c}{t^{15}} - \frac{723}{t^{14}} + \frac{924c}{t^{14}} + \frac{3818}{t^{13}} - \frac{4784c}{t^{13}} - \frac{15133}{t^{12}} + \frac{18588c}{t^{12}} + \frac{47848}{t^{11}} - \frac{57552c}{t^{11}} - \\
\frac{125539}{t^{10}} + \frac{147540c}{t^{10}} + \frac{281054}{t^9} - \frac{321552c}{t^9} - \frac{548129}{t^8} + \frac{606988c}{t^8} + \frac{945756}{t^7} - \frac{1004976c}{t^7} - \frac{1460263}{t^6} + \\
\frac{1469820c}{t^6} + \frac{2034106}{t^5} - \frac{1901560c}{t^5} - \frac{2570432}{t^4} + \frac{2163176c}{t^4} + \frac{2956518}{t^3} - \frac{2123520c}{t^3} - \frac{3099338}{t^2} + \\
\frac{1711728c}{t^2} + \frac{2958726}{t} - \frac{958272c}{t} + 2000454t + 958272ct - 1387610t^2 - 1711728ct^2 + 832998t^3 + \\
2123520ct^3 - 407256t^4 - 2163176ct^4 + 132546t^5 + 1901560ct^5 + 9557t^6 - 1469820ct^6 - 59220t^7 + \\
1004976ct^7 + 58859t^8 - 606988ct^8 - 40498t^9 + 321552ct^9 + 22001t^{10} - 147540ct^{10} - 9704t^{11} + \\
57552ct^{11} + 3455t^{12} - 18588ct^{12} - 966t^{13} + 4784ct^{13} + 201t^{14} - 924ct^{14} - 28t^{15} + 120ct^{15} + 2t^{16} - \\
8ct^{16} + 253564vw - \frac{8vw}{t^{16}} + \frac{112vw}{t^{15}} - \frac{812vw}{t^{14}} + \frac{3972vw}{t^{13}} - \frac{14616vw}{t^{12}} + \frac{42936vw}{t^{11}} - \frac{104604vw}{t^{10}} + \frac{216948vw}{t^9} - \\
\frac{390040vw}{t^8} + \frac{614936vw}{t^7} - \frac{854884vw}{t^6} + \frac{1046676vw}{t^5} - \frac{1116500vw}{t^4} + \frac{1007020vw}{t^3} - \frac{704708vw}{t^2} + \frac{253564vw}{t} - \\
704708tvw + 1007020t^2vw - 1116500t^3vw + 1046676t^4vw - 854884t^5vw + 614936t^6vw - 390040t^7vw + \\
216948t^8vw - 104604t^9vw + 42936t^{10}vw - 14616t^{11}vw + 3972t^{12}vw - 812t^{13}vw + 112t^{14}vw - 8t^{15}vw \left. \right\}$$

PrintProfile[];

Together4P: called 2765 times, time in 71.608/71.608

Parents:

( 380) 5.704/ 5.704 under CF

( 2385) 65.904/ 65.904 under CF4P

CF4P: called 100 times, time in 44.672/110.576

Parents:

( 100) 44.672/ 110.580 under CF2

Children:

( 2385) 65.904/ 65.904 above Together4P

DP: called 300 times, time in 1.329/1.329

Parents:

( 200) 1.078/ 1.078 under Ncx  
 ( 100) 0.251/ 0.251 under Nwv

CF: called 380 times, time in 0.596/6.3

Parents:

( 100) 0.111/ 1.048 under m  
 ( 200) 0.437/ 4.907 under Ncx  
 ( 80) 0.048/ 0.345 under z

Children:

( 380) 5.704/ 5.704 above Together4P

Ncx: called 200 times, time in 0.278/6.263

Parents:

( 200) 0.278/ 6.263 under m

Children:

( 200) 0.437/ 4.907 above CF  
 ( 200) 1.078/ 1.078 above DP

Nwv: called 100 times, time in 0.25/111.219

Parents:

( 100) 0.250/ 111.220 under m

Children:

( 100) 0.142/ 110.720 above CF2  
 ( 100) 0.251/ 0.251 above DP

m: called 100 times, time in 0.172/118.702

Parents:

( 100) 0.172/ 118.700 under z

Children:

( 100) 0.111/ 1.048 above CF  
 ( 200) 0.278/ 6.263 above Ncx  
 ( 100) 0.250/ 111.220 above Nwv

CF2: called 100 times, time in 0.142/110.718

Parents:

( 100) 0.142/ 110.720 under Nwv

Children:

( 100) 44.672/ 110.580 above CF4P

```
z: called 1 times, time in 0./119.047
```

```
Parents:
```

```
( 1) 0.000/ 119.050 under ProfileRoot
```

```
Children:
```

```
( 80) 0.048/ 0.345 above CF
```

```
( 100) 0.172/ 118.700 above m
```

```
ProfileRoot: called 0 times, time in 0./0.
```

```
Children:
```

```
( 1) 0.000/ 119.050 above z
```

## Testing $T_{9,5}$ ...

```
BeginProfile[];  
Timing[Z[TorusKnot[9, 5]]]  
EndProfile[];  
PrintProfile[];
```