

Pensieve header: Cheap CF optimization for the NOE1 program (V4.5, failed).

Initialization

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2016-12"];
Once[<< KnotTheory`];
Once[<< "../Projects/Profile/Profile.m"]
```

Loading KnotTheory` version of September 6, 2014, 13:37:37.2841.

Read more at <http://katlas.org/wiki/KnotTheory>.

This is Profile.m, Nov 2016 mods of July 1994 version

Rotational Virtual Knots

```
RVK::usage =
"RVK[xs, rots] represents a Rotational Virtual Knot with a list of n Xp/Xm crossings xs and
a length 2n list of rotation numbers rots. Crossing sites are indexed 1 through
2n, and rots[[k]] is the rotation between site k-1 and site k. RVK is also a casting
operator converting to the RVK presentation from other knot presentations.";
RVK[pd_PD] := Module[{n, xs, x, rots, front, k},
  n = Length[pd];
  xs = List@@pd /. x_X => If[PositiveQ[x], Xp[x[[4]], x[[1]], Xm[x[[2]], x[[1]]];
  rots = Table[0, {2 n});
  front = {0};
  For[k = 0, k < 2 n, ++k,
    If[k == 0 ∨ FreeQ[front, -k],
      front = Flatten[front /. k → Catch[xs /. {
        Xp[k + 1, L_] | Xm[L_, k + 1] => Throw[{L, k + 1, 1 - L]},
        Xp[L_, k + 1] | Xm[k + 1, L_] => ({++rots[[L]]; Throw[{1 - L, k + 1, L]})
      }]],
      If[MatchQ[front, {___, k, ___, -k, ___}], --rots[[k + 1]]
    ]
  ];
  RVK[xs, rots]
];
RVK[K_] := RVK[PD[K]];
```

NOE-It

1Gens

```
Ri_,j_+ := E[1, Log[ti cj, vi wj, vi ci wj + ci cj + vi2 wj2 / 4];
Ri_,j_- := E[1, -Log[ti cj, -ti-1 vi wj, -ci cj + ti-1 vi cj wj - ti-2 vi2 wj2 / 4];
(uri_ := E[ti-1/2, 0, 0, ci ti-2]; nri_ := E[ti1/2, 0, 0, -ci ti2];)
```

1DP

```
DPx_→Dα,y_→Dβ[P_][f_] := (* means P[∂α, ∂β][f] *)
PPDP@Total[CoefficientRules[P, {x, y}] /. ({m_, n_} → c_) => c D[f, {α, m}, {β, n}]]
```

1Util

```
CF[E[ω_, L_, Q_, P_]] :=
PPCF@E[Expand@Together@ω, Expand@Together@L, Expand@Together@Q, Expand@PPTogether4P@Together@P];
```

1Util

$$\mathbb{E} /: \mathbb{E}[\omega 1_, L1_, Q1_, P1_] \mathbb{E}[\omega 2_, L2_, Q2_, P2_] := \text{CF} @ \mathbb{E}[\omega 1 \omega 2, L1 + L2, \omega 2 Q1 + \omega 1 Q2, \omega 2^4 P1 + \omega 1^4 P2];$$

Logos

$$\Delta[k_, \omega_] := \left((t_k - 1) \left(2 (\alpha \beta + \omega^{-2} \delta \mu)^2 - \alpha^2 \beta^2 \right) - 4 v_k c_k w_k \omega^{-2} \delta^2 \mu^2 - \right. \\ \left. \omega^{-2} \delta (\omega + \mu) (w_k^2 \alpha^2 + v_k^2 \beta^2) - v_k^2 w_k^2 \omega^{-4} \delta^3 (\omega + 3 \mu) - 2 (\alpha \beta + 2 \omega^{-2} \delta \mu + v_k w_k \omega^{-3} \delta^2 (\omega + 2 \mu) + 2 c_k \omega^{-3} \delta \mu^2) \right. \\ \left. (w_k \alpha + v_k \beta) - 4 (c_k \omega^{-2} \mu^2 + v_k w_k \omega^{-2} \delta (\omega + \mu)) (\alpha \beta + \omega^{-2} \delta \mu) \right) (1 + t_k) / 4;$$

1NOuw

$$\mathbb{N}_{w_i v_j \rightarrow k}[\mathbb{E}[\omega_, L_, Q_, P_]] := \text{PP}_{\text{Nwv}} @ \text{With}[\{q = ((1 - t_k) \omega \alpha \beta + \omega \beta v_k + \delta v_k w_k + \omega \alpha w_k) / \mu\}, \text{CF}[\\ \mathbb{E}[\mu, L, \mu q + \omega^{-1} \mu (Q /. w_i | v_j \rightarrow \theta), \omega^{-4} \mu^4 (\text{DP}_{w_i \rightarrow D_\alpha, v_j \rightarrow D_\beta}[P][e^q] /. e \rightarrow 1) + \omega^4 \Delta[k, \omega]] /. \\ \mu \rightarrow \omega + (t_k - 1) \delta /. \{\alpha \rightarrow \omega^{-1} (\partial_{w_i} Q /. v_j \rightarrow \theta), \beta \rightarrow \omega^{-1} (\partial_{v_j} Q /. w_i \rightarrow \theta), \delta \rightarrow \partial_{w_i, v_j} Q\} \\]];$$

1NOc

$$\mathbb{N}_{c_j (x:v|w)_i \rightarrow k}[\mathbb{E}[\omega_, L_, Q_, P_]] := \text{PP}_{\text{Ncx}} @ \text{With}[\{q = e^\gamma \beta x_k + \gamma c_k\}, \text{CF}[\\ \mathbb{E}[\omega, \gamma c_k + (L /. c_j \rightarrow \theta), \omega e^\gamma \beta x_k + (Q /. x_i \rightarrow \theta), e^{-q} \text{DP}_{c_j \rightarrow D_\gamma, x_i \rightarrow D_\beta}[P][e^q]] /. \{\gamma \rightarrow \partial_{c_j} L, \beta \rightarrow \omega^{-1} \partial_{x_i} Q\}]];$$

1m

$$m_{i,j \rightarrow k}[Z_{\mathbb{E}}] := \text{PP}_m @ \text{Module}[\{x, z\}, \\ \text{CF}[(Z // \mathbb{N}_{w_i v_j \rightarrow x} // \mathbb{N}_{c_i v_x \rightarrow x} // \mathbb{N}_{w_x c_j \rightarrow x}) /. z_{-i|j|x} \rightarrow z_k]]$$

Z

$$\text{ul}_ = \text{nl}_ = \text{rot}[_ , \theta] = \mathbb{E}[1, \theta, \theta, \theta]; \\ \text{rot}[i_, 1] := \text{ur}_i; \\ \text{rot}[i_, n_Integer] /; n > 1 := \text{Module}[\{y\}, \text{rot}[i, n - 1] \text{rot}[y, 1] // m_{i,y \rightarrow i}]; \\ \text{rot}[i_, -1] := \text{nr}_i; \\ \text{rot}[i_, n_Integer] /; n < -1 := \text{Module}[\{y\}, \text{rot}[i, n + 1] \text{rot}[y, -1] // m_{i,y \rightarrow i}];$$

```

t_ = t;
Z[K_] := Z[RVK@K];
Z[rvk_RVK] := PPz@Module[{todo, n, rots,  $\xi$ , done, st, x,  $\xi_1$ , i, j, k, k1, k2, k3},
  {todo, rots} = List@@rvk;
  AppendTo[rots, 0];
  n = Length[todo];
   $\xi$  =  $\mathbb{E}$ [1, 0, 0, 0];
  done = {0};
  st = Range[0, 2 n + 1];
  While[todo != {},
    {x} = MaximalBy[todo, Length[done  $\cap$  {#[[1]], #[[2]], #[[1]] - 1, #[[2]] - 1}] &, 1];
    Z$todo = todo; Z$x = x;
    {i, j} = List@@x;
     $\xi_1$  = Switch[Head[x],
      Xp, mj,k→j [ $R_{i,j}^+$  ( $R_{k3,k}^-$  nrk1 ulk2 // mk,k1→k // mk,k2→k // mk,k3→k) ],
      Xm, mj,k→j [ $R_{i,j}^-$  ( $R_{k,k3}^+$  nrk1 ulk2 // mk,k1→k // mk,k2→k // mk,k3→k) ]
    ];
     $\xi_1$  = rot[k, rots[[i]]  $\xi_1$  // mk,i→i; rots[[i]] = 0;
     $\xi_1$  =  $\xi_1$  rot[k, rots[[i + 1]] // mi,k→i; rots[[i + 1]] = 0;
     $\xi_1$  = rot[k, rots[[j]]  $\xi_1$  // mk,j→j; rots[[j]] = 0;
     $\xi_1$  =  $\xi_1$  rot[k, rots[[j + 1]] // mj,k→j; rots[[j + 1]] = 0;
     $\xi$  *=  $\xi_1$ ;
    If[MemberQ[done, i],  $\xi$  =  $\xi$  // mi,i+1→i; st = st /. st[[i + 2]] → st[[i + 1]];
    If[MemberQ[done, i - 1],  $\xi$  =  $\xi$  // mst[[i],i→st[[i]]; st = st /. st[[i + 1]] → st[[i]];
    If[MemberQ[done, j],  $\xi$  =  $\xi$  // mj,j+1→j; st = st /. st[[j + 2]] → st[[j + 1]];
    If[MemberQ[done, j - 1],  $\xi$  =  $\xi$  // mst[[j],j→st[[j]]; st = st /. st[[j + 1]] → st[[j]];
    done = done  $\cup$  {i - 1, i, j - 1, j};
    todo = DeleteCases[todo, x]
  ];
   $\xi$  /. {V0 → V, C0 → C, W0 → W}
]

```

Timing[Z[Knot[3, 1]]]

KnotTheory: Loading precomputed data in PD4Knots`.

{1.70313,

$$\mathbb{E}\left[-1 + \frac{1}{t} + t, 0, 0, -\frac{264}{(1-2t+2t^2)^2} + \frac{168c}{(1-2t+2t^2)^2} - \frac{2}{t^4(1-2t+2t^2)^2} + \frac{2c}{t^4(1-2t+2t^2)^2} + \frac{15}{t^3(1-2t+2t^2)^2} - \frac{14c}{t^3(1-2t+2t^2)^2} - \frac{58}{t^2(1-2t+2t^2)^2} + \frac{50c}{t^2(1-2t+2t^2)^2} + \frac{146}{t(1-2t+2t^2)^2} - \frac{112c}{t(1-2t+2t^2)^2} + \frac{358t}{(1-2t+2t^2)^2} - \frac{160ct}{(1-2t+2t^2)^2} - \frac{372t^2}{(1-2t+2t^2)^2} + \frac{62ct^2}{(1-2t+2t^2)^2} + \frac{297t^3}{(1-2t+2t^2)^2} + \frac{78ct^3}{(1-2t+2t^2)^2} - \frac{180t^4}{(1-2t+2t^2)^2} - \frac{170ct^4}{(1-2t+2t^2)^2} + \frac{80t^5}{(1-2t+2t^2)^2} + \frac{168ct^5}{(1-2t+2t^2)^2} - \frac{24t^6}{(1-2t+2t^2)^2} - \frac{104ct^6}{(1-2t+2t^2)^2} + \frac{4t^7}{(1-2t+2t^2)^2} + \frac{40ct^7}{(1-2t+2t^2)^2} - \frac{8ct^8}{(1-2t+2t^2)^2} - \frac{94vw}{(1-2t+2t^2)^2} - \frac{187c vw}{(1-2t+2t^2)^2} - \frac{2vw}{t^4(1-2t+2t^2)^2} + \frac{12vw}{t^3(1-2t+2t^2)^2} + \frac{4c vw}{t^3(1-2t+2t^2)^2} - \frac{38vw}{t^2(1-2t+2t^2)^2} - \frac{25c vw}{t^2(1-2t+2t^2)^2} + \frac{74vw}{t(1-2t+2t^2)^2} + \frac{85c vw}{t(1-2t+2t^2)^2} + \frac{66t vw}{(1-2t+2t^2)^2} + \frac{285ct vw}{(1-2t+2t^2)^2} + \frac{4t^2 vw}{(1-2t+2t^2)^2} - \frac{289ct^2 vw}{(1-2t+2t^2)^2} - \frac{74t^3 vw}{(1-2t+2t^2)^2} + \frac{149ct^3 vw}{(1-2t+2t^2)^2} + \frac{96t^4 vw}{(1-2t+2t^2)^2} + \frac{76ct^4 vw}{(1-2t+2t^2)^2} - \frac{72t^5 vw}{(1-2t+2t^2)^2} - \frac{248ct^5 vw}{(1-2t+2t^2)^2} + \frac{32t^6 vw}{(1-2t+2t^2)^2} + \frac{280ct^6 vw}{(1-2t+2t^2)^2} - \frac{8t^7 vw}{(1-2t+2t^2)^2} - \frac{204ct^7 vw}{(1-2t+2t^2)^2} + \frac{111ct^8 vw}{(1-2t+2t^2)^2} - \frac{55ct^9 vw}{(1-2t+2t^2)^2} + \frac{29ct^{10} vw}{(1-2t+2t^2)^2} - \frac{15ct^{11} vw}{(1-2t+2t^2)^2} + \frac{5ct^{12} vw}{(1-2t+2t^2)^2} - \frac{ct^{13} vw}{(1-2t+2t^2)^2}\right]$$

Testing 10₁₀₀...

Timing[Z[Knot[10, 100]]]

$$\{118.266, \mathbb{E}\left[13 + \frac{1}{t^4} - \frac{4}{t^3} + \frac{9}{t^2} - \frac{12}{t} - 12t + 9t^2 - 4t^3 + t^4, 0, 0, -2563146 - \frac{6}{t^{16}} + \frac{8c}{t^{16}} + \frac{92}{t^{15}} - \frac{120c}{t^{15}} - \frac{723}{t^{14}} + \frac{924c}{t^{14}} + \frac{3818}{t^{13}} - \frac{4784c}{t^{13}} - \frac{15133}{t^{12}} + \frac{18588c}{t^{12}} + \frac{47848}{t^{11}} - \frac{57552c}{t^{11}} - \frac{125539}{t^{10}} + \frac{147540c}{t^{10}} + \frac{281054}{t^9} - \frac{321552c}{t^9} - \frac{548129}{t^8} + \frac{606988c}{t^8} + \frac{945756}{t^7} - \frac{1004976c}{t^7} - \frac{1460263}{t^6} + \frac{1469820c}{t^6} + \frac{2034106}{t^5} - \frac{1901560c}{t^5} - \frac{2570432}{t^4} + \frac{2163176c}{t^4} + \frac{2956518}{t^3} - \frac{2123520c}{t^3} - \frac{3099338}{t^2} + \frac{1711728c}{t^2} + \frac{2958726}{t} - \frac{958272c}{t} + 2000454t + 958272ct - 1387610t^2 - 1711728ct^2 + 832998t^3 + 2123520ct^3 - 407256t^4 - 2163176ct^4 + 132546t^5 + 1901560ct^5 + 9557t^6 - 1469820ct^6 - 59220t^7 + 1004976ct^7 + 58859t^8 - 606988ct^8 - 40498t^9 + 321552ct^9 + 22001t^{10} - 147540ct^{10} - 9704t^{11} + 57552ct^{11} + 3455t^{12} - 18588ct^{12} - 966t^{13} + 4784ct^{13} + 201t^{14} - 924ct^{14} - 28t^{15} + 120ct^{15} + 2t^{16} - 8ct^{16} + 253564vw - \frac{8vw}{t^{16}} + \frac{112vw}{t^{15}} - \frac{812vw}{t^{14}} + \frac{3972vw}{t^{13}} - \frac{14616vw}{t^{12}} + \frac{42936vw}{t^{11}} - \frac{104604vw}{t^{10}} + \frac{216948vw}{t^9} - \frac{390040vw}{t^8} + \frac{614936vw}{t^7} - \frac{854884vw}{t^6} + \frac{1046676vw}{t^5} - \frac{1116500vw}{t^4} + \frac{1007020vw}{t^3} - \frac{704708vw}{t^2} + \frac{253564vw}{t} + 704708t vw + 1007020t^2 vw - 1116500t^3 vw + 1046676t^4 vw - 854884t^5 vw + 614936t^6 vw - 390040t^7 vw + 216948t^8 vw - 104604t^9 vw + 42936t^{10} vw - 14616t^{11} vw + 3972t^{12} vw - 812t^{13} vw + 112t^{14} vw - 8t^{15} vw\right]\}$$

```
BeginProfile [];
Timing[Z[Knot[10, 100]]]
EndProfile [];
```

$$\left\{ 119.047, \mathbb{E} \left[13 + \frac{1}{t^4} - \frac{4}{t^3} + \frac{9}{t^2} - \frac{12}{t} - 12t + 9t^2 - 4t^3 + t^4, 0, 0, \right. \right.$$

$$\left. -2563146 - \frac{6}{t^{16}} + \frac{8c}{t^{16}} + \frac{92}{t^{15}} - \frac{120c}{t^{15}} - \frac{723}{t^{14}} + \frac{924c}{t^{14}} + \frac{3818}{t^{13}} - \frac{4784c}{t^{13}} - \frac{15133}{t^{12}} + \frac{18588c}{t^{12}} + \frac{47848}{t^{11}} - \frac{57552c}{t^{11}} - \right.$$

$$\left. \frac{125539}{t^{10}} + \frac{147540c}{t^{10}} + \frac{281054}{t^9} - \frac{321552c}{t^9} - \frac{548129}{t^8} + \frac{606988c}{t^8} + \frac{945756}{t^7} - \frac{1004976c}{t^7} - \frac{1460263}{t^6} + \right.$$

$$\left. \frac{1469820c}{t^6} + \frac{2034106}{t^5} - \frac{1901560c}{t^5} - \frac{2570432}{t^4} + \frac{2163176c}{t^4} + \frac{2956518}{t^3} - \frac{2123520c}{t^3} - \frac{3099338}{t^2} + \right.$$

$$\left. \frac{1711728c}{t^2} + \frac{2958726}{t} - \frac{958272c}{t} + 2000454t + 958272ct - 1387610t^2 - 1711728ct^2 + 832998t^3 + \right.$$

$$\left. 2123520ct^3 - 407256t^4 - 2163176ct^4 + 132546t^5 + 1901560ct^5 + 9557t^6 - 1469820ct^6 - 59220t^7 + \right.$$

$$\left. 1004976ct^7 + 58859t^8 - 606988ct^8 - 40498t^9 + 321552ct^9 + 22001t^{10} - 147540ct^{10} - 9704t^{11} + \right.$$

$$\left. 57552ct^{11} + 3455t^{12} - 18588ct^{12} - 966t^{13} + 4784ct^{13} + 201t^{14} - 924ct^{14} - 28t^{15} + 120ct^{15} + 2t^{16} - \right.$$

$$\left. 8ct^{16} + 253564vw - \frac{8vw}{t^{16}} + \frac{112vw}{t^{15}} - \frac{812vw}{t^{14}} + \frac{3972vw}{t^{13}} - \frac{14616vw}{t^{12}} + \frac{42936vw}{t^{11}} - \frac{104604vw}{t^{10}} + \frac{216948vw}{t^9} - \right.$$

$$\left. \frac{390040vw}{t^8} + \frac{614936vw}{t^7} - \frac{854884vw}{t^6} + \frac{1046676vw}{t^5} - \frac{1116500vw}{t^4} + \frac{1007020vw}{t^3} - \frac{704708vw}{t^2} + \frac{253564vw}{t} - \right.$$

$$\left. 704708tvw + 1007020t^2vw - 1116500t^3vw + 1046676t^4vw - 854884t^5vw + 614936t^6vw - 390040t^7vw + \right.$$

$$\left. 216948t^8vw - 104604t^9vw + 42936t^{10}vw - 14616t^{11}vw + 3972t^{12}vw - 812t^{13}vw + 112t^{14}vw - 8t^{15}vw \right\}$$

```
PrintProfile [];
```

Together4P: called 2765 times, time in 71.608/71.608

Parents:

(380) 5.704/ 5.704 under CF

(2385) 65.904/ 65.904 under CF4P

CF4P: called 100 times, time in 44.672/110.576

Parents:

(100) 44.672/ 110.580 under CF2

Children:

(2385) 65.904/ 65.904 above Together4P

DP: called 300 times, time in 1.329/1.329

Parents:

(200) 1.078/ 1.078 under Ncx

(100) 0.251/ 0.251 under Nwv

CF: called 380 times, time in 0.596/6.3

Parents:

(100) 0.111/ 1.048 under m

(200) 0.437/ 4.907 under Ncx

(80) 0.048/ 0.345 under z

Children:

(380) 5.704/ 5.704 above Together4P

Ncx: called 200 times, time in 0.278/6.263

Parents:

(200) 0.278/ 6.263 under m

Children:

(200) 0.437/ 4.907 above CF

(200) 1.078/ 1.078 above DP

Nwv: called 100 times, time in 0.25/111.219

Parents:

(100) 0.250/ 111.220 under m

Children:

(100) 0.142/ 110.720 above CF2

(100) 0.251/ 0.251 above DP

m: called 100 times, time in 0.172/118.702

Parents:

(100) 0.172/ 118.700 under z

Children:

(100) 0.111/ 1.048 above CF

(200) 0.278/ 6.263 above Ncx

(100) 0.250/ 111.220 above Nwv

CF2: called 100 times, time in 0.142/110.718

Parents:

(100) 0.142/ 110.720 under Nwv

Children:

(100) 44.672/ 110.580 above CF4P

z: called 1 times, time in 0./119.047

Parents:

(1) 0.000/ 119.050 under ProfileRoot

Children:

(80) 0.048/ 0.345 above CF

(100) 0.172/ 118.700 above m

ProfileRoot: called 0 times, time in 0./0.

Children:

(1) 0.000/ 119.050 above z

Testing $T_{9,5}$...

```
BeginProfile[];  
Timing[Z[TorusKnot[9, 5]]]  
EndProfile[];  
  
PrintProfile[];
```