

Pensieve header: Cheap CF optimization for the NOE1 program (V3, use "Cancel", failed).

## Initialization

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2016-12"];
Once[<< KnotTheory`];
Once[<< "../Projects/Profile/Profile.m"]
```

Loading KnotTheory` version of September 6, 2014, 13:37:37.2841.

Read more at <http://katlas.org/wiki/KnotTheory>.

This is Profile.m, Nov 2016 mods of July 1994 version

## Rotational Virtual Knots

```
RVK::usage =
"RVK[xs, rots] represents a Rotational Virtual Knot with a list of n Xp/Xm crossings xs and
a length 2n list of rotation numbers rots. Crossing sites are indexed 1 through
2n, and rots[[k]] is the rotation between site k-1 and site k. RVK is also a casting
operator converting to the RVK presentation from other knot presentations.";
RVK[pd_PD] := Module[{n, xs, x, rots, front, k},
n = Length[pd];
xs = List @@ pd /. x_X => If[PositiveQ[x], Xp[x[[4]], x[[1]], Xm[x[[2]], x[[1]]]];
rots = Table[0, {2 n}];
front = {0};
For[k = 0, k < 2 n, ++k,
If[k == 0 || FreeQ[front, -k],
front = Flatten[front /. k -> Catch[xs /. {
Xp[k + 1, L_] | Xm[L_, k + 1] => Throw[{L, k + 1, 1 - L}],
Xp[L_, k + 1] | Xm[k + 1, L_] => (++rots[[L]]; Throw[{1 - L, k + 1, L})}]]],
If[MatchQ[front, {___, k, ___, -k, ___}], --rots[[k + 1]]]
];
RVK[xs, rots]
];
RVK[K_] := RVK[PD[K]];
```

## NOE-It

Logos

$$\Delta[k_] := \left( (t_k - 1) (2 (\alpha \beta + \delta \mu)^2 - \alpha^2 \beta^2) - 4 v_k c_k w_k \delta^2 \mu^2 - \delta (1 + \mu) (w_k^2 \alpha^2 + v_k^2 \beta^2) - v_k^2 w_k^2 \delta^3 (1 + 3 \mu) - 2 (\alpha \beta + 2 \delta \mu + v_k w_k \delta^2 (1 + 2 \mu) + 2 c_k \delta \mu^2) (w_k \alpha + v_k \beta) - 4 (c_k \mu^2 + v_k w_k \delta (1 + \mu)) (\alpha \beta + \delta \mu) (1 + t_k) \right) / 4;$$

1Gens

$$\begin{aligned} R_{i,j}^+ &:= \mathbb{E} \left[ 1, \text{Log}[t_i] c_j, v_i w_j, v_i c_i w_j + c_i c_j + v_i^2 w_j^2 / 4 \right]; \\ R_{i,j}^- &:= \mathbb{E} \left[ 1, -\text{Log}[t_i] c_j, -t_i^{-1} v_i w_j, -c_i c_j + t_i^{-1} v_i c_j w_j - t_i^{-2} v_i^2 w_j^2 / 4 \right]; \\ (ur_{i-} &:= \mathbb{E} [t_i^{-1/2}, 0, 0, c_i t_i^{-2}]; nr_{i-} := \mathbb{E} [t_i^{1/2}, 0, 0, -c_i t_i^2];) \end{aligned}$$

1DP

$$\begin{aligned} DP_{x \rightarrow \partial_\alpha, y \rightarrow \partial_\beta} [P_] [f_] &:= (* \text{ means } P[\partial_\alpha, \partial_\beta] [f] *) \\ PP_{DP} @ \text{Total} [\text{CoefficientRules} [P, \{x, y\}] /. (\{m_, n_\} \rightarrow c_) => c D[f, \{\alpha, m\}, \{\beta, n\}]] \end{aligned}$$

1Util

```

CF[E[ω-, L-, Q-, P-]] :=
  PPCF@E[ExpandTogether@ω, ExpandTogether@L, ExpandTogether@Q, Expand@PPCancel14P@Cancel@P];
CF2[ωθ-, E[ω1-, L-, Q-, P-]] := PPCF2@E[ExpandTogether@ (ω1 / ω → ωθ),
  ExpandTogether@ (L / ω → ωθ), ExpandTogether@ (Q / ω → ωθ), PPCF4P@Module[{vars},
  vars = Union@Cases[P, (c | v | w)-, ∞];
  Total[CoefficientRules[P, vars] /.
  (p- → cc-) ⇒ (Expand@PPCancel14P@Cancel@ (cc / ω → ωθ)) (Times@@ (varsp))]
  ]];

```

1Util

```

E /: E[ω1-, L1-, Q1-, P1-] E[ω2-, L2-, Q2-, P2-] := CF@E[ω1 ω2, L1 + L2, ω2 Q1 + ω1 Q2, ω24 P1 + ω14 P2];

```

1NOuw

```

Nwi vj → k[E[ωθ-, L-, Q-, P-]] := PPNwv@With[{q = ((1 - tk) α β + β vk + δ vk wk + α wk}) / μ},
  CF2[ωθ,
  E[μ ω, L, μ ω q + μ (Q / wi | vj → θ), μ4 (DPwi → Dα, vj → Dβ}[P][eq] / e- → 1) + ω4 Δ[k]] / μ → 1 + (tk - 1) δ /
  {α → ω-1 (∂wi Q / vj → θ), β → ω-1 (∂vj Q / wi → θ), δ → ω-1 ∂wi, vj Q}
  ]];

```

1NOc

```

Ncj (x: v | w)i → k[E[ω-, L-, Q-, P-]] := PPNcx@With[{q = eγ β xk + γ ck}, CF[
  E[ω, γ ck + (L / cj → θ), ω eγ β xk + (Q / xi → θ), e-q DPcj → Dγ, xi → Dβ}[P][eq]] / {γ → ∂cj L, β → ω-1 ∂xi Q}]];

```

1m

```

mi, j → k[ZE] := PPm@Module[{x, z},
  CF[(Z // Nwi vj → x // Nci vx → x // Nwx cj → x) / z-i | j | x → zk]]

```

## Z

```

ul- = nl- = rot[_ , 0] = E[1, 0, 0, 0];
rot[i-, 1] := uri;
rot[i-, n_Integer] /; n > 1 := Module[{y}, rot[i, n - 1] rot[y, 1] // mi, y → i];
rot[i-, -1] := nri;
rot[i-, n_Integer] /; n < -1 := Module[{y}, rot[i, n + 1] rot[y, -1] // mi, y → i];

```

```

t_ = t;
Z[K_] := Z[RVK@K];
Z[rvk_RVK] := PPz@Module[{todo, n, rots, ζ, done, st, x, ζ1, i, j, k, k1, k2, k3},
  {todo, rots} = List@@rvk;
  AppendTo[rots, 0];
  n = Length[todo];
  ζ = E[1, 0, 0, 0];
  done = {0};
  st = Range[0, 2 n + 1];
  While[todo != {},
    {x} = MaximalBy[todo, Length[done ∩ {#[[1]], #[[2]], #[[1]] - 1, #[[2]] - 1}] &, 1];
    Z$todo = todo; Z$x = x;
    {i, j} = List@@x;
    ζ1 = Switch[Head[x],
      Xp, m_{j,k→j} [R_{i,j}^+ (R_{k3,k}^- nr_{k1} ul_{k2} // m_{k,k1→k} // m_{k,k2→k} // m_{k,k3→k})],
      Xm, m_{j,k→j} [R_{i,j}^- (R_{k,k3}^+ nr_{k1} ul_{k2} // m_{k,k1→k} // m_{k,k2→k} // m_{k,k3→k})]
    ];
    ζ1 = rot[k, rots[[i]] ζ1 // m_{k,i→i}; rots[[i]] = 0;
    ζ1 = ζ1 rot[k, rots[[i + 1]] // m_{i,k→i}; rots[[i + 1]] = 0;
    ζ1 = rot[k, rots[[j]] ζ1 // m_{k,j→j}; rots[[j]] = 0;
    ζ1 = ζ1 rot[k, rots[[j + 1]] // m_{j,k→j}; rots[[j + 1]] = 0;
    ζ *= ζ1;
    If[MemberQ[done, i], ζ = ζ // m_{i,i+1→i}; st = st /. st[[i + 2]] → st[[i + 1]];
    If[MemberQ[done, i - 1], ζ = ζ // m_{st[[i],i→st[[i]]}; st = st /. st[[i + 1]] → st[[i]];
    If[MemberQ[done, j], ζ = ζ // m_{j,j+1→j}; st = st /. st[[j + 2]] → st[[j + 1]];
    If[MemberQ[done, j - 1], ζ = ζ // m_{st[[j],j→st[[j]]}; st = st /. st[[j + 1]] → st[[j]];
    done = done ∪ {i - 1, i, j - 1, j};
    todo = DeleteCases[todo, x]
  ];
  ζ /. {V_0 → V, C_0 → C, W_0 → W}
]

```

Timing[Z[Knot[3, 1]]]

KnotTheory: Loading precomputed data in PD4Knots`.

$$\left\{ 5.48438, \mathbb{E} \left[ -1 + \frac{1}{t} + t, 0, 0, -16 - \frac{2}{t^4} + \frac{2c}{t^4} + \frac{7}{t^3} - \frac{6c}{t^3} - \frac{14}{t^2} + \frac{10c}{t^2} + \frac{18}{t} - \frac{8c}{t} + 10t + 8ct - 4t^2 - 10ct^2 + t^3 + 6ct^3 - 2ct^4 + 2vw - \frac{2vw}{t^4} + \frac{4vw}{t^3} - \frac{6vw}{t^2} + \frac{2vw}{t} - 6tvw + 4t^2vw - 2t^3vw \right] \right\}$$

Testing  $10_{100}$ ...

Timing[Z[Knot[10, 100]]]

$$\left\{ 185.016, \mathbb{E} \left[ 13 + \frac{1}{t^4} - \frac{4}{t^3} + \frac{9}{t^2} - \frac{12}{t} - 12t + 9t^2 - 4t^3 + t^4, 0, 0, \right. \right. \\
- 2563146 - \frac{6}{t^{16}} + \frac{8c}{t^{16}} + \frac{92}{t^{15}} - \frac{120c}{t^{15}} - \frac{723}{t^{14}} + \frac{924c}{t^{14}} + \frac{3818}{t^{13}} - \frac{4784c}{t^{13}} - \frac{15133}{t^{12}} + \frac{18588c}{t^{12}} + \frac{47848}{t^{11}} - \frac{57552c}{t^{11}} - \\
\frac{125539}{t^{10}} + \frac{147540c}{t^{10}} + \frac{281054}{t^9} - \frac{321552c}{t^9} - \frac{548129}{t^8} + \frac{606988c}{t^8} + \frac{945756}{t^7} - \frac{1004976c}{t^7} - \frac{1460263}{t^6} + \\
\frac{1469820c}{t^6} + \frac{2034106}{t^5} - \frac{1901560c}{t^5} - \frac{2570432}{t^4} + \frac{2163176c}{t^4} + \frac{2956518}{t^3} - \frac{2123520c}{t^3} - \frac{3099338}{t^2} + \\
\frac{1711728c}{t^2} + \frac{2958726}{t} - \frac{958272c}{t} + 2000454t + 958272ct - 1387610t^2 - 1711728ct^2 + 832998t^3 + \\
2123520ct^3 - 407256t^4 - 2163176ct^4 + 132546t^5 + 1901560ct^5 + 9557t^6 - 1469820ct^6 - 59220t^7 + \\
1004976ct^7 + 58859t^8 - 606988ct^8 - 40498t^9 + 321552ct^9 + 22001t^{10} - 147540ct^{10} - 9704t^{11} + \\
57552ct^{11} + 3455t^{12} - 18588ct^{12} - 966t^{13} + 4784ct^{13} + 201t^{14} - 924ct^{14} - 28t^{15} + 120ct^{15} + 2t^{16} - \\
8ct^{16} + 253564vw - \frac{8vw}{t^{16}} + \frac{112vw}{t^{15}} - \frac{812vw}{t^{14}} + \frac{3972vw}{t^{13}} - \frac{14616vw}{t^{12}} + \frac{42936vw}{t^{11}} - \frac{104604vw}{t^{10}} + \frac{216948vw}{t^9} - \\
\frac{390040vw}{t^8} + \frac{614936vw}{t^7} - \frac{854884vw}{t^6} + \frac{1046676vw}{t^5} - \frac{1116500vw}{t^4} + \frac{1007020vw}{t^3} - \frac{704708vw}{t^2} + \frac{253564vw}{t} - \\
704708tvw + 1007020t^2vw - 1116500t^3vw + 1046676t^4vw - 854884t^5vw + 614936t^6vw - 390040t^7vw + \\
216948t^8vw - 104604t^9vw + 42936t^{10}vw - 14616t^{11}vw + 3972t^{12}vw - 812t^{13}vw + 112t^{14}vw - 8t^{15}vw \left. \right\}$$

BeginProfile[];

Timing[Z[Knot[10, 100]]]

EndProfile[];

$$\left\{ 192.484, \mathbb{E} \left[ 13 + \frac{1}{t^4} - \frac{4}{t^3} + \frac{9}{t^2} - \frac{12}{t} - 12t + 9t^2 - 4t^3 + t^4, 0, 0, \right. \right. \\
- 2563146 - \frac{6}{t^{16}} + \frac{8c}{t^{16}} + \frac{92}{t^{15}} - \frac{120c}{t^{15}} - \frac{723}{t^{14}} + \frac{924c}{t^{14}} + \frac{3818}{t^{13}} - \frac{4784c}{t^{13}} - \frac{15133}{t^{12}} + \frac{18588c}{t^{12}} + \frac{47848}{t^{11}} - \frac{57552c}{t^{11}} - \\
\frac{125539}{t^{10}} + \frac{147540c}{t^{10}} + \frac{281054}{t^9} - \frac{321552c}{t^9} - \frac{548129}{t^8} + \frac{606988c}{t^8} + \frac{945756}{t^7} - \frac{1004976c}{t^7} - \frac{1460263}{t^6} + \\
\frac{1469820c}{t^6} + \frac{2034106}{t^5} - \frac{1901560c}{t^5} - \frac{2570432}{t^4} + \frac{2163176c}{t^4} + \frac{2956518}{t^3} - \frac{2123520c}{t^3} - \frac{3099338}{t^2} + \\
\frac{1711728c}{t^2} + \frac{2958726}{t} - \frac{958272c}{t} + 2000454t + 958272ct - 1387610t^2 - 1711728ct^2 + 832998t^3 + \\
2123520ct^3 - 407256t^4 - 2163176ct^4 + 132546t^5 + 1901560ct^5 + 9557t^6 - 1469820ct^6 - 59220t^7 + \\
1004976ct^7 + 58859t^8 - 606988ct^8 - 40498t^9 + 321552ct^9 + 22001t^{10} - 147540ct^{10} - 9704t^{11} + \\
57552ct^{11} + 3455t^{12} - 18588ct^{12} - 966t^{13} + 4784ct^{13} + 201t^{14} - 924ct^{14} - 28t^{15} + 120ct^{15} + 2t^{16} - \\
8ct^{16} + 253564vw - \frac{8vw}{t^{16}} + \frac{112vw}{t^{15}} - \frac{812vw}{t^{14}} + \frac{3972vw}{t^{13}} - \frac{14616vw}{t^{12}} + \frac{42936vw}{t^{11}} - \frac{104604vw}{t^{10}} + \frac{216948vw}{t^9} - \\
\frac{390040vw}{t^8} + \frac{614936vw}{t^7} - \frac{854884vw}{t^6} + \frac{1046676vw}{t^5} - \frac{1116500vw}{t^4} + \frac{1007020vw}{t^3} - \frac{704708vw}{t^2} + \frac{253564vw}{t} - \\
704708tvw + 1007020t^2vw - 1116500t^3vw + 1046676t^4vw - 854884t^5vw + 614936t^6vw - 390040t^7vw + \\
216948t^8vw - 104604t^9vw + 42936t^{10}vw - 14616t^{11}vw + 3972t^{12}vw - 812t^{13}vw + 112t^{14}vw - 8t^{15}vw \left. \right\}$$

PrintProfile[];

Cancel4P: called 2772 times, time in 142.432/142.432

Parents:

( 380) 53.937/ 53.937 under CF

( 2392) 88.495/ 88.495 under CF4P

CF4P: called 100 times, time in 33.741/122.236

Parents:

( 100) 33.741/ 122.236 under CF2

Children:

( 2392) 88.495/ 88.495 above Cancel4P

DP: called 300 times, time in 13.135/13.135

Parents:

( 200) 12.920/ 12.920 under Ncx  
 ( 100) 0.215/ 0.215 under Nwv

Ncx: called 200 times, time in 1.423/68.138

Parents:

( 200) 1.423/ 68.138 under m

Children:

( 200) 0.451/ 53.795 above CF  
 ( 200) 12.920/ 12.920 above DP

m: called 100 times, time in 0.877/192.235

Parents:

( 100) 0.877/ 192.235 under z

Children:

( 100) 0.079/ 0.485 above CF  
 ( 200) 1.423/ 68.138 above Ncx  
 ( 100) 0.189/ 122.735 above Nwv

CF: called 380 times, time in 0.577/54.514

Parents:

( 100) 0.079/ 0.485 under m  
 ( 200) 0.451/ 53.795 under Ncx  
 ( 80) 0.047/ 0.234 under z

Children:

( 380) 53.937/ 53.937 above Cancel4P

Nwv: called 100 times, time in 0.189/122.735

Parents:

( 100) 0.189/ 122.735 under m

Children:

( 100) 0.095/ 122.331 above CF2  
 ( 100) 0.215/ 0.215 above DP

CF2: called 100 times, time in 0.095/122.331

Parents:

( 100) 0.095/ 122.331 under Nwv

Children:

( 100) 33.741/ 122.236 above CF4P

```
z: called 1 times, time in 0.015/192.484
```

```
Parents:
```

```
( 1) 0.015/ 192.484 under ProfileRoot
```

```
Children:
```

```
( 80) 0.047/ 0.234 above CF
```

```
( 100) 0.877/ 192.235 above m
```

```
ProfileRoot: called 0 times, time in 0./0.
```

```
Children:
```

```
( 1) 0.015/ 192.484 above z
```

## Testing $T_{9,5}$ ...

```
BeginProfile[];  
Timing[Z[TorusKnot[9, 5]]]  
EndProfile[];  
PrintProfile[];
```