

Pensieve header: Cheap CF optimization for the NOE1 program (V2, still failed).

Initialization

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2016-12"];
Once[<< KnotTheory`];
Once[<< "../Projects/Profile/Profile.m"]
```

Loading KnotTheory` version of September 6, 2014, 13:37:37.2841.

Read more at <http://katlas.org/wiki/KnotTheory>.

This is Profile.m, Nov 2016 mods of July 1994 version

Rotational Virtual Knots

```
RVK::usage =
  "RVK[xs, rots] represents a Rotational Virtual Knot with a list of n Xp/Xm crossings xs and
  a length 2n list of rotation numbers rots. Crossing sites are indexed 1 through
  2n, and rots[[k]] is the rotation between site k-1 and site k. RVK is also a casting
  operator converting to the RVK presentation from other knot presentations.";
RVK[pd_PD] := Module[{n, xs, x, rots, front, k},
  n = Length[pd];
  xs = List@@pd /. x_X => If[PositiveQ[x], Xp[x[[4]], x[[1]], Xm[x[[2]], x[[1]]];
  rots = Table[0, {2 n};
  front = {0};
  For[k = 0, k < 2 n, ++k,
  If[k == 0 ∨ FreeQ[front, -k],
  front = Flatten[front /. k → Catch[xs /. {
    Xp[k + 1, L_] | Xm[L_, k + 1] => Throw[{L, k + 1, 1 - L]},
    Xp[L_, k + 1] | Xm[k + 1, L_] => (++rots[[L]]; Throw[{1 - L, k + 1, L})
  }]],
  If[MatchQ[front, {___, k, ___, -k, ___}], --rots[[k + 1]]
  ]
  ];
  RVK[xs, rots]
  ];
RVK[K_] := RVK[PD[K]];
```

NOE-It

Logos

$$\Delta[k_] := \left((t_k - 1) (2(\alpha\beta + \delta\mu)^2 - \alpha^2\beta^2) - 4v_k c_k w_k \delta^2 \mu^2 - \delta(1 + \mu)(w_k^2 \alpha^2 + v_k^2 \beta^2) - v_k^2 w_k^2 \delta^3 (1 + 3\mu) - 2(\alpha\beta + 2\delta\mu + v_k w_k \delta^2 (1 + 2\mu) + 2c_k \delta \mu^2)(w_k \alpha + v_k \beta) - 4(c_k \mu^2 + v_k w_k \delta(1 + \mu))(\alpha\beta + \delta\mu)(1 + t_k) \right) / 4;$$

1Gens

$$\begin{aligned} R_{i,j}^+ &:= \mathbb{E}\left[1, \text{Log}[t_i] c_j, v_i w_j, v_i c_i w_j + c_i c_j + v_i^2 w_j^2 / 4\right]; \\ R_{i,j}^- &:= \mathbb{E}\left[1, -\text{Log}[t_i] c_j, -t_i^{-1} v_i w_j, -c_i c_j + t_i^{-1} v_i c_j w_j - t_i^{-2} v_i^2 w_j^2 / 4\right]; \\ (ur_{i-} &:= \mathbb{E}[t_i^{-1/2}, 0, 0, c_i t_i^{-2}]; nr_{i-} := \mathbb{E}[t_i^{1/2}, 0, 0, -c_i t_i^2];) \end{aligned}$$

1DP

$$\begin{aligned} DP_{x \rightarrow d_\alpha, y \rightarrow d_\beta}[P_][f_] &:= (* \text{ means } P[\partial_\alpha, \partial_\beta][f] *) \\ PP_{DP} @ \text{Total}[\text{CoefficientRules}[P, \{x, y\}] /. (\{m_, n_\} \rightarrow c_) => c D[f, \{\alpha, m\}, \{\beta, n\}]] \end{aligned}$$

1Util

```

CF[E[ω_, L_, Q_, P_]] :=
  PPCF@E[Expand@Together@ω, Expand@Together@L, Expand@Together@Q, Expand@PPTogether4P@Together@P];
CF2[ω_, E[ω1_, L_, Q_, P_]] := PPCF2@E[Expand@Together@(ω1 / ω → ωθ),
  Expand@Together@(L / ω → ωθ), Expand@Together@(Q / ω → ωθ), PPCF4P@Module[{vars},
  vars = Union@Cases[P, (c | v | w)_, ∞];
  Total[CoefficientRules[P, vars] /.
  (p_ → cc_) ⇒ (Expand@PPTogether4P@Together@(cc / ω → ωθ)) (Times@@(vars^p))]]];

```

1Util

```

E /: E[ω1_, L1_, Q1_, P1_] E[ω2_, L2_, Q2_, P2_] := CF@E[ω1 ω2, L1 + L2, ω2 Q1 + ω1 Q2, ω2^4 P1 + ω1^4 P2];

```

1NOuw

```

Nw_i v_j → k_ [E[ωθ_, L_, Q_, P_]] := PPNwv@With[{q = ((1 - t_k) α β + β v_k + δ v_k w_k + α w_k) / μ},
  CF2[ωθ,
  E[μ ω, L, μ ω q + μ (Q / w_i | v_j → θ), μ^4 (DPw_i → D_α, v_j → D_β [P][e^q] / e → 1) + ω^4 Δ[k] / μ → 1 + (t_k - 1) δ /
  {α → ω^-1 (∂w_i Q / v_j → θ), β → ω^-1 (∂v_j Q / w_i → θ), δ → ω^-1 ∂w_i v_j Q}]]];

```

1NOc

```

Nc_j (x:v|w)_i → k_ [E[ω_, L_, Q_, P_]] := PPNcx@With[{q = e^γ β x_k + γ c_k}, CF[
  E[ω, γ c_k + (L / c_j → θ), ω e^γ β x_k + (Q / x_i → θ), e^-q DPc_j → D_γ, x_i → D_β [P][e^q]] / {γ → ∂c_j L, β → ω^-1 ∂x_i Q}]]];

```

1m

```

mi_,j → k_ [Z_E] := PPm@Module[{x, z},
  CF[(Z // Nw_i v_j → x // Nc_i v_x → x // Nw_x c_j → x) / z_{-i|j|x} → z_k]]];

```

Z

```

ul_ = n1_ = rot[_ , 0] = E[1, 0, 0, 0];
rot[i_, 1] := ur_i;
rot[i_, n_Integer] /; n > 1 := Module[{y}, rot[i, n - 1] rot[y, 1] // m_{i,y → i}];
rot[i_, -1] := nr_i;
rot[i_, n_Integer] /; n < -1 := Module[{y}, rot[i, n + 1] rot[y, -1] // m_{i,y → i}];

```

```

t_ = t;
Z[K_] := Z[RVK@K];
Z[rvk_RVK] := PPz@Module[{todo, n, rots, ζ, done, st, x, ζ1, i, j, k, k1, k2, k3},
  {todo, rots} = List@@rvk;
  AppendTo[rots, 0];
  n = Length[todo];
  ζ = E[1, 0, 0, 0];
  done = {0};
  st = Range[0, 2 n + 1];
  While[todo != {},
    {x} = MaximalBy[todo, Length[done ∩ {#[[1]], #[[2]], #[[1]] - 1, #[[2]] - 1}] &, 1];
    Z$todo = todo; Z$x = x;
    {i, j} = List@@x;
    ζ1 = Switch[Head[x],
      Xp, m_{j,k→j} [R_{i,j}^+ (R_{k3,k}^- nr_{k1} ul_{k2} // m_{k,k1→k} // m_{k,k2→k} // m_{k,k3→k})],
      Xm, m_{j,k→j} [R_{i,j}^- (R_{k,k3}^+ nr_{k1} ul_{k2} // m_{k,k1→k} // m_{k,k2→k} // m_{k,k3→k})]
    ];
    ζ1 = rot[k, rots[[i]] ζ1 // m_{k,i→i}; rots[[i]] = 0;
    ζ1 = ζ1 rot[k, rots[[i + 1]] // m_{i,k→i}; rots[[i + 1]] = 0;
    ζ1 = rot[k, rots[[j]] ζ1 // m_{k,j→j}; rots[[j]] = 0;
    ζ1 = ζ1 rot[k, rots[[j + 1]] // m_{j,k→j}; rots[[j + 1]] = 0;
    ζ *= ζ1;
    If[MemberQ[done, i], ζ = ζ // m_{i,i+1→i}; st = st /. st[[i + 2]] → st[[i + 1]];
    If[MemberQ[done, i - 1], ζ = ζ // m_{st[[i],i→st[[i]]}; st = st /. st[[i + 1]] → st[[i]];
    If[MemberQ[done, j], ζ = ζ // m_{j,j+1→j}; st = st /. st[[j + 2]] → st[[j + 1]];
    If[MemberQ[done, j - 1], ζ = ζ // m_{st[[j],j→st[[j]]}; st = st /. st[[j + 1]] → st[[j]];
    done = done ∪ {i - 1, i, j - 1, j};
    todo = DeleteCases[todo, x]
  ];
  ζ /. {V_0 → V, C_0 → C, W_0 → W}
]

```

Timing[Z[Knot[3, 1]]]

KnotTheory: Loading precomputed data in PD4Knots`.

$$\left\{ 3.07813, \mathbb{E} \left[-1 + \frac{1}{t} + t, 0, 0, -16 - \frac{2}{t^4} + \frac{2c}{t^4} + \frac{7}{t^3} - \frac{6c}{t^3} - \frac{14}{t^2} + \frac{10c}{t^2} + \frac{18}{t} - \frac{8c}{t} + 10t + 8ct - 4t^2 - 10ct^2 + t^3 + 6ct^3 - 2ct^4 + 2vw - \frac{2vw}{t^4} + \frac{4vw}{t^3} - \frac{6vw}{t^2} + \frac{2vw}{t} - 6tvw + 4t^2vw - 2t^3vw \right] \right\}$$

Testing 10_{100} ...

Timing[Z[Knot[10, 100]]]

$$\left\{ 98.7344, \mathbb{E} \left[13 + \frac{1}{t^4} - \frac{4}{t^3} + \frac{9}{t^2} - \frac{12}{t} - 12t + 9t^2 - 4t^3 + t^4, 0, 0, \right. \right. \\
- 2563146 - \frac{6}{t^{16}} + \frac{8c}{t^{16}} + \frac{92}{t^{15}} - \frac{120c}{t^{15}} - \frac{723}{t^{14}} + \frac{924c}{t^{14}} + \frac{3818}{t^{13}} - \frac{4784c}{t^{13}} - \frac{15133}{t^{12}} + \frac{18588c}{t^{12}} + \frac{47848}{t^{11}} - \frac{57552c}{t^{11}} - \\
\frac{125539}{t^{10}} + \frac{147540c}{t^{10}} + \frac{281054}{t^9} - \frac{321552c}{t^9} - \frac{548129}{t^8} + \frac{606988c}{t^8} + \frac{945756}{t^7} - \frac{1004976c}{t^7} - \frac{1460263}{t^6} + \\
\frac{1469820c}{t^6} + \frac{2034106}{t^5} - \frac{1901560c}{t^5} - \frac{2570432}{t^4} + \frac{2163176c}{t^4} + \frac{2956518}{t^3} - \frac{2123520c}{t^3} - \frac{3099338}{t^2} + \\
\frac{1711728c}{t^2} + \frac{2958726}{t} - \frac{958272c}{t} + 2000454t + 958272ct - 1387610t^2 - 1711728ct^2 + 832998t^3 + \\
2123520ct^3 - 407256t^4 - 2163176ct^4 + 132546t^5 + 1901560ct^5 + 9557t^6 - 1469820ct^6 - 59220t^7 + \\
1004976ct^7 + 58859t^8 - 606988ct^8 - 40498t^9 + 321552ct^9 + 22001t^{10} - 147540ct^{10} - 9704t^{11} + \\
57552ct^{11} + 3455t^{12} - 18588ct^{12} - 966t^{13} + 4784ct^{13} + 201t^{14} - 924ct^{14} - 28t^{15} + 120ct^{15} + 2t^{16} - \\
8ct^{16} + 253564vw - \frac{8vw}{t^{16}} + \frac{112vw}{t^{15}} - \frac{812vw}{t^{14}} + \frac{3972vw}{t^{13}} - \frac{14616vw}{t^{12}} + \frac{42936vw}{t^{11}} - \frac{104604vw}{t^{10}} + \frac{216948vw}{t^9} - \\
\frac{390040vw}{t^8} + \frac{614936vw}{t^7} - \frac{854884vw}{t^6} + \frac{1046676vw}{t^5} - \frac{1116500vw}{t^4} + \frac{1007020vw}{t^3} - \frac{704708vw}{t^2} + \frac{253564vw}{t} - \\
704708tvw + 1007020t^2vw - 1116500t^3vw + 1046676t^4vw - 854884t^5vw + 614936t^6vw - 390040t^7vw + \\
216948t^8vw - 104604t^9vw + 42936t^{10}vw - 14616t^{11}vw + 3972t^{12}vw - 812t^{13}vw + 112t^{14}vw - 8t^{15}vw \left. \right\}$$

BeginProfile[];

Timing[Z[Knot[10, 100]]]

EndProfile[];

$$\left\{ 99.4375, \mathbb{E} \left[13 + \frac{1}{t^4} - \frac{4}{t^3} + \frac{9}{t^2} - \frac{12}{t} - 12t + 9t^2 - 4t^3 + t^4, 0, 0, \right. \right. \\
- 2563146 - \frac{6}{t^{16}} + \frac{8c}{t^{16}} + \frac{92}{t^{15}} - \frac{120c}{t^{15}} - \frac{723}{t^{14}} + \frac{924c}{t^{14}} + \frac{3818}{t^{13}} - \frac{4784c}{t^{13}} - \frac{15133}{t^{12}} + \frac{18588c}{t^{12}} + \frac{47848}{t^{11}} - \frac{57552c}{t^{11}} - \\
\frac{125539}{t^{10}} + \frac{147540c}{t^{10}} + \frac{281054}{t^9} - \frac{321552c}{t^9} - \frac{548129}{t^8} + \frac{606988c}{t^8} + \frac{945756}{t^7} - \frac{1004976c}{t^7} - \frac{1460263}{t^6} + \\
\frac{1469820c}{t^6} + \frac{2034106}{t^5} - \frac{1901560c}{t^5} - \frac{2570432}{t^4} + \frac{2163176c}{t^4} + \frac{2956518}{t^3} - \frac{2123520c}{t^3} - \frac{3099338}{t^2} + \\
\frac{1711728c}{t^2} + \frac{2958726}{t} - \frac{958272c}{t} + 2000454t + 958272ct - 1387610t^2 - 1711728ct^2 + 832998t^3 + \\
2123520ct^3 - 407256t^4 - 2163176ct^4 + 132546t^5 + 1901560ct^5 + 9557t^6 - 1469820ct^6 - 59220t^7 + \\
1004976ct^7 + 58859t^8 - 606988ct^8 - 40498t^9 + 321552ct^9 + 22001t^{10} - 147540ct^{10} - 9704t^{11} + \\
57552ct^{11} + 3455t^{12} - 18588ct^{12} - 966t^{13} + 4784ct^{13} + 201t^{14} - 924ct^{14} - 28t^{15} + 120ct^{15} + 2t^{16} - \\
8ct^{16} + 253564vw - \frac{8vw}{t^{16}} + \frac{112vw}{t^{15}} - \frac{812vw}{t^{14}} + \frac{3972vw}{t^{13}} - \frac{14616vw}{t^{12}} + \frac{42936vw}{t^{11}} - \frac{104604vw}{t^{10}} + \frac{216948vw}{t^9} - \\
\frac{390040vw}{t^8} + \frac{614936vw}{t^7} - \frac{854884vw}{t^6} + \frac{1046676vw}{t^5} - \frac{1116500vw}{t^4} + \frac{1007020vw}{t^3} - \frac{704708vw}{t^2} + \frac{253564vw}{t} - \\
704708tvw + 1007020t^2vw - 1116500t^3vw + 1046676t^4vw - 854884t^5vw + 614936t^6vw - 390040t^7vw + \\
216948t^8vw - 104604t^9vw + 42936t^{10}vw - 14616t^{11}vw + 3972t^{12}vw - 812t^{13}vw + 112t^{14}vw - 8t^{15}vw \left. \right\}$$

PrintProfile[];

Together4P: called 2772 times, time in 65.435/65.435

Parents:

(380) 4.939/ 4.939 under CF

(2392) 60.496/ 60.496 under CF4P

CF4P: called 100 times, time in 31.817/92.313

Parents:

(100) 31.817/ 92.313 under CF2

Children:

(2392) 60.496/ 60.496 above Together4P

DP: called 300 times, time in 1.269/1.269

Parents:

(200) 1.017/ 1.017 under Ncx
 (100) 0.252/ 0.252 under Nwv

CF: called 380 times, time in 0.405/5.344

Parents:

(100) 0.092/ 0.826 under m
 (200) 0.282/ 4.251 under Ncx
 (80) 0.031/ 0.267 under z

Children:

(380) 4.939/ 4.939 above Together4P

m: called 100 times, time in 0.154/99.17

Parents:

(100) 0.154/ 99.170 under z

Children:

(100) 0.092/ 0.826 above CF
 (200) 0.138/ 5.406 above Ncx
 (100) 0.125/ 92.784 above Nwv

Ncx: called 200 times, time in 0.138/5.406

Parents:

(200) 0.138/ 5.406 under m

Children:

(200) 0.282/ 4.251 above CF
 (200) 1.017/ 1.017 above DP

Nwv: called 100 times, time in 0.125/92.784

Parents:

(100) 0.125/ 92.784 under m

Children:

(100) 0.094/ 92.407 above CF2
 (100) 0.252/ 0.252 above DP

CF2: called 100 times, time in 0.094/92.407

Parents:

(100) 0.094/ 92.407 under Nwv

Children:

(100) 31.817/ 92.313 above CF4P

```
z: called 1 times, time in 0./99.437
```

```
Parents:
```

```
( 1) 0.000/ 99.437 under ProfileRoot
```

```
Children:
```

```
( 80) 0.031/ 0.267 above CF
```

```
( 100) 0.154/ 99.170 above m
```

```
ProfileRoot: called 0 times, time in 0./0.
```

```
Children:
```

```
( 1) 0.000/ 99.437 above z
```

Testing $T_{9,5}$...

```
BeginProfile[];  
Timing[Z[TorusKnot[9, 5]]]  
EndProfile[];  
PrintProfile[];
```