Sottile@Colloq: Galois groups in Enumerative Geometry and Applications

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1870: on a smooth cubic $F=0$ in $\mathbb{CP}^3$ there are 27 lines, in a remarkable configuration.

If $F/\mathbb{Q}$, the lines are defined over some extension K of $\mathbb{Q}$. $\mathrm{Gal}(K/\mathbb{Q})$ acts on the lines preserving the config, so $\mathrm{Gal} \subset E_6 =$ symm. of config.

$$E_6 \subset S_{27}$$

Modern view

$\mathbb{P}^{19}$: space of cubics F $\quad$ $\mathbb{G}$: lines in $\mathbb{P}^3$

$$\mathcal{L} \subset \{(F,\ell) \in \mathbb{P}^{19} \times \mathbb{G} : F(\ell) = 0\}$$

$$\mathcal{L} \xrightarrow{\pi} \mathbb{P}^{19} \qquad |\pi^{-1}(F)| = 27$$

Galois = monodromy