Getzler: Introduction to the BV formalism, with examples including the spinning particle

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What's the purpose? Why are we doing this? As always, BV is ad-hoc.

Felder & Kazhdan recent paper:

V an algebraic variety $\subset \mathbb{C}^n$

Tate, 1957: dg-scheme, all singularity in the differential.

Take $I = I(V)$, find generators $\{f_1 \dots f_n\}$ in $\mathbb{C}[x_1 \dots x_n]$. Introduce y_i s.t. $\delta y_i = f_i$, consider $\mathbb{C}[x_i]$ tensored w/ the free graded commutative algebra w/ the y_i in degree (-1) [that is, it an exterior alg]

Now

$$H^0(\mathbb{C}[x_i, y_i]) \cong I(V)$$

and H^{-1} is f.d.. Introduce generators in degree -2 to kill H^{-1}, and keep going.

<u>Thm</u> Any two such resolutions are quasi-iso.

BV: suppose that V is the critical locus of a function f; $I(V) = \frac{\partial f}{\partial x_i}$

Introduce a dual basis x_i^+ in deg (-1) w/

$$\delta x_i^+ = \frac{\partial f}{\partial x_i}$$

Introduce a sequence of anti-fields in degrees < -1 to kill cohomology in neg. deg.. Simult. introduce their canonical conj. fields w/

$$\deg(y_i) + \deg(y_i^+) = -1$$

-2	-1	0	1
anti-fields of ghosts	anti fields	field	ghosts

$$\Omega = \sum_i d\phi_i \wedge d\phi_i^+ \qquad \phi_i \in \{x_i, y_i\}$$

$\Rightarrow$ there's a Poisson bracket of deg $+1$.

BV call it "the anti-bracket": $(-,-)$

Let $F = F_0$ & find $F = f_0 + f_1 + \ldots$

s.t. f_k is deg k in anti-fields and $(F,F)=0$

and $\delta = (F, -)$.

<u>Free Particle</u> $x^\mu(t)$ in Minkowski space

$\eta_{\mu\nu}$: The Lorentz metric

Introduce $p_\mu(t)$, and action

$$S = \int \left(p_\mu \partial x^\mu - \frac{1}{2}\eta^{\mu\nu} p_\mu p_\nu \right) dt$$

$$\delta x_\mu^+ = (S, x_\mu^+) = -\partial p_\mu$$

$$\delta p^{+\mu} = (S, p^{+\mu}) = \partial x^\mu - \eta^{\mu\nu} p_\nu$$

Introduce $e(t)$, a frame of the cotangent bundle of the world line

$$S \longmapsto \int \left(p_\mu \partial x^\mu dt - \frac{1}{2}\eta^{\mu\nu} e\, p_\mu p_\nu \right)$$

$$\delta c^+ = (S, c^+) = -\frac{1}{2}\eta^{\mu\nu} p_\mu p_\nu$$

The BV crowd simply does not want to be understood.