Figuring out NOE

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Differential Polynomials

$$\text{DP}_{x\_\to D_{\alpha\_},\, y\_\to D_{\beta\_}}[P\_][f\_] := \text{(* means } P[\partial_\alpha,\partial_\beta][f] \text{ *)}$$
$$\quad \text{Total}[\text{CoefficientRules}[P, \{x, y\}] \;/.$$
$$\qquad (\{m\_, n\_\} \to c\_) :\!\to c\, \text{D}[f, \{\alpha, m\}, \{\beta, n\}]]$$

Utilities

$$\text{CF}[\mathcal{E}\_\mathbb{E}] := \text{Expand} /@ \text{Together} /@ \mathcal{E};$$
$$\mathbb{E} /: \mathbb{E}[\omega1\_, L1\_, Q1\_, P1\_]\, \mathbb{E}[\omega2\_, L2\_, Q2\_, P2\_] :=$$
$$\quad \text{CF}@\mathbb{E}\left[\omega1\,\omega2,\, L1 + L2,\, \omega2\, Q1 + \omega1\, Q2,\, \omega2^4\, P1 + \omega1^4\, P2\right];$$

Normal Ordering Operators

$$N_{c_{j\_}\,(x:v|w)_{i\_}\to k\_}[\mathbb{E}[\omega\_, L\_, Q\_, P\_]] := \text{With}\left[\{q = e^{\gamma}\beta x_k + \gamma c_k\},\, \text{CF}\right[$$
$$\quad \mathbb{E}\left[\omega,\, \gamma c_k + (L /. c_j \to 0),\, \omega e^{\gamma}\beta x_k + (Q /. x_i \to 0),\right.$$
$$\qquad \left. e^{-q}\, \text{DP}_{c_j\to D_\gamma,\, x_i\to D_\beta}[P][e^q]\right] /. \{\gamma \to \partial_{c_j} L,\, \beta \to \omega^{-1}\partial_{x_i} Q\}]];$$
$$N_{w_{i\_}\, v_{j\_}\to k\_}[\mathbb{E}[\omega\_, L\_, Q\_, P\_]] :=$$
$$\quad \text{With}\left[\{q = ((1 - t_k)\alpha\beta + \beta v_k + \delta v_k w_k + \alpha w_k)\, / \mu\},\, \text{CF}\right[\quad \text{D}$$
$$\qquad \mathbb{E}\left[\mu\,\omega,\, \text{B},\, \mu\,\omega q + \mu\,(Q /. w_i \,|\, v_j \to 0),\right. \quad \text{C}$$
$$\text{A} \qquad \boxed{\mu^4\, \boxed{e^{-q}}\, \text{DP}_{w_i\to D_\alpha,\, v_j\to D_\beta}[P][e^q]} + \boxed{\omega^4\, \Lambda[k]}\left.\right] /. \mu \to 1 + (t_k - 1)\,\delta \;/.$$
$$\qquad \{\alpha \to \omega^{-1}(\partial_{w_i} Q /. v_j \to 0),\, \beta \to \omega^{-1}(\partial_{v_j} Q /. w_i \to 0),$$
$$\qquad\quad \delta \to \omega^{-1}\partial_{w_i, v_j} Q\}]];$$

Stitching

$$m_{i\_, j\_\to k\_}[Z\_] := \text{Module}\left[\{x, z\},\, \text{CF}\right[$$
$$\quad Z \,//\, N_{w_i\, v_j\to x} \,//\, N_{c_i\, v_x\to x} \,//\, N_{w_x\, c_j\to x} \,//\, \text{ReplaceAll}[z\_{i|j|x} \to z_k]]]$$

2016-11/TracingNOE1@vcb.nb discoveries:

In $\mathbb{E}(w, L, Q, P)$, the coeff of $(vw)^k c^l$ is divisible by w^{2+l-k}, and this is true for the Λ terms & the DP terms independently.

P may contain: $1, c, cc, vw, cvw, vwvw$.

D: In conjunction with $\mu = 1 + (t_k - 1)\delta$ & $\delta = w^{-1} Q^{w_i v_j}$, leads to complicated denominators.

A: An Expand here may cancel many μ's. B: Sep optimization trick applies.

$$\Lambda[k\_] :=$$
$$\left((t_k - 1)\left(2(\alpha\beta + \delta\mu)^2 - \overset{4}{\alpha^2\beta^2}\right) - 4 v_k c_k w_k \overset{4}{\delta^2}\mu^2 -\right.$$
$$\quad \delta(\overset{3}{1} + \overset{4}{\mu})\left(w_k^2\alpha^2 + v_k^2\beta^2\right) - v_k^2 w_k^2 \delta^3 (\overset{3}{1} + 3\overset{4}{\mu}) -$$
$$\quad 2\left(\alpha\overset{3}{\beta} + 2\delta\overset{3}{\mu} + v_k w_k \delta^2 (\overset{3}{1} + 2\overset{4}{\mu}) + 2 c_k \overset{4}{\delta}\mu^2\right)(w_k\alpha + v_k\beta) -$$
$$\quad \left. 4\left(c_k \underset{4}{\mu^2} + v_k w_k \delta (\underset{3}{1} + \underset{4}{\mu})\right)(\alpha\beta + \delta\mu)\right)(1 + t_k)/4;$$

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Green: $\alpha, \beta, \delta, \mu$ degree.

C: Bring the ω into Λ and get rid of all divisions.

$$\mathbb{O}\left(\epsilon P(u, w) e^{\alpha w + \beta u + \delta u w} | wu\right) = \mathbb{O}\left(\epsilon P(\partial_\beta, \partial_\alpha) v e^{\nu(-b\alpha\beta + \alpha w + \beta u + \delta u w)} | cuw\right)$$

perhaps the key is "our Z's always represent an automorphism of $\mathfrak{g}_1$; that constrains the relationship between w, L, Q, and P." That might even be the key to the polynomiality of Q. — Probably junk

Is "gain one w" a general property or is it only for our specific P's?