Comparing g_0 and qg_0

November 17, 2016 12:29 PM

All with $t = e^b$, $\zeta = \frac{t-1}{b}$

$$\mathfrak{g}_0 \xrightarrow{\rho} q\mathfrak{g}_0$$

$\mathfrak{g}_0$	$q\mathfrak{g}_0$
$[c,u]=u$, $[c,w]=-w$	$[c,u]=u$ $[c,w]=-w$
$[u,w]=b$	$[u,w]=t-1$
b,c,u,w are primitive	b,c are primitive
$\Delta'_{12}u = t_2u_1+u_2$	$\Delta_{12}(u) = t_2u_1+u_2$
$\Delta'_{12}w = t_2^{-1}w_1+w_2$	$\Delta_{12}(w) = w_1+w_2$

In $q\mathfrak{g}_0$: $\Delta(t-1) = t_1t_2-1$

Aside: $(\Delta\otimes 1)(\Delta u) = (\Delta\otimes 1)(t_2u_1+u_2) = t_3(t_2u_1+u_2)+u_3$

$(1\otimes\Delta)\Delta u = (1\otimes\Delta)(t_2u_1+u_2) = t_2t_3u_1+t_3u_2+u_3$ ✓

$$[\Delta u,\Delta w] = [t_2u_1+u_2, w_1+w_2] = t_2(t_1-1)+t_2-1 = t_1t_2-1$$

The map $\rho: \mathfrak{g}_0 \to q\mathfrak{g}_0$ fixes everything except $\rho(u) = \zeta^{-1}u$:

$$\rho[u,w] = b \qquad [\rho u,\rho w] = [\zeta^{-1}u, w] = \left(\frac{t-1}{b}\right)^{-1}(t-1) = b$$

$$u//\rho^{-1}//\Delta//\rho\otimes\rho = \zeta u//\Delta//\rho\otimes\rho = \frac{t_1t_2-1}{b_1+b_2}(u_1+u_2)//\rho\otimes\rho$$

$$= \frac{t_1t_2-1}{b_1+b_2}\left(\frac{b_1}{t_1-1}u_1 + \frac{b_2}{t_2-1}u_2\right)$$

$$u//\rho^{-1}//\Delta'//\rho\otimes\rho = \zeta u//\Delta'//\rho\otimes\rho = \frac{t_1t_2-1}{b_1+b_2}(t_2u_1+u_2)//\rho\otimes\rho$$

$$= \frac{t_1t_2-1}{b_1+b_2}\left(\frac{t_2b_1}{t_1-1}u_1 + \frac{b_2}{t_2-1}u_2\right)$$

$$u//\rho//\Delta//\rho^{-1}\otimes\rho^{-1} = \zeta^{-1}u//\Delta//\rho^{-1}\otimes\rho^{-1} = \frac{b_1+b_2}{t_1t_2-1}(t_2u_1+u_2)//\rho^{-1}\otimes\rho^{-1}$$

$$= \frac{b_1+b_2}{t_1t_2-1}\left(\frac{t_2(t_1-1)}{b_1}u_1 + \frac{t_2-1}{b_2}u_2\right)$$

Q. What are all the co-associative coproducts on $\mathcal{U}(b, u)$?