Pensieve header: Implementing $\mathbb O$.

```
Evaluate[Deg /@ {b, c, u, w, ϵ}] = {1, 0, 1, 0, 1}
```

```
{1, 0, 1, 0, 1}
```

```
Deg[b]
```

```
1
```

```
Clear[x];
CoefficientRules[Normal[Series[Sin[x], {x, 0, 5}]], {x}]
```

$$\left\{\{5\} \to \frac{1}{120}, \{3\} \to -\frac{1}{6}, \{1\} \to 1\right\}$$

```
𝕆[n_, poly_, specs___] := Module[{vs, us, h},
  vs = Join @@ (First /@ {specs});
  us = Join @@ ({specs} /. (l_ → s_) :> (l /. x_i_ :> x_s));
  Total[
   CoefficientRules[
     Normal@Series[poly /. b_i_ :> h b_i /. Thread[vs → h^(Deg/@vs) vs], {h, 0, n}] /. h → 1, vs
    ] /. (p_ → c_) :> c UU @@ (us^p)
  ]
 ]
```

$$\left\{n = 3,\ poly = \text{Exp}\left[b_1 c_2 + \frac{\text{e}^{b_1} - 1}{b_1} u_1 w_2\right],\ specs = \text{Sequence}[\{u_1\} \to 1, \{c_2, w_2\} \to 3]\right\}$$

$$\left\{3, \text{e}^{b_1 c_2 + \frac{\left(-1+\text{e}^{b_1}\right) u_1 w_2}{b_1}}, \{u_1\} \to 1, \{c_2, w_2\} \to 3\right\}$$

```
vs = Join @@ (First /@ {specs})
```

$$\{u_1, c_2, w_2\}$$

```
us = Join @@ ({specs} /. (l_ → s_) :> (l /. x_i_ :> x_s))
```

$$\{u_1, c_3, w_3\}$$

```
Thread[vs → h^(Deg/@vs) vs]
```

$$\{u_1 \to h\, u_1,\ c_2 \to c_2,\ w_2 \to w_2\}$$

```
CoefficientRules[
  Normal@Series[poly /. Thread[vs → h^(Deg/@vs) vs], {h, 0, n}] /. h → 1, vs
 ] /. (p_ → c_) :> c UU @@ (us^p)
```

$$\left\{\left(\text{e}^{b_1 c_2} + \frac{\text{e}^{b_1 c_2}\left(-1+\text{e}^{b_1}\right) u_1 w_2}{b_1} + \frac{\text{e}^{b_1 c_2}\left(-1+\text{e}^{b_1}\right)^2 u_1^2 w_2^2}{2 b_1^2} + \frac{\text{e}^{b_1 c_2}\left(-1+\text{e}^{b_1}\right)^3 u_1^3 w_2^3}{6 b_1^3}\right) U[\,]\right\}$$

$$\mathcal{O}\left[5,\ \text{Exp}\left[b_1 c_2 + \frac{e^{b_1}-1}{b_1} u_1 w_2\right],\ \{u_1\}\to 1,\ \{c_2, w_2\}\to 3\right]$$

$$U[] + b_1 U[c_3] + \frac{1}{2} b_1^2 U[c_3, c_3] + \left(1 + \frac{b_1}{2} + \frac{b_1^2}{6} + \frac{b_1^3}{24} + \frac{b_1^4}{120}\right) U[u_1, w_3] +$$
$$\frac{1}{6} b_1^3 U[c_3, c_3, c_3] + \left(b_1 + \frac{b_1^2}{2} + \frac{b_1^3}{6} + \frac{b_1^4}{24}\right) U[u_1, c_3, w_3] + \frac{1}{24} b_1^4 U[c_3, c_3, c_3, c_3] +$$
$$\left(\frac{b_1^2}{2} + \frac{b_1^3}{4} + \frac{b_1^4}{12}\right) U[u_1, c_3, c_3, w_3] + \left(\frac{1}{2} + \frac{b_1}{2} + \frac{7 b_1^2}{24} + \frac{b_1^3}{8}\right) U[u_1, u_1, w_3, w_3] +$$
$$\frac{1}{120} b_1^5 U[c_3, c_3, c_3, c_3, c_3] + \left(\frac{b_1^3}{6} + \frac{b_1^4}{12}\right) U[u_1, c_3, c_3, c_3, w_3] +$$
$$\left(\frac{b_1}{2} + \frac{b_1^2}{2} + \frac{7 b_1^3}{24}\right) U[u_1, u_1, c_3, w_3, w_3] + \frac{1}{24} b_1^4 U[u_1, c_3, c_3, c_3, c_3, w_3] +$$
$$\left(\frac{b_1^2}{4} + \frac{b_1^3}{4}\right) U[u_1, u_1, c_3, c_3, w_3, w_3] + \left(\frac{1}{6} + \frac{b_1}{4} + \frac{5 b_1^2}{24}\right) U[u_1, u_1, u_1, w_3, w_3, w_3] +$$
$$\frac{1}{12} b_1^3 U[u_1, u_1, c_3, c_3, c_3, w_3, w_3] + \left(\frac{b_1}{6} + \frac{b_1^2}{4}\right) U[u_1, u_1, u_1, c_3, w_3, w_3, w_3] +$$
$$\frac{1}{12} b_1^2 U[u_1, u_1, u_1, c_3, c_3, w_3, w_3, w_3] + \left(\frac{1}{24} + \frac{b_1}{12}\right) U[u_1, u_1, u_1, u_1, w_3, w_3, w_3, w_3] +$$
$$\frac{1}{24} b_1 U[u_1, u_1, u_1, u_1, c_3, w_3, w_3, w_3, w_3] + \frac{1}{120} U[u_1, u_1, u_1, u_1, u_1, w_3, w_3, w_3, w_3, w_3]$$