

Pensieve header: Tracing NOE-1 using vcw conventions

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SetDirectory["C:\\drorbn\\AcademicPensieve\\2016-11"];
Once[<< KnotTheory`]
```

Loading KnotTheory` version of September 6, 2014, 13:37:37.2841.

Read more at <http://katlas.org/wiki/KnotTheory>.

NOE-I Programs

$$\Delta[k_-] := -\frac{\lambda(1+t_k)}{4} \left(4 v_k c_k w_k \delta^2 \mu^2 + \delta(1+\mu) (w_k^2 \alpha^2 + v_k^2 \beta^2) + v_k^2 w_k^2 \delta^3 (1+3\mu) + (1-t_k) (2(\alpha\beta + \delta\mu)^2 - \alpha^2 \beta^2) + 2(\alpha\beta + 2\delta\mu + v_k w_k \delta^2 (1+2\mu) + 2c_k \delta \mu^2) (w_k \alpha + v_k \beta) + 4(c_k \mu^2 + v_k w_k \delta(1+\mu)) (\alpha\beta + \delta\mu) \right);$$

```
DPx→Dα, y→Dβ[P_-][f_-] := (* means P[∂α, ∂β][f] *)
Total[CoefficientRules[P, {x, y}] /. ({m_-, n_} → c_-) ⇒ c D[f, {α, m}, {β, n}]]
```

```
CF[ $\mathbb{E}[\omega_-, L_-, Q_-, P_-]$ ] := Expand /@ Together /@
 $\mathbb{E}[\omega / . b_l \Rightarrow \text{Log}[t_l], L, Q / . b_l \Rightarrow \text{Log}[t_l], P / . b_l \Rightarrow \text{Log}[t_l]]$ ;
 $\mathbb{E} / : \mathbb{E}[\omega 1_-, L 1_-, Q 1_-, P 1_-] \mathbb{E}[\omega 2_-, L 2_-, Q 2_-, P 2_-] := \text{CF} @ \mathbb{E}[\omega 1 \omega 2, L 1 + L 2, \omega 2 Q 1 + \omega 1 Q 2, \omega 2^4 P 1 + \omega 1^4 P 2]$ ;
 $\mathbb{E} / : \mathbb{E}[\omega 1_-, L 1_-, Q 1_-, P 1_-] \equiv \mathbb{E}[\omega 2_-, L 2_-, Q 2_-, P 2_-] := (\omega 1 = \omega 2 \wedge L 1 = L 2 \wedge Q 1 = Q 2 \wedge P 1 = P 2)$ ;
```

```
Ncj (x:v|w)i→k[ $\mathbb{E}[\omega_-, L_-, Q_-, P_-]$ ] := With[{q = eγ β xk + γ ck}, CF[
 $\mathbb{E}[\omega, \gamma c_k + (L / . c_j \rightarrow \theta), \omega e^\gamma \beta x_k + (Q / . x_i \rightarrow \theta), e^{-q} \text{DP}_{c_j \rightarrow D_\gamma, x_i \rightarrow D_\beta}[P][e^q]] / . \{\gamma \rightarrow \partial_{c_j} L, \beta \rightarrow \omega^{-1} \partial_{x_i} Q\}$ ];
```

```
Nwi vj→k[ $\mathbb{E}[\omega_-, L_-, Q_-, P_-]$ ] := With[{q = ((1 - tk) α β + β vk + δ vk wk + α wk) / μ}, CF[
 $\mathbb{E}[\mu \omega, L, \mu \omega q + \mu (Q / . w_i | v_j \rightarrow \theta), \mu^4 e^{-q} \text{DP}_{w_i \rightarrow D_\alpha, v_j \rightarrow D_\beta}[P][e^q] + \omega^4 \Delta[k]] / . \mu \rightarrow 1 + (t_k - 1) \delta / .$ 
 $\{\alpha \rightarrow \omega^{-1} (\partial_{w_i} Q / . v_j \rightarrow \theta), \beta \rightarrow \omega^{-1} (\partial_{v_j} Q / . w_i \rightarrow \theta), \delta \rightarrow \omega^{-1} \partial_{w_i, v_j} Q\}$ ];
```

```
mi,j→k[Z_-] := Module[{x, z},
Z // Nwi vj→x // Nci vx→x // Nwx cj→x // ReplaceAll[z-i|j|x → zk] // CF]
```

```
Ri,j+ :=  $\mathbb{E}[1, b_i c_j, v_i w_j, v_i c_i w_j + c_i c_j + v_i^2 w_j^2 / 4]$ ;
Ri,j- :=  $\mathbb{E}[1, -b_i c_j, -t_i^{-1} v_i w_j, -c_i c_j + t_i^{-1} v_i c_j w_j - t_i^{-2} v_i^2 w_j^2 / 4]$ ;
uri :=  $\mathbb{E}[t_i^{-1/2}, \theta, \theta, c_i t_i^2]$ ; nri :=  $\mathbb{E}[t_i^{1/2}, \theta, \theta, -c_i t_i^2]$ ;
ul- = nl- =  $\mathbb{E}[1, \theta, \theta, \theta]$ ;
```

Computing $\text{tr}Z_1$

```

RVK::usage =
  "RVK[xs, rots] represents a Rotational Virtual Knot with a list of n Xp/Xm crossings xs and
  a length 2n list of rotation numbers rots. Crossing sites are indexed 1 through
  2n, and rots[[k]] is the rotation between site k-1 and site k. RVK is also a casting
  operator converting to the RVK presentation from other knot presentations.";
RVK[pd_PD] := Module[{n, xs, x, rots, front, k},
  n = Length[pd];
  xs = List@@pd /. x_X => If[PositiveQ[x], Xp[x[[4]], x[[1]], Xm[x[[2]], x[[1]]];
  rots = Table[0, {2 n});
  front = {0};
  For[k = 0, k < 2 n, ++k,
  If[k == 0 ∨ FreeQ[front, -k],
  front = Flatten[front /. k → Catch[xs /. {
    Xp[k + 1, L_] | Xm[L_, k + 1] => Throw[{L, k + 1, 1 - L]},
    Xp[L_, k + 1] | Xm[k + 1, L_] => (++)rots[[L]; Throw[{1 - L, k + 1, L}]
  }]],
  If[MatchQ[front, {___, k, ___, -k, ___}], --rots[[k + 1]]
  ];
  RVK[xs, rots]
];
RVK[K_] := RVK[PD[K]];

```

```

rot[_, 0] = E[1, 0, 0, 0];
rot[i_, 1] := ur_i;
rot[i_, n_Integer] /; n > 1 := Module[{y}, rot[i, n - 1] rot[y, 1] // mi,y→i;
rot[i_, -1] := nr_i;
rot[i_, n_Integer] /; n < -1 := Module[{y}, rot[i, n + 1] rot[y, -1] // mi,y→i;

```

```
{rot[i, 2], rot[i, -2]}
```

```
{E[ $\frac{1}{t_i}$ , 0, 0,  $\frac{2c_i}{t_i^4}$ ], E[ti, 0, 0, -2 ci ti4]}
```

```

trZ[K_] := trZ[RVK@K];
trZ[rvk_RVK] := Module[{tr, todo, n, rots,  $\zeta$ , done, st, x,  $\zeta_1$ , i, j, k, k1, k2, k3},
  {todo, rots} = List@@rvk;
  AppendTo[rots, 0];
  n = Length[todo];
  tr = { $\zeta$  =  $\mathbb{E}$ [1, 0, 0, 0]};
  done = {0};
  st = Range[0, 2 n + 1];
  While[todo != {},
    {x} = MaximalBy[todo, Length[done  $\cap$  {#[[1]], #[[2]], #[[1]] - 1, #[[2]] - 1}] &, 1];
    Z$x = x; Z$ $\zeta$  =  $\zeta$ ;
    {i, j} = List@@x;
     $\zeta_1$  = Switch[Head[x],
      Xp,  $m_{j,k \rightarrow j} [R_{i,j}^+ (R_{k3,k}^- n r_{k1} u_{k2} // m_{k,k1 \rightarrow k} // m_{k,k2 \rightarrow k} // m_{k,k3 \rightarrow k})]$ ,
      Xm,  $m_{j,k \rightarrow j} [R_{i,j}^- (R_{k,k3}^+ n r_{k1} u_{k2} // m_{k,k1 \rightarrow k} // m_{k,k2 \rightarrow k} // m_{k,k3 \rightarrow k})]$ 
    ];
     $\zeta_1$  = rot[k, rots[[i]]  $\zeta_1$  //  $m_{k,i \rightarrow i}$ ; rots[[i]] = 0;
     $\zeta_1$  =  $\zeta_1$  rot[k, rots[[i + 1]] //  $m_{i,k \rightarrow i}$ ; rots[[i + 1]] = 0;
     $\zeta_1$  = rot[k, rots[[j]]]  $\zeta_1$  //  $m_{k,j \rightarrow j}$ ; rots[[j]] = 0;
     $\zeta_1$  =  $\zeta_1$  rot[k, rots[[j + 1]]] //  $m_{j,k \rightarrow j}$ ; rots[[j + 1]] = 0;
     $\zeta$  *=  $\zeta_1$ ;
    If[MemberQ[done, i],  $\zeta$  =  $\zeta$  //  $m_{i,i+1 \rightarrow i}$ ; st = st /. st[[i + 2]]  $\rightarrow$  st[[i + 1]];
    If[MemberQ[done, i - 1],  $\zeta$  =  $\zeta$  //  $m_{st[[i], i \rightarrow st[[i]]}$ ; st = st /. st[[i + 1]]  $\rightarrow$  st[[i]];
    If[MemberQ[done, j],  $\zeta$  =  $\zeta$  //  $m_{j,j+1 \rightarrow j}$ ; st = st /. st[[j + 2]]  $\rightarrow$  st[[j + 1]];
    If[MemberQ[done, j - 1],  $\zeta$  =  $\zeta$  //  $m_{st[[j], j \rightarrow st[[j]]}$ ; st = st /. st[[j + 1]]  $\rightarrow$  st[[j]];
    done = done  $\cup$  {i - 1, i, j - 1, j};
    todo = DeleteCases[todo, x];
    AppendTo[tr,  $\zeta$ ];
  ];
  tr
]

```

Some Traces

 $t_ = t$; $trZ[\text{Knot}[3, 1]]$

KnotTheory: Loading precomputed data in PD4Knots`.

$$\{E[1, \theta, \theta, \theta], E[t, b_0 c_0 - b_4 c_0, v_0 w_0 - v_4 w_0, -t^4 c_0 + t^4 c_0^2 - t^4 c_4 - t^4 c_0 c_4 + t^3 c_0 v_0 w_0 - t^3 c_4 v_0 w_0 + t^3 v_4 w_0 - t^4 c_0 v_4 w_0 + t^3 \lambda c_0 v_4 w_0 + t^4 \lambda c_0 v_4 w_0 + \frac{1}{4} t^2 v_0^2 w_0^2 + \frac{1}{2} t^2 v_0 v_4 w_0^2 - \frac{1}{2} t^3 v_0 v_4 w_0^2 + \frac{1}{2} t^2 \lambda v_0 v_4 w_0^2 + \frac{1}{2} t^3 \lambda v_0 v_4 w_0^2 + \frac{1}{2} t^3 v_4^2 w_0^2 + \frac{1}{4} t^4 v_4^2 w_0^2 - \frac{1}{4} t^2 \lambda v_4^2 w_0^2 - \frac{1}{2} t^3 \lambda v_4^2 w_0^2 - \frac{1}{4} t^4 \lambda v_4^2 w_0^2], E[t^{3/2}, b_0 c_0 - b_4 c_0 - b_0 c_4 + b_4 c_4, \sqrt{t} v_0 w_0 - \sqrt{t} v_4 w_0 - \frac{v_0 w_4}{\sqrt{t}} + \frac{v_4 w_0}{\sqrt{t}}, -t^6 c_0 + t^6 c_0^2 - 2 t^6 c_4 - 2 t^6 c_0 c_4 + t^6 c_4^2 + t^5 c_0 v_0 w_0 - 2 t^5 c_4 v_0 w_0 + t^5 v_4 w_0 - t^6 c_0 v_4 w_0 + t^5 \lambda c_0 v_4 w_0 + t^6 \lambda c_0 v_4 w_0 + t^5 c_4 v_4 w_0 + \frac{1}{4} t^4 v_0^2 w_0^2 + \frac{1}{2} t^4 v_0 v_4 w_0^2 - \frac{1}{2} t^5 v_0 v_4 w_0^2 + \frac{1}{2} t^4 \lambda v_0 v_4 w_0^2 + \frac{1}{2} t^5 \lambda v_0 v_4 w_0^2 + \frac{1}{2} t^5 v_4^2 w_0^2 + \frac{1}{4} t^6 v_4^2 w_0^2 - \frac{1}{4} t^4 \lambda v_4^2 w_0^2 - \frac{1}{2} t^5 \lambda v_4^2 w_0^2 - \frac{1}{4} t^6 \lambda v_4^2 w_0^2 - t^4 c_0 v_0 w_4 - t^5 c_0 v_0 w_4 + t^4 \lambda c_0 v_0 w_4 + t^5 \lambda c_0 v_0 w_4 + t^4 c_4 v_0 w_4 - t^5 c_4 v_0 w_4 + t^4 \lambda c_4 v_0 w_4 - t^5 \lambda c_4 v_0 w_4 - t^4 v_4 w_4 - t^5 c_0 v_4 w_4 - t^6 c_0 v_4 w_4 - 2 t^4 \lambda c_0 v_4 w_4 - t^5 \lambda c_0 v_4 w_4 + t^6 \lambda c_0 v_4 w_4 + 2 t^5 c_4 v_4 w_4 - t^6 c_4 v_4 w_4 - t^4 \lambda c_4 v_4 w_4 + t^6 \lambda c_4 v_4 w_4 - \frac{1}{2} t^3 v_0^2 w_0 w_4 - \frac{1}{2} t^4 v_0^2 w_0 w_4 + \frac{1}{2} t^3 \lambda v_0^2 w_0 w_4 + \frac{1}{2} t^4 \lambda v_0^2 w_0 w_4 - t^3 v_0 v_4 w_0 w_4 - 2 t^3 \lambda v_0 v_4 w_0 w_4 - 2 t^4 \lambda v_0 v_4 w_0 w_4 + \frac{1}{2} t^5 v_4^2 w_0 w_4 + \frac{1}{2} t^6 v_4^2 w_0 w_4 + t^3 \lambda v_4^2 w_0 w_4 + t^4 \lambda v_4^2 w_0 w_4 - \frac{1}{2} t^5 \lambda v_4^2 w_0 w_4 - \frac{1}{2} t^6 \lambda v_4^2 w_0 w_4 + \frac{1}{4} t^2 v_0^2 w_4^2 + t^3 v_0^2 w_4^2 + \frac{1}{2} t^4 v_0^2 w_4^2 - \frac{1}{2} t^2 \lambda v_0^2 w_4^2 - t^3 \lambda v_0^2 w_4^2 - \frac{1}{2} t^4 \lambda v_0^2 w_4^2 + \frac{1}{2} t^2 v_0 v_4 w_4^2 - t^3 v_0 v_4 w_4^2 + \frac{1}{2} t^4 v_0 v_4 w_4^2 + t^5 v_0 v_4 w_4^2 + \frac{3}{2} t^2 \lambda v_0 v_4 w_4^2 + 2 t^3 \lambda v_0 v_4 w_4^2 - \frac{1}{2} t^4 \lambda v_0 v_4 w_4^2 - t^5 \lambda v_0 v_4 w_4^2 + \frac{1}{2} t^3 v_4^2 w_4^2 - \frac{1}{4} t^4 v_4^2 w_4^2 - \frac{1}{2} t^5 v_4^2 w_4^2 + \frac{1}{2} t^6 v_4^2 w_4^2 - \frac{3}{4} t^2 \lambda v_4^2 w_4^2 - \frac{1}{2} t^3 \lambda v_4^2 w_4^2 + \frac{5}{4} t^4 \lambda v_4^2 w_4^2 + \frac{1}{2} t^5 \lambda v_4^2 w_4^2 - \frac{1}{2} t^6 \lambda v_4^2 w_4^2], E[-1 + \frac{1}{t} + t, \theta, \theta, -8 + \frac{1}{t^4} - \frac{3}{t^3} + \frac{11}{2 t^2} - \frac{3}{t} + 24 t - \frac{67 t^2}{2} + 27 t^3 - \frac{21 t^4}{2} - 4 t^5 + 8 t^6 - 5 t^7 + \frac{3 t^8}{2} - 8 \lambda - \frac{3 \lambda}{t^4} + \frac{10 \lambda}{t^3} - \frac{39 \lambda}{2 t^2} + \frac{21 \lambda}{t} - 14 t \lambda + \frac{59 t^2 \lambda}{2} - 26 t^3 \lambda + \frac{21 t^4 \lambda}{2} + 4 t^5 \lambda - 8 t^6 \lambda + 5 t^7 \lambda - \frac{3 t^8 \lambda}{2} - 33 c_0 - \frac{4 c_0}{t^4} + \frac{15 c_0}{t^3} - \frac{34 c_0}{t^2} + \frac{44 c_0}{t} - 2 t c_0 + 34 t^2 c_0 - 45 t^3 c_0 + 32 t^4 c_0 - 14 t^5 c_0 + 3 t^6 c_0 + 33 \lambda c_0 + \frac{6 \lambda c_0}{t^4} - \frac{21 \lambda c_0}{t^3} + \frac{44 \lambda c_0}{t^2} - \frac{52 \lambda c_0}{t} + 10 t \lambda c_0 - 44 t^2 \lambda c_0 + 51 t^3 \lambda c_0 - 34 t^4 \lambda c_0 + 14 t^5 \lambda c_0 - 3 t^6 \lambda c_0 + 17 v_0 w_0 + \frac{4 v_0 w_0}{t^4} - \frac{15 v_0 w_0}{t^3} + \frac{31 v_0 w_0}{t^2} - \frac{36 v_0 w_0}{t} + 17 t v_0 w_0 - 42 t^2 v_0 w_0 + 37 t^3 v_0 w_0 - 17 t^4 v_0 w_0 - 2 t^5 v_0 w_0 + 5 t^6 v_0 w_0 - 3 t^7 v_0 w_0 - 15 \lambda v_0 w_0 - \frac{6 \lambda v_0 w_0}{t^4} + \frac{19 \lambda v_0 w_0}{t^3} - \frac{37 \lambda v_0 w_0}{t^2} + \frac{38 \lambda v_0 w_0}{t} - 23 t \lambda v_0 w_0 + 46 t^2 \lambda v_0 w_0 - 39 t^3 \lambda v_0 w_0 + 17 t^4 \lambda v_0 w_0 + 2 t^5 \lambda v_0 w_0 - 5 t^6 \lambda v_0 w_0 + 3 t^7 \lambda v_0 w_0 - 9 c_0 v_0 w_0 - \frac{3 c_0 v_0 w_0}{t^4} + \frac{9 c_0 v_0 w_0}{t^3} - \frac{18 c_0 v_0 w_0}{t^2} + \frac{18 c_0 v_0 w_0}{t} - 9 t c_0 v_0 w_0 + 18 t^2 c_0 v_0 w_0 - 18 t^3 c_0 v_0 w_0 + 9 t^4 c_0 v_0 w_0 - 3 t^5 c_0 v_0 w_0 + 9 \lambda c_0 v_0 w_0 + \frac{3 \lambda c_0 v_0 w_0}{t^4} - \frac{9 \lambda c_0 v_0 w_0}{t^3} + \frac{18 \lambda c_0 v_0 w_0}{t^2} - \frac{18 \lambda c_0 v_0 w_0}{t} + 9 t \lambda c_0 v_0 w_0 - 18 t^2 \lambda c_0 v_0 w_0 + 18 t^3 \lambda c_0 v_0 w_0 - 9 t^4 \lambda c_0 v_0 w_0 + 3 t^5 \lambda c_0 v_0 w_0 + \frac{9}{4} v_0^2 w_0^2 + \frac{9 v_0^2 w_0^2}{4 t^4} - \frac{6 v_0^2 w_0^2}{t^3} + \frac{45 v_0^2 w_0^2}{4 t^2} - \frac{9 v_0^2 w_0^2}{t} + 9 t v_0^2 w_0^2 - \frac{45}{4} t^2 v_0^2 w_0^2 + 9 t^3 v_0^2 w_0^2 - \frac{9}{4} t^4 v_0^2 w_0^2 + \frac{3}{4} t^6 v_0^2 w_0^2 - \frac{9}{4} \lambda v_0^2 w_0^2 - \frac{9 \lambda v_0^2 w_0^2}{4 t^4} + \frac{6 \lambda v_0^2 w_0^2}{t^3} - \frac{45 \lambda v_0^2 w_0^2}{4 t^2} + \frac{9 \lambda v_0^2 w_0^2}{t} - 9 t \lambda v_0^2 w_0^2 + \frac{45}{4} t^2 \lambda v_0^2 w_0^2 - 9 t^3 \lambda v_0^2 w_0^2 + \frac{9}{4} t^4 \lambda v_0^2 w_0^2 - \frac{3}{4} t^6 \lambda v_0^2 w_0^2]]$$

 $t_ = t$; $tr = trZ[\text{Knot}[10, 160]]$

$$\{E[1, \theta, \theta, \theta], \dots, E[3 - \frac{1}{t^3} + \frac{4}{t^2} - \frac{4}{t} - 4 t + 4 t^2 - t^3, \theta, \theta, \frac{77711}{2} - \frac{5}{2 t^{12}} + \frac{26}{t^{11}} - \frac{76}{t^{10}} - \frac{160}{t^9} + \frac{1801}{t^8} - \frac{6356}{t^7} + \frac{14373}{t^6} - \frac{25252}{t^5} + \frac{75153}{2 t^4} - \frac{48192}{t^3} + \frac{53022}{t^2} - \frac{49889}{t} - 22035 t + \frac{8643 t^2}{2} + \dots + \frac{2064 \lambda v_0^2 w_0^2}{t} - 1958 t \lambda v_0^2 w_0^2 + \frac{6509}{2} t^2 \lambda v_0^2 w_0^2 - 3792 t^3 \lambda v_0^2 w_0^2 + 4128 t^4 \lambda v_0^2 w_0^2 - 3824 t^5 \lambda v_0^2 w_0^2 + 2769 t^6 \lambda v_0^2 w_0^2 - 1740 t^7 \lambda v_0^2 w_0^2 + 1059 t^8 \lambda v_0^2 w_0^2 - 510 t^9 \lambda v_0^2 w_0^2 + 112 t^{10} \lambda v_0^2 w_0^2 + 30 t^{11} \lambda v_0^2 w_0^2 - \frac{51}{2} t^{12} \lambda v_0^2 w_0^2 + 6 t^{13} \lambda v_0^2 w_0^2 - \frac{1}{2} t^{14} \lambda v_0^2 w_0^2]\}$$

large output

show less

show more

show all

set size limit...

tr[[1 ;; 3]]

$$\left\{ \mathbb{E}[1, 0, 0, 0], \mathbb{E}[1, b_0 c_4 - b_4 c_4, v_0 w_4 - v_4 w_4, \frac{1}{2} - \frac{t^2}{2} - \frac{\lambda}{2} + \frac{t^2 \lambda}{2} - c_4 - t c_4 + \lambda c_4 + t \lambda c_4 + c_0 c_4 - c_4^2 - t v_0 w_4 - t^2 v_0 w_4 + t \lambda v_0 w_4 + t^2 \lambda v_0 w_4 + c_0 v_0 w_4 - c_4 v_0 w_4 - t c_4 v_0 w_4 + \lambda c_4 v_0 w_4 + t \lambda c_4 v_0 w_4 + 2 v_4 w_4 + 2 t v_4 w_4 + t^2 v_4 w_4 - \lambda v_4 w_4 - 2 t \lambda v_4 w_4 - t^2 \lambda v_4 w_4 - c_0 v_4 w_4 + 2 c_4 v_4 w_4 + t c_4 v_4 w_4 - \lambda c_4 v_4 w_4 - t \lambda c_4 v_4 w_4 - \frac{1}{2} t v_0^2 w_4^2 - \frac{1}{4} t^2 v_0^2 w_4^2 + \frac{1}{4} \lambda v_0^2 w_4^2 + \frac{1}{2} t \lambda v_0^2 w_4^2 + \frac{1}{4} t^2 \lambda v_0^2 w_4^2 + v_0 v_4 w_4^2 + \frac{3}{2} t v_0 v_4 w_4^2 + \frac{1}{2} t^2 v_0 v_4 w_4^2 - \lambda v_0 v_4 w_4^2 - \frac{3}{2} t \lambda v_0 v_4 w_4^2 - \frac{1}{2} t^2 \lambda v_0 v_4 w_4^2 - v_4^2 w_4^2 - t v_4^2 w_4^2 - \frac{1}{4} t^2 v_4^2 w_4^2 + \frac{3}{4} \lambda v_4^2 w_4^2 + t \lambda v_4^2 w_4^2 + \frac{1}{4} t^2 \lambda v_4^2 w_4^2], \mathbb{E}\left[\frac{1}{t}, b_0 c_3 - b_3 c_3 + b_3 c_{12} - b_{12} c_{12}, \frac{v_0 w_3}{t} - \frac{v_3 w_3}{t} + \frac{v_3 w_{12}}{t} - \frac{v_{12} w_{12}}{t}, \frac{1}{t^4} - \frac{1}{t^2} - \frac{\lambda}{t^4} + \frac{\lambda}{t^2} - \frac{c_3}{t^4} - \frac{c_3}{t^3} + \frac{\lambda c_3}{t^4} + \frac{\lambda c_3}{t^3} + \frac{c_0 c_3}{t^4} - \frac{c_3^2}{t^4} + \frac{c_{12}}{t^4} - \frac{c_{12}}{t^3} + \frac{\lambda c_{12}}{t^4} + \frac{\lambda c_{12}}{t^3} + \frac{c_3 c_{12}}{t^4} - \frac{c_{12}^2}{t^4} - \frac{v_0 w_3}{t^3} - \frac{v_0 w_3}{t^2} + \frac{\lambda v_0 w_3}{t^3} + \frac{\lambda v_0 w_3}{t^2} + \frac{c_0 v_0 w_3}{t^4} - \frac{c_3 v_0 w_3}{t^4} - \frac{c_3 v_0 w_3}{t^3} + \frac{\lambda c_3 v_0 w_3}{t^4} + \frac{\lambda c_3 v_0 w_3}{t^3} + \frac{2 v_3 w_3}{t^4} + \frac{2 v_3 w_3}{t^3} + \frac{v_3 w_3}{t^2} - \frac{\lambda v_3 w_3}{t^4} - \frac{2 \lambda v_3 w_3}{t^3} - \frac{\lambda v_3 w_3}{t^2} - \frac{c_0 v_3 w_3}{t^4} + \frac{2 c_3 v_3 w_3}{t^4} + \frac{c_3 v_3 w_3}{t^3} - \frac{\lambda c_3 v_3 w_3}{t^4} - \frac{\lambda c_3 v_3 w_3}{t^3} - \frac{c_{12} v_3 w_3}{t^4} - \frac{v_0^2 w_3^2}{2 t^3} - \frac{v_0^2 w_3^2}{4 t^2} + \frac{\lambda v_0^2 w_3^2}{4 t^4} + \frac{\lambda v_0^2 w_3^2}{2 t^3} + \frac{\lambda v_0^2 w_3^2}{4 t^2} + \frac{v_0 v_3 w_3^2}{t^4} + \frac{3 v_0 v_3 w_3^2}{2 t^3} + \frac{v_0 v_3 w_3^2}{2 t^2} - \frac{\lambda v_0 v_3 w_3^2}{t^4} - \frac{3 \lambda v_0 v_3 w_3^2}{2 t^3} - \frac{\lambda v_0 v_3 w_3^2}{2 t^2} - \frac{v_3^2 w_3^2}{t^4} - \frac{v_3^2 w_3^2}{t^3} - \frac{v_3^2 w_3^2}{4 t^2} + \frac{3 \lambda v_3^2 w_3^2}{4 t^4} + \frac{\lambda v_3^2 w_3^2}{t^3} + \frac{\lambda v_3^2 w_3^2}{4 t^2} + \frac{v_3 w_{12}}{t^4} - \frac{v_3 w_{12}}{t^3} - \frac{v_3 w_{12}}{t^2} + \frac{\lambda v_3 w_{12}}{t^3} + \frac{\lambda v_3 w_{12}}{t^2} + \frac{c_3 v_3 w_{12}}{t^4} - \frac{c_{12} v_3 w_{12}}{t^4} - \frac{c_{12} v_3 w_{12}}{t^3} + \frac{\lambda c_{12} v_3 w_{12}}{t^4} + \frac{\lambda c_{12} v_3 w_{12}}{t^3} + \frac{2 v_{12} w_{12}}{t^3} + \frac{v_{12} w_{12}}{t^2} - \frac{\lambda v_{12} w_{12}}{t^4} - \frac{2 \lambda v_{12} w_{12}}{t^3} - \frac{\lambda v_{12} w_{12}}{t^2} - \frac{c_3 v_{12} w_{12}}{t^4} + \frac{2 c_{12} v_{12} w_{12}}{t^4} + \frac{c_{12} v_{12} w_{12}}{t^3} - \frac{\lambda c_{12} v_{12} w_{12}}{t^4} - \frac{\lambda c_{12} v_{12} w_{12}}{t^3} - \frac{v_3^2 w_3 w_{12}}{t^4} + \frac{v_3 v_{12} w_3 w_{12}}{t^4} - \frac{v_3^2 w_{12}^2}{2 t^3} - \frac{v_3^2 w_{12}^2}{4 t^2} + \frac{\lambda v_3^2 w_{12}^2}{4 t^4} + \frac{\lambda v_3^2 w_{12}^2}{2 t^3} + \frac{\lambda v_3^2 w_{12}^2}{4 t^2} + \frac{v_3 v_{12} w_{12}^2}{t^4} + \frac{3 v_3 v_{12} w_{12}^2}{2 t^3} + \frac{v_3 v_{12} w_{12}^2}{2 t^2} - \frac{\lambda v_3 v_{12} w_{12}^2}{t^4} - \frac{3 \lambda v_3 v_{12} w_{12}^2}{2 t^3} - \frac{\lambda v_3 v_{12} w_{12}^2}{2 t^2} - \frac{v_{12}^2 w_{12}^2}{t^4} - \frac{v_{12}^2 w_{12}^2}{t^3} - \frac{v_{12}^2 w_{12}^2}{4 t^2} + \frac{3 \lambda v_{12}^2 w_{12}^2}{4 t^4} + \frac{\lambda v_{12}^2 w_{12}^2}{t^3} + \frac{\lambda v_{12}^2 w_{12}^2}{4 t^2}] \right\}$$

```
tr [[8 ;;]] /.  $\mathcal{E}_{-E} \Rightarrow \{$ 
  vars = Union@Cases[ $\mathcal{E}$ , (c | v | w)_,  $\infty$ ],
  al = Numerator@Factor@ $\mathcal{E}$ [[1]]; al /= Coefficient[al, t, 0],
   $\omega$ s = CoefficientRules[ $\frac{\mathcal{E}[[4]]}{al^5}$ , vars] /. (p_ -> c_) -> (Factor[c] /. a_. al^m. ->  $\omega^{m+5}$ ) Times@@ (vars^p),
   $\omega$ s /. {c_ ->  $\frac{1}{2}$ ,  $\omega$  | w_ -> 2}
}
```

$\{ \{ \{ c_0, c_9, c_{16}, v_0, v_9, v_{16}, w_0, w_9, w_{16} \}, \frac{2-t}{2}, \{ \frac{64 c_0^2}{(-2+t) t^{12}}, -\frac{64 c_0 c_9}{(-2+t) t^{12}}, \dots 78 \dots, -\frac{16 (-1+t) (-24+15 t^2-5 t^3+22 \lambda+2 t \lambda-15 t^2 \lambda+5 t^3 \lambda)}{(-2+t)^3 t^{12}} \}, \{ \frac{16}{(-2+t) t^{12}}, -\frac{16}{(-2+t) t^{12}}, -\frac{16}{(-2+t) t^{12}}, \dots 76 \dots, \frac{1}{(-2+t)^4 t^{12}} \}, 64 (20 - 32 t + 39 t^2 - 33 t^3 + 29 t^4 - 29 t^6 + 20 t^7 - 4 t^8 - 12 \lambda + 10 t \lambda - 15 t^2 \lambda + 10 t^3 \lambda - 13 t^4 \lambda - 6 t^5 \lambda + 30 t^6 \lambda - 20 t^7 \lambda + 4 t^8 \lambda) v_{16}, -\frac{16 (-1+t) (-24+15 t^2-5 t^3+22 \lambda+2 t \lambda-15 t^2 \lambda+5 t^3 \lambda)}{(-2+t)^3 t^{12}} \}, \{ \dots 1 \dots \}, \{ \dots 1 \dots \}, \{ \{ c_0, v_0, w_0 \}, 1 - 4 t + \dots 6 \dots + t^6, \{ \dots 1 \dots \}, \{ 16 v_0, 4, 64 v_0^2, 16 v_0, 4 \} \}$

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```
tr [[1 ;;]] /.  $\mathcal{E}_{-E} \Rightarrow \{$ 
  vars = Union@Cases[ $\mathcal{E}$ , (c | v | w)_,  $\infty$ ],
  al = Numerator@Factor@ $\mathcal{E}$ [[1]],
  CoefficientRules[ $\mathcal{E}$ [[4]], vars] /. (p_ -> cc_) -> (mon = Times@@ (vars^p);
  Denominator@Factor[ $\frac{al^{\text{Exponent}[mon/.v.->v, cc]} cc}{al^{\text{Exponent}[mon/.c.->c, c]} al^2}$ ] mon
}
```



```

tr [[7 ;;]] /.  $\mathcal{E}_{\mathbb{E}}$  => {
  vars = Union@Cases[ $\mathcal{E}$ , (c | v | w)_,  $\infty$ ],
  a1 = Numerator@Factor@ $\mathcal{E}$ [[1],
  CoefficientRules[ $\mathcal{E}$ [[4]], vars] /. (p_ -> cc_) => (mon = Times @@ (varsp);
  Factor[ $\frac{a1^{\text{Exponent}[mon/.v->v, v]} cc}{a1^{\text{Exponent}[mon/.c->c, c]} a1^2}$ )] mon
}

```

$\{ \{ \{ C_0, C_9, C_{16}, V_0, V_9, V_{16}, W_0, W_9, W_{16} \}, 2 - t, \left\{ -\frac{2c_0^2}{t^{10}}, \frac{2c_0 c_9}{t^{10}}, \frac{c_0 c_{16}}{t^{10}}, -\frac{(-2+t)^2 (1+t) (-4+t-3t^2+\lambda+3t^2\lambda) c_0 v_0 w_0}{t^{10}}, \right.$
 $\left. \frac{\dots 61 \dots}{t^{10}}, \frac{(8-26t+32t^2-14t^3-9t^4+9t^5-2t^6+2\lambda+8t\lambda-12t^2\lambda+4t^3\lambda+11t^4\lambda-9t^5\lambda+2t^6\lambda) v_{16} w_0}{t^{10}}, \frac{(4-2t+t^2-2\lambda-2t\lambda) v_{16} w_{16}}{t^{10}}, \right.$
 $\left. \frac{(-1+t) (-10+6t^2-2t^3+9\lambda+t\lambda-6t^2\lambda+2t^3\lambda)}{t^{10}} \right\}, \dots 3 \dots, \{ \{ C_0, V_0, W_0 \}, -1 + \dots 9 \dots, \left.$
 $\left. \left\{ \frac{2(1+t) (\dots 9 \dots + t^6)^2 (-1+\lambda) c_0 v_0 w_0}{t^{12}}, \frac{(1+t) (\dots 1 \dots) c_0}{t^{12}}, -\frac{\dots 1 \dots}{2 (\dots 1 \dots)}, -\frac{\dots 1 \dots}{t^{12}}, -\frac{(-1+t) (-5+\dots 45 \dots + 2t^{15}\lambda)}{2t^{12}} \right\} \right\}$

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