

```

GroebnerBasis[
  {x1 + x2, x1 x2, y1 + y2, y1 y2, x1 y2 + x2 y1},
  {x1, x2, y1, y2}
]
{y2^2, y1 + y2, x2 y2, x2^2, x1 + x2}

eqns = {
  b1 b2 t2 (-1 + t1^2 t2^2)^2 h[b2, b1, t2, t1]^2 +
    (b1 + b2) (-t2 (b2 (-1 + t1^2)^2 t2 + b1 t1^3 (-1 + t2^2)^2) + b1^2 t1 t2 (-1 + t2^2) (-1 + t1^2 t2^2)
      g11[b1, b2, t1, t2] + b2^2 (-1 + t1^2) (-1 + t1^2 t2^2) g11[b2, b1, t2, t1]) == 0,
  -b1 b2 t2 (-1 + t1^2 t2^2)^2 h[b2, b1, t2, t1]^2 - (b1 + b2)
    (-t2 (b2 (-1 + t1^2)^2 t2 + b1 t1^3 (-1 + t2^2)^2) + b1^2 t1 t2 (-1 + t2^2) (-1 + t1^2 t2^2)
      g11[b1, b2, t1, t2] + b2^2 (-1 + t1^2) (-1 + t1^2 t2^2) g11[b2, b1, t2, t1]) == 0,
  b1 b2 t2 (-1 + t1^2 t2^2)^2 h[b1, b2, t1, t2]^2 + (b1 + b2)
    (-t2 (b2 (-1 + t1^2)^2 t2 + b1 t1^3 (-1 + t2^2)^2) + b1^2 t1 t2 (-1 + t2^2) (-1 + t1^2 t2^2)
      g11[b1, b2, t1, t2] + b2^2 (-1 + t1^2) (-1 + t1^2 t2^2) g11[b2, b1, t2, t1]) == 0,
  -b1 b2 t2 (-1 + t1^2 t2^2)^2 h[b1, b2, t1, t2]^2 - (b1 + b2)
    (-t2 (b2 (-1 + t1^2)^2 t2 + b1 t1^3 (-1 + t2^2)^2) + b1^2 t1 t2 (-1 + t2^2) (-1 + t1^2 t2^2)
      g11[b1, b2, t1, t2] + b2^2 (-1 + t1^2) (-1 + t1^2 t2^2) g11[b2, b1, t2, t1]) == 0
}

{b1 b2 t2 (-1 + t1^2 t2^2)^2 h[b2, b1, t2, t1]^2 +
  (b1 + b2) (-t2 (b2 (-1 + t1^2)^2 t2 + b1 t1^3 (-1 + t2^2)^2) + b1^2 t1 t2 (-1 + t2^2) (-1 + t1^2 t2^2)
    g11[b1, b2, t1, t2] + b2^2 (-1 + t1^2) (-1 + t1^2 t2^2) g11[b2, b1, t2, t1]) == 0,
  -b1 b2 t2 (-1 + t1^2 t2^2)^2 h[b2, b1, t2, t1]^2 - (b1 + b2)
    (-t2 (b2 (-1 + t1^2)^2 t2 + b1 t1^3 (-1 + t2^2)^2) + b1^2 t1 t2 (-1 + t2^2) (-1 + t1^2 t2^2)
      g11[b1, b2, t1, t2] + b2^2 (-1 + t1^2) (-1 + t1^2 t2^2) g11[b2, b1, t2, t1]) == 0,
  b1 b2 t2 (-1 + t1^2 t2^2)^2 h[b1, b2, t1, t2]^2 + (b1 + b2)
    (-t2 (b2 (-1 + t1^2)^2 t2 + b1 t1^3 (-1 + t2^2)^2) + b1^2 t1 t2 (-1 + t2^2) (-1 + t1^2 t2^2)
      g11[b1, b2, t1, t2] + b2^2 (-1 + t1^2) (-1 + t1^2 t2^2) g11[b2, b1, t2, t1]) == 0,
  -b1 b2 t2 (-1 + t1^2 t2^2)^2 h[b1, b2, t1, t2]^2 - (b1 + b2)
    (-t2 (b2 (-1 + t1^2)^2 t2 + b1 t1^3 (-1 + t2^2)^2) + b1^2 t1 t2 (-1 + t2^2) (-1 + t1^2 t2^2)
      g11[b1, b2, t1, t2] + b2^2 (-1 + t1^2) (-1 + t1^2 t2^2) g11[b2, b1, t2, t1]) == 0}

Solve[eqns ∪ (eqns /. {b1 → b2, b2 → b1, t1 → t2, t2 → t1}) /. h[___] ⇒ h,
  {g11[b1, b2, t1, t2], g11[b2, b1, t2, t1]}
]
{{g11[b1, b2, t1, t2] → -((-b2 h^2 t1 + b1 t1^2 + b2 t1^2 - b1 t1^2 t2^2 - b2 t1^2 t2^2 + b2 h^2 t1^3 t2^2) /
  (b1 (b1 + b2) (-1 + t1^2 t2^2))),
  g11[b2, b1, t2, t1] → -((-b1 h^2 t2 + b1 t2^2 + b2 t2^2 - b1 t1^2 t2^2 - b2 t1^2 t2^2 + b1 h^2 t1^2 t2^3) /
  (b2 (b1 + b2) (-1 + t1^2 t2^2)))}}

```

$$\begin{aligned}
 \text{err} = & (-b_2^2 b_3 (b_2 + b_3) (-1 + t_1^2) t_2 \\
 & (b_3 ((-1 + t_2^2 t_3^2) u_2 w_1 + u_1 ((-1 + t_2^2 t_3^2 + t_2 (-1 + t_3^2)) w_1 + (-1 + t_2^2 t_3^2) w_2)) + \\
 & b_2 (t_2 (-1 + t_3^2) u_1 w_1 + u_3 w_1 + u_1 w_3 - t_2^2 t_3^2 (u_3 w_1 + u_1 w_3))) + \\
 & b_1^3 (b_2^2 t_2 (-1 + t_3^2) ((-1 + t_1^2 t_2^2) u_2 w_3 + u_3 ((-1 + t_1^2 t_2^2) w_2 + (1 + t_2) (-1 + t_1^2 t_2) w_3)) + \\
 & b_3^3 (-1 + t_2^2) ((-1 + t_1^2 t_2^2 t_3^2) u_3 w_2 + u_2 ((-1 + t_1^2 t_2) (1 + t_2 t_3^2) w_2 + (-1 + t_1^2 t_2^2 t_3^2) w_3)) + \\
 & b_2 b_3 (-u_3 w_3 - t_1^2 t_2^4 t_3^2 u_3 w_3 + t_2 ((t_1^2 - t_3^2) u_3 (w_2 + w_3) + u_2 ((-1 + t_3^2) w_2 + (t_1^2 - t_3^2) w_3)) + \\
 & t_2^2 (-(-1 + t_1^2) (-1 + t_3^2) u_2 (w_2 + w_3) + u_3 (-(-1 + t_1^2) (-1 + t_3^2) w_2 + (1 + t_1^2 t_3^2) w_3)) + \\
 & t_3^2 (t_3^2 (u_2 w_3 + u_3 (w_2 + w_3)) - t_1^2 (u_3 (w_2 + w_3) + u_2 ((-1 + t_3^2) w_2 + w_3)))) + \\
 & b_1 b_2 (b_3^3 (u_1 ((1 + t_1^2 t_2) (-1 + t_2^2) (-1 + t_2 t_3^2) w_1 + t_2 (-t_3^2 + t_2^2 t_3^2 - t_2 (-1 + t_3^2) - \\
 & t_1^2 (-1 + t_2^2 - t_2 (-1 + t_3^2))) w_2) + t_2 u_2 (-t_3^2 w_1 + t_2^2 t_3^2 (w_1 - w_2) + w_2 - t_2 (-1 + t_3^2) \\
 & (w_1 + w_2) + t_1^2 (-(-1 + t_2^2 - t_2 (-1 + t_3^2)) w_1 + (-1 + t_2^2 t_3^2 + t_2 (-1 + t_3^2)) w_2))) - \\
 & b_2^3 t_2 (-1 + t_3^2) ((-1 + t_1^2 t_2^2) u_1 w_3 + u_3 ((-1 + t_1^2 t_2^2) w_1 - (-1 + t_1^2) t_2 w_3)) + b_2^2 b_3 t_2 \\
 & ((1 + t_2) u_2 ((-1 + t_1^2 t_2) (-1 + t_3^2) w_1 - (-1 + t_1^2) (-1 + t_2 t_3^2) w_3) + u_1 ((-1 + t_2) (1 + t_1^2 t_2) \\
 & (-1 + t_3^2) w_1 + (1 + t_2) ((-1 + t_1^2 t_2) (-1 + t_3^2) w_2 + 2 (-1 + t_2) (t_1^2 - t_3^2) w_3)) + \\
 & u_3 (-w_2 - t_2 w_2 - w_3 + t_2 w_3 + t_1^2 (2 (-1 + t_2^2) w_1 + (1 + t_2 - t_2 t_3^2 - t_2^2 t_3^2) w_2 - (-1 + \\
 & t_2 - t_2 t_3^2 + t_2^2 t_3^2) w_3) + t_3^2 (-2 (-1 + t_2^2) w_1 + t_2 ((1 + t_2) w_2 + (-1 + t_2) w_3)))) + \\
 & b_2 b_3^2 (u_1 ((-1 + t_2) (1 + t_1^2 t_2) (-1 + t_2^2 t_3^2 + 2 t_2 (-1 + t_3^2)) w_1 + \\
 & t_2 ((1 - 2 t_3^2 + t_2^2 t_3^2 - 2 t_2 (-1 + t_3^2) + t_1^2 (1 + t_2^2 (-2 + t_3^2) + 2 t_2 (-1 + t_3^2))) w_2 + \\
 & (1 + (1 - 2 t_2^2) t_3^2 + t_1^2 (-2 + t_2^2 (1 + t_3^2))) w_3)) + \\
 & t_2 (u_3 ((1 + (1 - 2 t_2^2) t_3^2 + t_1^2 (-2 + t_2^2 (1 + t_3^2))) w_1 - (-1 + t_1^2) t_2 (-1 + t_3^2) w_2) + \\
 & u_2 ((1 - 2 t_3^2 + t_2^2 t_3^2 - 2 t_2 (-1 + t_3^2) + t_1^2 (1 + t_2^2 (-2 + t_3^2) + 2 t_2 (-1 + t_3^2))) w_1 + \\
 & (-1 + t_1^2) ((-2 + 2 t_2^2 t_3^2 + t_2 (-1 + t_3^2)) w_2 - t_2 (-1 + t_3^2) w_3)))) + \\
 & b_1^2 (-b_3^3 (-1 + t_2^2) ((-1 + t_1^2 t_2^2 t_3^2) u_1 w_2 + u_2 ((-1 + t_1^2 t_2^2 t_3^2) w_1 + (1 + t_1^2 t_2) (-1 + t_2 t_3^2) w_2)) - \\
 & b_3^2 t_2 (-1 + t_3^2) ((-1 + t_1^2 t_2^2) (u_1 - u_2) w_3 + \\
 & u_3 ((-1 + t_1^2 t_2^2) w_1 + (1 - t_1^2 t_2^2) w_2 + (1 - 2 (-1 + t_1^2) t_2 - t_1^2 t_2^2) w_3)) + \\
 & b_2 b_3^2 (t_2 t_3^2 u_3 w_1 - t_3^2 t_3^2 u_3 w_1 - u_1 w_2 + 2 t_2^2 u_1 w_2 - t_2^2 t_3^2 u_1 w_2 + u_3 w_2 - \\
 & 2 t_2^2 u_3 w_2 + t_2^2 t_3^2 u_3 w_2 + t_2 t_3^2 u_1 w_3 - t_3^2 t_3^2 u_1 w_3 + t_1^2 t_2 \\
 & (u_3 ((-1 + t_2^2) w_1 + t_2 (1 + (-2 + t_2^2) t_3^2) w_2) - u_1 (t_3^2 t_3^2 w_2 + t_2 (1 - 2 t_3^2) w_2 + w_3 - t_2^2 w_3)) + \\
 & u_2 (- (1 + t_1^2 t_2^4 t_3^2 + t_2^2 (-2 + t_3^2 + t_1^2 (1 - 2 t_3^2))) w_1 + 3 t_2 t_3^2 w_2 - 3 t_3^2 t_3^2 w_2 + \\
 & w_3 - 2 t_2^2 w_3 + t_2^2 t_3^2 w_3 + t_1^2 t_2 (3 (-1 + t_2^2) w_2 + t_2 (1 + (-2 + t_2^2) t_3^2) w_3))) - \\
 & b_2^2 b_3 (u_3 w_3 + t_1^2 t_2^4 t_3^2 u_3 w_3 + t_2^2 (-(-1 + t_1^2) (-1 + t_3^2) u_1 w_2 - (-1 + t_1^2) (-1 + t_3^2) u_2 \\
 & (w_1 - w_2 - 2 w_3) + u_3 (2 (-1 + t_1^2) (-1 + t_3^2) w_2 + (-2 + t_3^2 + t_1^2 (1 - 2 t_3^2)) w_3)) + \\
 & t_2 ((1 + t_1^2 - 2 t_3^2) u_1 w_3 + u_2 (-2 (-1 + t_3^2) w_2 + (1 - 2 t_1^2 + t_3^2) w_3) + \\
 & u_3 ((1 + t_1^2 - 2 t_3^2) w_1 - (-1 + 2 t_1^2 - t_3^2) (w_2 + w_3))) + \\
 & t_3^2 (t_3^2 ((u_1 - 2 u_2) w_3 + u_3 (w_1 - 2 (w_2 + w_3))) + t_1^2 ((-2 + t_3^2) u_1 w_3 + \\
 & u_2 (2 (-1 + t_3^2) w_2 + (1 + t_3^2) w_3) + u_3 ((-2 + t_3^2) w_1 + (1 + t_3^2) (w_2 + w_3)))))) / \\
 & (b_1 b_2 (b_1 + b_2) b_3 (b_2 + b_3) (b_1 + b_2 + b_3) (-1 + t_1^2 t_2^2) (-1 + t_2^2 t_3^2))
 \end{aligned}$$

$$\begin{aligned}
 & (-b_2^2 b_3 (b_2 + b_3) (-1 + t_1^2) t_2 \\
 & \quad (b_3 ((-1 + t_2^2 t_3^2) u_2 w_1 + u_1 ((-1 + t_2^2 t_3^2 + t_2 (-1 + t_3^2)) w_1 + (-1 + t_2^2 t_3^2) w_2)) + \\
 & \quad b_2 (t_2 (-1 + t_3^2) u_1 w_1 + u_3 w_1 + u_1 w_3 - t_2^2 t_3^2 (u_3 w_1 + u_1 w_3))) + \\
 & b_1^3 (b_2^2 t_2 (-1 + t_3^2) ((-1 + t_1^2 t_2^2) u_2 w_3 + u_3 ((-1 + t_1^2 t_2^2) w_2 + (1 + t_2) (-1 + t_1^2 t_2) w_3)) + \\
 & \quad b_3^2 (-1 + t_2^2) ((-1 + t_1^2 t_2^2 t_3^2) u_3 w_2 + u_2 ((-1 + t_1^2 t_2) (1 + t_2 t_3^2) w_2 + (-1 + t_1^2 t_2^2 t_3^2) w_3)) + \\
 & \quad b_2 b_3 (-u_3 w_3 - t_1^2 t_2^4 t_3^2 u_3 w_3 + t_2 ((t_1^2 - t_3^2) u_3 (w_2 + w_3) + u_2 ((-1 + t_3^2) w_2 + (t_1^2 - t_3^2) w_3))) + \\
 & \quad t_2^2 ((1 - t_1^2) (-1 + t_3^2) u_2 (w_2 + w_3) + u_3 ((1 - t_1^2) (-1 + t_3^2) w_2 + (1 + t_1^2 t_3^2) w_3)) + \\
 & \quad t_3^2 (t_3^2 (u_2 w_3 + u_3 (w_2 + w_3)) - t_1^2 (u_3 (w_2 + w_3) + u_2 ((-1 + t_3^2) w_2 + w_3)))) + \\
 & b_1 b_2 (b_3^3 (u_1 ((1 + t_1^2 t_2) (-1 + t_2^2) (-1 + t_2 t_3^2) w_1 + t_2 (-t_3^2 + t_2^2 t_3^2 - t_2 (-1 + t_3^2) - t_1^2 \\
 & \quad (-1 + t_2^2 - t_2 (-1 + t_3^2))) w_2) + t_2 u_2 (-t_3^2 w_1 + t_2^2 t_3^2 (w_1 - w_2) + w_2 - \\
 & \quad t_2 (-1 + t_3^2) (w_1 + w_2) + t_1^2 ((1 - t_2^2 + t_2 (-1 + t_3^2)) w_1 + (-1 + t_2^2 t_3^2 + t_2 (-1 + t_3^2)) w_2))) - \\
 & b_2^3 t_2 (-1 + t_3^2) ((-1 + t_1^2 t_2^2) u_1 w_3 + u_3 ((-1 + t_1^2 t_2^2) w_1 - (-1 + t_1^2) t_2 w_3)) + b_2^2 b_3 t_2 \\
 & \quad ((1 + t_2) u_2 ((-1 + t_1^2 t_2) (-1 + t_3^2) w_1 - (-1 + t_1^2) (-1 + t_2 t_3^2) w_3) + u_1 ((-1 + t_2) (1 + t_1^2 t_2) \\
 & \quad (-1 + t_3^2) w_1 + (1 + t_2) ((-1 + t_1^2 t_2) (-1 + t_3^2) w_2 + 2 (-1 + t_2) (t_1^2 - t_3^2) w_3)) + \\
 & \quad u_3 (-w_2 - t_2 w_2 - w_3 + t_2 w_3 + t_1^2 (2 (-1 + t_2^2) w_1 + (1 + t_2 - t_2 t_3^2 - t_2^2 t_3^2) w_2 - (-1 + \\
 & \quad t_2 - t_2 t_3^2 + t_2^2 t_3^2) w_3) + t_3^2 (-2 (-1 + t_2^2) w_1 + t_2 ((1 + t_2) w_2 + (-1 + t_2) w_3)))) + \\
 & b_2 b_3^2 (u_1 ((-1 + t_2) (1 + t_1^2 t_2) (-1 + t_2^2 t_3^2 + 2 t_2 (-1 + t_3^2)) w_1 + \\
 & \quad t_2 ((1 - 2 t_3^2 + t_2^2 t_3^2 - 2 t_2 (-1 + t_3^2) + t_1^2 (1 + t_2^2 (-2 + t_3^2) + 2 t_2 (-1 + t_3^2))) \\
 & \quad w_2 + (1 + (1 - 2 t_2^2) t_3^2 + t_1^2 (-2 + t_2^2 (1 + t_3^2))) w_3)) + \\
 & \quad t_2 (u_3 ((1 + (1 - 2 t_2^2) t_3^2 + t_1^2 (-2 + t_2^2 (1 + t_3^2))) w_1 - (-1 + t_1^2) t_2 (-1 + t_3^2) w_2) + \\
 & \quad u_2 ((1 - 2 t_3^2 + t_2^2 t_3^2 - 2 t_2 (-1 + t_3^2) + t_1^2 (1 + t_2^2 (-2 + t_3^2) + 2 t_2 (-1 + t_3^2))) \\
 & \quad w_1 + (-1 + t_1^2) ((-2 + 2 t_2^2 t_3^2 + t_2 (-1 + t_3^2)) w_2 - t_2 (-1 + t_3^2) w_3)))) + \\
 & b_1^2 (-b_3^3 (-1 + t_2^2) ((-1 + t_1^2 t_2^2 t_3^2) u_1 w_2 + u_2 ((-1 + t_1^2 t_2^2 t_3^2) w_1 + (1 + t_1^2 t_2) (-1 + t_2 t_3^2) w_2)) - \\
 & \quad b_2^3 t_2 (-1 + t_3^2) ((-1 + t_1^2 t_2^2) (u_1 - u_2) w_3 + \\
 & \quad u_3 ((-1 + t_1^2 t_2^2) w_1 + (1 - t_1^2 t_2^2) w_2 + (1 - 2 (-1 + t_1^2) t_2 - t_1^2 t_2^2) w_3)) + \\
 & b_2 b_3^2 (t_2 t_3^2 u_3 w_1 - t_2^3 t_3^2 u_3 w_1 - u_1 w_2 + 2 t_2^2 u_1 w_2 - t_2^2 t_3^2 u_1 w_2 + u_3 w_2 - \\
 & \quad 2 t_2^2 u_3 w_2 + t_2^2 t_3^2 u_3 w_2 + t_2 t_3^2 u_1 w_3 - t_2^3 t_3^2 u_1 w_3 + t_1^2 t_2 \\
 & \quad (u_3 ((-1 + t_2^2) w_1 + t_2 (1 + (-2 + t_2^2) t_3^2) w_2) - u_1 (t_2^2 t_3^2 w_2 + t_2 (1 - 2 t_3^2) w_2 + w_3 - t_2^2 w_3)) + \\
 & \quad u_2 ((-1 - t_1^2 t_2^4 t_3^2 - t_2^2 (-2 + t_3^2 + t_1^2 (1 - 2 t_3^2))) w_1 + 3 t_2 t_3^2 w_2 - 3 t_2^3 t_3^2 w_2 + w_3 - \\
 & \quad 2 t_2^2 w_3 + t_2^2 t_3^2 w_3 + t_1^2 t_2 (3 (-1 + t_2^2) w_2 + t_2 (1 + (-2 + t_2^2) t_3^2) w_3))) - \\
 & b_2^2 b_3 (u_3 w_3 + t_1^2 t_2^4 t_3^2 u_3 w_3 + t_2^2 ((1 - t_1^2) (-1 + t_3^2) u_1 w_2 - (-1 + t_1^2) (-1 + t_3^2) u_2 (w_1 - w_2 - 2 w_3) + \\
 & \quad u_3 (2 (-1 + t_1^2) (-1 + t_3^2) w_2 + (-2 + t_3^2 + t_1^2 (1 - 2 t_3^2)) w_3)) + \\
 & \quad t_2 ((1 + t_1^2 - 2 t_3^2) u_1 w_3 + u_2 (-2 (-1 + t_3^2) w_2 + (1 - 2 t_1^2 + t_3^2) w_3) + \\
 & \quad u_3 ((1 + t_1^2 - 2 t_3^2) w_1 - (-1 + 2 t_1^2 - t_3^2) (w_2 + w_3))) + \\
 & \quad t_2^3 (t_3^2 ((u_1 - 2 u_2) w_3 + u_3 (w_1 - 2 (w_2 + w_3))) + t_1^2 ((-2 + t_3^2) u_1 w_3 + u_2 \\
 & \quad (2 (-1 + t_3^2) w_2 + (1 + t_3^2) w_3) + u_3 ((-2 + t_3^2) w_1 + (1 + t_3^2) (w_2 + w_3)))))) / \\
 & (b_1 b_2 (b_1 + b_2) b_3 (b_2 + b_3) (b_1 + b_2 + b_3) (-1 + t_1^2 t_2^2) \\
 & \quad (-1 + \\
 & \quad t_2^2 t_3^2))
 \end{aligned}$$

CoefficientRules[err, {u₁, u₂, u₃, w₁, w₂, w₃}] /.

(p₋ → c₋) ⇒ **Simplify**[**Series**[c /. t_i ⇒ e^{b_i/2} /. {b_i ⇒ ħ b_i, u_i ⇒ ħ u_i}, {ħ, 0, 1}]]

$$\left\{ \frac{b_2 (2 b_1 + b_2) b_3 \hbar}{12 (b_1 + b_2) (b_1 + b_2 + b_3)} + O[\hbar]^2, -\frac{(b_3 (b_1^2 + b_1 b_3 + b_2 b_3)) \hbar}{12 ((b_1 + b_2) (b_1 + b_2 + b_3))} + O[\hbar]^2, \right.$$

$$\frac{b_2 (-b_1 + b_3) \hbar}{12 (b_1 + b_2 + b_3)} + O[\hbar]^2, -\frac{(b_3 (b_1^2 + b_1 b_3 + b_2 b_3)) \hbar}{12 ((b_1 + b_2) (b_1 + b_2 + b_3))} + O[\hbar]^2,$$

$$-\frac{(b_1 b_2 (b_1 - b_3) b_3) \hbar}{12 ((b_1 + b_2) (b_2 + b_3) (b_1 + b_2 + b_3))} + O[\hbar]^2, \frac{b_1 (b_3^2 + b_1 (b_2 + b_3)) \hbar}{12 (b_2 + b_3) (b_1 + b_2 + b_3)} + O[\hbar]^2,$$

$$\left. \frac{b_2 (-b_1 + b_3) \hbar}{12 (b_1 + b_2 + b_3)} + O[\hbar]^2, \frac{b_1 (b_3^2 + b_1 (b_2 + b_3)) \hbar}{12 (b_2 + b_3) (b_1 + b_2 + b_3)} + O[\hbar]^2, -\frac{(b_1 b_2 (b_2 + 2 b_3)) \hbar}{12 ((b_2 + b_3) (b_1 + b_2 + b_3))} + O[\hbar]^2 \right\}$$

Series[

$$b_1 b_2^2 b_3^2 + b_1 b_2 b_3^3 + b_1 b_3^2 b_3 t_2 + b_1 b_2^2 b_3^2 t_2 - b_3^2 b_3^2 t_2 - b_2^2 b_3^3 t_2 + b_1 b_2^2 b_3^2 t_1^2 t_2 + b_2^2 b_3^2 t_1^2 t_2 + b_1 b_2 b_3^3 t_1^2 t_2 +$$

$$b_2^2 b_3^3 t_1^2 t_2 - b_1 b_3^2 b_3 t_2^2 - b_2^4 b_3 t_2^2 - 2 b_1 b_2^2 b_3^2 t_2^2 - 2 b_2^2 b_3^3 t_2^2 - b_1 b_2 b_3^3 t_2^2 - b_2^2 b_3^3 t_2^2 + b_1 b_3^2 b_3 t_1^2 t_2^2 +$$

$$b_2^4 b_3 t_1^2 t_2^2 + b_1 b_2^2 b_3^2 t_1^2 t_2^2 + 2 b_2^2 b_3^3 t_1^2 t_2^2 + b_2^2 b_3^3 t_1^2 t_2^2 - b_1 b_3^2 b_3 t_1^2 t_2^3 - 2 b_1 b_2^2 b_3^2 t_1^2 t_2^3 -$$

$$b_1 b_2 b_3^3 t_1^2 t_2^3 - b_1 b_3^2 b_3 t_2 t_2^3 - 2 b_1 b_2^2 b_3^2 t_2 t_2^3 - b_1 b_2 b_3^3 t_2 t_2^3 + b_1 b_3^2 b_3 t_2^2 t_2^3 +$$

$$b_2^4 b_3 t_2^2 t_2^3 + b_1 b_2^2 b_3^2 t_2^2 t_2^3 + 2 b_2^2 b_3^3 t_2^2 t_2^3 + b_2^2 b_3^3 t_2^2 t_2^3 - b_1 b_3^2 b_3 t_1^2 t_2^2 t_2^3 - b_2^4 b_3 t_1^2 t_2^2 t_2^3 -$$

$$2 b_1 b_2^2 b_3^2 t_1^2 t_2^2 t_2^3 - 2 b_2^2 b_3^3 t_1^2 t_2^2 t_2^3 - b_1 b_2 b_3^3 t_1^2 t_2^2 t_2^3 - b_2^2 b_3^3 t_1^2 t_2^2 t_2^3 + b_1 b_2^2 b_3^2 t_2^2 t_2^3 +$$

$$b_3^2 b_3^2 t_2^2 t_2^3 + b_1 b_2 b_3^3 t_2^2 t_2^3 + b_2^2 b_3^3 t_2^2 t_2^3 + b_1 b_3^2 b_3 t_1^2 t_2^3 t_2^3 + b_1 b_2^2 b_3^2 t_1^2 t_2^3 t_2^3 - b_3^2 b_3^2 t_1^2 t_2^3 t_2^3 -$$

$$b_2^2 b_3^3 t_1^2 t_2^3 t_2^3 + b_1 b_2^2 b_3^2 t_1^2 t_2^4 t_2^3 + b_1 b_2 b_3^3 t_1^2 t_2^4 t_2^3 /. t_{i-} \Rightarrow e^{\alpha b_i} /. b_{i-} \Rightarrow \hbar b_i, \{\hbar, 0, 9\}]$$

$$\frac{4}{3} (2 \alpha^4 b_1^3 b_2^4 b_3^3 + 3 \alpha^4 b_1^2 b_2^5 b_3^3 + \alpha^4 b_1 b_2^6 b_3^3 + 4 \alpha^4 b_1^3 b_2^3 b_3^3 +$$

$$6 \alpha^4 b_1^2 b_2^4 b_3^3 + 2 \alpha^4 b_1 b_2^5 b_3^3 + 2 \alpha^4 b_1^3 b_2^2 b_3^4 + 3 \alpha^4 b_1^2 b_2^3 b_3^4 + \alpha^4 b_1 b_2^4 b_3^4) \hbar^9 + O[\hbar]^{10}$$

Series[$\frac{b_1 c_2}{2} +$

$$\left((b_2 t_1 - b_1 t_1^2 - b_2 t_1^2 + b_1 t_1^2 t_2^2 + b_2 t_1^2 t_2^2 - b_2 t_1^3 t_2^2) u_1 w_1 \right) / (-b_1^2 - b_1 b_2 + b_1^2 t_1^2 t_2^2 + b_1 b_2 t_1^2 t_2^2) +$$

$$\left((-b_2 + b_1 t_1 + b_2 t_1 - b_1 t_1 t_2^2 - b_2 t_1 t_2^2 + b_2 t_1^2 t_2^2) u_2 w_1 \right) / (-b_1 b_2 - b_2^2 + b_1 b_2 t_1^2 t_2^2 + b_2^2 t_1^2 t_2^2) +$$

$$\left((b_1 + b_2 - b_1 t_1 - b_1 t_1^2 - b_2 t_1^2 + b_1 t_1^3 t_2^2) u_1 w_2 \right) / (-b_1^2 - b_1 b_2 + b_1^2 t_1^2 t_2^2 + b_1 b_2 t_1^2 t_2^2) +$$

$$\left((-b_2 + b_1 t_1 + b_2 t_1 - b_1 t_1 t_2^2 - b_2 t_1 t_2^2 + b_2 t_1^2 t_2^2) u_2 w_2 \right) / (-b_1 b_2 - b_2^2 + b_1 b_2 t_1^2 t_2^2 + b_2^2 t_1^2 t_2^2) /. t_{i-} \Rightarrow e^{b_i/2} /. \{b_{i-} \Rightarrow \hbar b_i, u_{i-} \Rightarrow \hbar u_i\}, \{\hbar, 0, 2\}] // \text{Simplify}$$

$$\frac{1}{2} (b_1 c_2 + u_1 w_2) \hbar + \frac{1}{24 (b_1 + b_2)}$$

$$(-2 b_2^2 u_1 (w_1 + w_2) + b_1^2 (3 u_1 w_2 + u_2 (w_1 + w_2)) + b_1 b_2 (-u_1 (w_1 - 2 w_2) + 2 u_2 (w_1 + w_2))) \hbar^2 + O[\hbar]^3$$

$$\frac{1}{24 (b_1 + b_2)}$$

$$(-2 b_2^2 u_1 (w_1 + w_2) + b_1^2 (3 u_1 w_2 + u_2 (w_1 + w_2)) + b_1 b_2 (-u_1 (w_1 - 2 w_2) + 2 u_2 (w_1 + w_2))) \hbar^2 // \text{Together}$$

$$\frac{1}{24 (b_1 + b_2)} \hbar^2 (-b_1 b_2 u_1 w_1 - 2 b_2^2 u_1 w_1 + b_1^2 u_2 w_1 +$$

$$2 b_1 b_2 u_2 w_1 + 3 b_1^2 u_1 w_2 + 2 b_1 b_2 u_1 w_2 - 2 b_2^2 u_1 w_2 + b_1^2 u_2 w_2 + 2 b_1 b_2 u_2 w_2)$$

Series [

$$\begin{aligned} & (-b_1^3 b_2 c_2 - b_1^2 b_2^2 c_2 + b_1^3 b_2 c_2 t_1^2 t_2^2 + b_1^2 b_2^2 c_2 t_1^2 t_2^2 + 2 h^2 b_2^2 t_1 u_1 w_1 - 2 b_1 b_2 t_1^2 u_1 w_1 - 2 b_2^2 t_1^2 u_1 w_1 + \\ & 2 b_1 b_2 t_1^2 t_2^2 u_1 w_1 + 2 b_2^2 t_1^2 t_2^2 u_1 w_1 - 2 h^2 b_2^2 t_1^3 t_2^2 u_1 w_1 - 2 h^2 b_1 b_2 u_2 w_1 + 2 b_1^2 t_1 u_2 w_1 + \\ & 2 b_1 b_2 t_1 u_2 w_1 - 2 b_1^2 t_1 t_2^2 u_2 w_1 - 2 b_1 b_2 t_1 t_2^2 u_2 w_1 + 2 h^2 b_1 b_2 t_1^2 t_2^2 u_2 w_1 + 2 b_1 b_2 u_1 w_2 + \\ & 2 b_2^2 u_1 w_2 - 2 h^2 b_1 b_2 t_1 u_1 w_2 - 2 b_1 b_2 t_1^2 u_1 w_2 - 2 b_2^2 t_1^2 u_1 w_2 + 2 h^2 b_1 b_2 t_1^3 t_2^2 u_1 w_2 - \\ & 2 b_1^2 u_2 w_2 + 2 h^2 b_1^2 u_2 w_2 - 2 b_1 b_2 u_2 w_2 + 2 b_1^2 t_1 u_2 w_2 + 2 b_1 b_2 t_1 u_2 w_2 - 2 b_1^2 t_1 t_2^2 u_2 w_2 - \\ & 2 b_1 b_2 t_1 t_2^2 u_2 w_2 + 2 b_1^2 t_1^2 t_2^2 u_2 w_2 - 2 h^2 b_1^2 t_1^2 t_2^2 u_2 w_2 + 2 b_1 b_2 t_1^2 t_2^2 u_2 w_2) / \\ & (-2 b_1^2 b_2 - 2 b_1 b_2^2 + 2 b_1^2 b_2 t_1^2 t_2^2 + 2 b_1 b_2^2 t_1^2 t_2^2) /. \end{aligned}$$

$$h \rightarrow \text{Sum}[h_{j,d-j} b_1^j b_2^{d-j}, \{d, 0, 3\}, \{j, 0, d\}] /. t_{i_} \Rightarrow e^{b_i/2} /.$$

$$\{b_{i_} \Rightarrow \hbar b_i, u_{i_} \Rightarrow \hbar u_i\} /. \{h_{0,0} \rightarrow 1\}, \{\hbar, 0, 1\} // \text{Simplify}$$

$$\frac{1}{2 b_1 b_2 (b_1 + b_2)}$$

$$\begin{aligned} & (-4 b_2^3 u_1 w_1 h_{0,1} + b_1^3 (b_2 c_2 - 4 u_2 w_2 h_{1,0}) + b_1 b_2^2 (4 u_2 w_1 h_{0,1} + u_1 (w_2 (1 + 4 h_{0,1}) - 4 w_1 h_{1,0})) + \\ & b_1^2 b_2 (b_2 c_2 + u_1 w_2 (1 + 4 h_{1,0}) + 4 u_2 (-w_2 h_{0,1} + w_1 h_{1,0}))) \hbar + O[\hbar]^2 \end{aligned}$$