Pensieve header: The non-existence of null twists in Aw.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2016-11"];
<< ../Projects/WKO4/FreeLie.m;
<< ../Projects/WKO4/AwCalculus.m;
$SeriesShowDegree = 2;
```

```
FreeLie` implements / extends
 {*, +, **, $SeriesShowDegree, ⟨⟩, ∫, ≡, ad, Ad, adSeries, AllCyclicWords, AllLyndonWords,
  AllWords, Arbitrator, ASeries, AW, b, BCH, BooleanSequence, BracketForm, BS, CC, Crop, cw,
  CW, CWS, CWSeries, D, Deg, DegreeScale, DerivationSeries, div, DK, DKS, DKSeries, EulerE,
  Exp, Inverse, j, J, JA, LieDerivation, LieMorphism, LieSeries, LS, LW, LyndonFactorization,
  Morphism, New, RandomCWSeries, Randomizer, RandomLieSeries, RC, SeriesSolve, Support,
  t, tb, TopBracketForm, tr, UndeterminedCoefficients, αMap, Γ, ι, Λ, σ, ħ, ⟝, ⟞}.

FreeLie` is in the public domain. Dror Bar-Natan is committed
  to support it within reason until July 15, 2022. This is version 150814.

AwCalculus` implements / extends {*, **, ≡, dA, dc, deg, dm, dS, dΔ, dη, dσ, El, Es, hA,
  hm, hS, hΔ, hη, hσ, RandomElSeries, RandomEsSeries, tA, tha, tm, tS, tΔ, tη, tσ, Γ, Λ}.

AwCalculus` is in the public domain. Dror Bar-Natan is committed
  to support it within reason until July 15, 2022. This is version 150909.
```

```
x = LW["x"]; y = LW["y"]; z = LW["z"];
αs["x"] = αs["y"] = βs["y"] = γs["x"] = γs["y"] = 0;
βs["x"] = 1;
α = LS[{x, y}, αs]; β = LS[{x, y}, βs]; γ = CWS[{x, y}, γs];
F = Es[⟨x → α, y → β⟩, γ];
```

```
msgs = SeriesSolve[{α, β, γ},
   F ** (F // dσ[y, z] // dΔ[x, x, y]) ≡ (F // dσ[{x, y} -> {y, z}]) ** (F // dΔ[y, y, z])];
F@
 {4}
```

SeriesSolve: No solution in degree 2.

```
$Aborted
```

```
F ** (F // dσ[y, z] // dΔ[x, x, y])
```

$$\mathrm{Es}\Big[\big\langle \overline{x} \to \mathrm{LS}\big[0, \overline{xy}\,\alpha s[x,y] + \overline{xz}\,\alpha s[x,y] + \overline{yz}\,\alpha s[x,y], \ldots\big],\ \overline{y} \to \mathrm{LS}\big[\overline{x}, \overline{xz}\,\alpha s[x,y] + \overline{yz}\,\alpha s[x,y] + \overline{xy}\,\beta s[x,y], \ldots\big],\ \overline{z} \to \mathrm{LS}\big[\overline{x}+\overline{y}, \overline{xz}\,\beta s[x,y] + \overline{yz}\,\beta s[x,y], \ldots\big]\big\rangle,\ \mathrm{CWS}\big[0, 2\,\widehat{xx}\,\gamma s[x,x] + 2\,\widehat{xy}\,\gamma s[x,x] + \widehat{yy}\,\gamma s[x,x] + \widehat{xy}\,\gamma s[x,y] + \widehat{xz}\,\gamma s[x,y] + \widehat{yz}\,\gamma s[x,y] + \widehat{yy}\,\gamma s[y,y] + \widehat{zz}\,\gamma s[y,y], \ldots\big]\Big]$$

```
(F // dσ[{x, y} -> {y, z}]) ** (F // dΔ[y, y, z])
```

$$\text{Es}\left[\left\langle \overline{x} \to \text{LS}\left[0, \overline{xy}\,\alpha s[x, y] + \overline{xz}\,\alpha s[x, y], \ldots\right],\right.$$
$$\overline{y} \to \text{LS}\left[\overline{x}, \overline{yz}\,\alpha s[x, y] + \overline{xy}\,\beta s[x, y] + \overline{xz}\,\beta s[x, y], \ldots\right],$$
$$\left.\overline{z} \to \text{LS}\left[\overline{x} + \overline{y}, \frac{\overline{xy}}{2} + \overline{xy}\,\beta s[x, y] + \overline{xz}\,\beta s[x, y] + \overline{yz}\,\beta s[x, y], \ldots\right]\right\rangle,$$
$$\text{CWS}\left[0, \widehat{xx}\,\gamma s[x, x] + \widehat{yy}\,\gamma s[x, x] + \widehat{xy}\,\gamma s[x, y] + \widehat{xz}\,\gamma s[x, y] +\right.$$
$$\left.\widehat{yz}\,\gamma s[x, y] + \widehat{yy}\,\gamma s[y, y] + 2\,\widehat{yz}\,\gamma s[y, y] + 2\,\widehat{zz}\,\gamma s[y, y], \ldots\right]\Big]$$

```
F1 = Es[⟨x → LS[0], y → LS[x]⟩, CWS[0]];
F1@{4}
```

$$\text{Es}\left[\left\langle \overline{x} \to \text{LS}[0, 0, 0, 0, \ldots], \overline{y} \to \text{LS}\left[\overline{x}, 0, 0, 0, \ldots\right]\right\rangle, \text{CWS}[0, 0, 0, 0, \ldots]\right]$$

```
(F1 // dσ[{x, y} -> {y, z}])@{3}
```

$$\text{Es}\left[\left\langle \overline{y} \to \text{LS}[0, 0, 0, \ldots], \overline{z} \to \text{LS}\left[\overline{y}, 0, 0, \ldots\right]\right\rangle, \text{CWS}[0, 0, 0, \ldots]\right]$$

```
(F1 // dΔ[y, y, z])@{3}
```

$$\text{Es}\left[\left\langle \overline{x} \to \text{LS}[0, 0, 0, \ldots], \overline{y} \to \text{LS}\left[\overline{x}, 0, 0, \ldots\right], \overline{z} \to \text{LS}\left[\overline{x}, 0, 0, \ldots\right]\right\rangle, \text{CWS}[0, 0, 0, \ldots]\right]$$

```
t1 = (F1 // dσ[{x, y} -> {y, z}]) ** (F1 // dΔ[y, y, z])
```

$$\text{Es}\left[\left\langle \overline{x} \to \text{LS}[0, 0, \ldots], \overline{y} \to \text{LS}\left[\overline{x}, 0, \ldots\right], \overline{z} \to \text{LS}\left[\overline{x} + \overline{y}, \overline{\frac{xy}{2}}, \ldots\right]\right\rangle, \text{CWS}[0, 0, \ldots]\right]$$

```
t1@{3}
```

$$\text{Es}\left[\left\langle \overline{x} \to \text{LS}[0, 0, 0, \ldots], \overline{y} \to \text{LS}\left[\overline{x}, 0, 0, \ldots\right],\right.\right.$$
$$\left.\left.\overline{z} \to \text{LS}\left[\overline{x} + \overline{y}, \overline{\frac{xy}{2}}, \frac{1}{12}\,\overline{x\,\overline{xy}} + \frac{1}{12}\,\overline{\overline{xy}\,y}, \ldots\right]\right\rangle, \text{CWS}[0, 0, 0, \ldots]\right]$$

```
Λ[t1]@{4}
```

$$\text{El}\left[\left\langle \overline{x} \to \text{LS}[0, 0, 0, 0, \ldots], \overline{y} \to \text{LS}\left[\overline{x}, 0, 0, 0, \ldots\right], \overline{z} \to \text{LS}\left[\overline{x} + \overline{y}, 0, 0, 0, \ldots\right]\right\rangle,\right.$$
$$\left.\text{CWS}[0, 0, 0, 0, \ldots]\right]$$

```
t2 = (F1 // dΔ[y, y, z]) ** (F1 // dσ[{x, y} -> {y, z}])
```

$$\text{Es}\left[\left\langle \overline{x} \to \text{LS}[0, 0, \ldots], \overline{y} \to \text{LS}\left[\overline{x}, 0, \ldots\right], \overline{z} \to \text{LS}\left[\overline{x} + \overline{y}, \overline{\frac{xy}{2}}, \ldots\right]\right\rangle, \text{CWS}[0, 0, \ldots]\right]$$

```
(t1 ≡ t2)@{6}
```

$$\text{BS}[7\ \text{True}, \ldots]$$

```
F2 = Λ[F1]
```

$$\text{El}\left[\left\langle \overline{x} \to \text{LS}[0, 0, \ldots], \overline{y} \to \text{LS}\left[\overline{x}, 0, \ldots\right]\right\rangle, \text{CWS}[0, 0, \ldots]\right]$$

```
t3 = (F2 // dσ[{x, y} -> {y, z}]) ** (F2 // dΔ[y, y, z])
```

$$h\sigma\left[\left\{\overline{x}, \overline{y}\right\} \to \left\{\overline{y}, \overline{z}\right\}\right]\left[\right.$$
$$t\sigma\left[\left\{\overline{x}, \overline{y}\right\} \to \left\{\overline{y}, \overline{z}\right\}\right]\left[\text{El}\left[\left\langle \overline{x} \to \text{LS}[0, 0, \ldots], \overline{y} \to \text{LS}\left[\overline{x}, 0, \ldots\right]\right\rangle, \text{CWS}[0, 0, \ldots]\right]\right]\Big] **$$
$$\text{El}\left[\left\langle \overline{x} \to \text{LS}[0, 0, \ldots], \overline{y} \to \text{LS}\left[\overline{x}, 0, \ldots\right], \overline{z} \to \text{LS}\left[\overline{x}, 0, \ldots\right]\right\rangle, \text{CWS}[0, 0, \ldots]\right]$$