Pensieve header: Finding a V leading to a trivial Φ, with Huan Vo.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2016-11"];
```

```
CF[expr_] := (expr /. e^x_ :> e^Simplify[x /. b_i_ :> 2 Log[t_i]]) // Together // ExpandNumerator // ExpandDenominator;
𝔼 /: CF[𝔼[ω_, Q_]] := 𝔼[CF[ω], CF[Q]];
𝔼 /: 𝔼[ω1_, Q1_] 𝔼[ω2_, Q2_] := CF@𝔼[ω1 ω2, Q1 + Q2];
𝔼[ω1_, Q1_] ≡ 𝔼[ω2_, Q2_] := CF[ω1 == ω2 ∧ Q1 == Q2];
```

```
N_{u_i_ c_j_→k_}[𝔼[ω_, Q_]] := CF[
    𝔼[ω, e^-γ β u_k + γ c_k + (Q /. c_j | u_i → 0)] /. {γ → ∂_{c_j} Q, β → ∂_{u_i} Q}];
N_{w_i_ c_j_→k_}[𝔼[ω_, Q_]] := CF[
    𝔼[ω, e^γ α w_k + γ c_k + (Q /. c_j | w_i → 0)] /. {γ → ∂_{c_j} Q, α → ∂_{w_i} Q}];
N_{w_i_ u_j_→k_}[𝔼[ω_, Q_]] := CF[
    𝔼[ν ω, -b_k ν α β + ν β u_k + ν δ u_k w_k + ν α w_k + (Q /. w_i | u_j → 0)] /. ν → (1 + b_k δ)^-1 /.
     {α → ∂_{w_i} Q /. u_j → 0, β → ∂_{u_j} Q /. w_i → 0, δ → ∂_{w_i,u_j} Q}];
```

```
m_i_,j_→k_[𝔼[ω_, Q_]] := CF[Module[{x},
    (𝔼[ω, Q] /. {b_i|j → b_k, t_i|j → t_k} // N_{w_i c_j→x} // N_{u_i c_x→x} // N_{w_x u_j→x}) /. {c_i → c_k, w_j → w_k, y_x :> y_k}]]
```

```
R_i_,j_[p_] := 𝔼[1, p b_i c_j + b_i^-1 (e^p b_i - 1) u_i w_j] // CF;
R^+_i_,j_ := R_i,j[1]; R^-_i_,j_ := R_i,j[-1];
```

```
𝔼 /: (e_𝔼)^σ[s___List] := CF[e /. Flatten@Table[{
        b_k → Total[b_# & /@ {s}[[k]]], c_k → Total[c_# & /@ {s}[[k]]],
        u_k → Total[u_# & /@ {s}[[k]]], w_k → Total[w_# & /@ {s}[[k]]], t_k → Times @@ (t_# & /@ {s}[[k]])
       }, {k, 1, Length@{s}}]];
σ[l___, k_Integer, r___] := σ[l, {k}, r];
```

```
𝔼 /: e1_𝔼 ** e2_𝔼 := Module[{λ, ν, e},
  λ = (Union@Cases[e1, (b | t | c | u | w)_k_ :> k, ∞]) ⋂ (Union@Cases[e2, (b | t | c | u | w)_k_ :> k, ∞]);
  ν = Table[Unique[], {Length@λ}];
  e = e1 (e2 /.
      Flatten@Table[{b_λ[[k]] → b_ν[[k]], t_λ[[k]] → t_ν[[k]], c_λ[[k]] → c_ν[[k]], u_λ[[k]] → u_ν[[k]], w_λ[[k]] → w_ν[[k]]}, {k, Length@λ}]);
  Do[e = m_{λ[[k]],ν[[k]]→λ[[k]]}[e], {k, Length@λ}];
  e
 ]
```

```
A[n_][e_𝔼] := (e /. {c_n → -c_n, w_n → -w_n}) // N_{u_n c_n→n} // N_{w_n c_n→n} // N_{w_n u_n→n};
A[][e_𝔼] := e;
A[n_, r___][e_𝔼] := e // A[n] // A[r];
```

```
Θ[i_][e_𝔼] := (e /. {b_i :> -b_i, c_i :> -c_i, u_i :> w_i, w_i :> u_i, t_i :> t_i^-1}) // CF // N_{w_i u_i→i};
Θ[][e_𝔼] := e;
Θ[n_, r___][e_𝔼] := e // Θ[n] // Θ[r];
```

```
f_11 = f_21 = f_22 = 0; f_12 = 1/2;
V = 𝔼[h[b_1, b_2, t_1, t_2], b_1 c_1 f_11 + b_1 c_2 f_12 + b_2 c_1 f_21 + b_2 c_2 f_22 + u_1 w_1 g_11[b_1, b_2, t_1, t_2] +
     u_1 w_2 g_12[b_1, b_2, t_1, t_2] + u_2 w_1 g_21[b_1, b_2, t_1, t_2] + u_2 w_2 g_22[b_1, b_2, t_1, t_2]];
fi_11 = fi_21 = fi_22 = 0; fi_12 = -1/2;
Vi = 𝔼[hi[b_1, b_2, t_1, t_2], b_1 c_1 fi_11 + b_1 c_2 fi_12 + b_2 c_1 fi_21 + b_2 c_2 fi_22 + u_1 w_1 gi_11[b_1, b_2, t_1, t_2] +
     u_1 w_2 gi_12[b_1, b_2, t_1, t_2] + u_2 w_1 gi_21[b_1, b_2, t_1, t_2] + u_2 w_2 gi_22[b_1, b_2, t_1, t_2]];
```

$$\left\{g_{12} \to \frac{gi_{12}\, t_1 \left(-1 + b_1\, b_2\, g_{21}\, gi_{12}\, t_1\right)}{\left(-1 + b_1\, gi_{11}\right)\left(-1 + b_2\, gi_{22}\right)},\ g_{22} \to \frac{gi_{22} - b_1\, g_{21}\, gi_{12}\, t_1}{-1 + b_2\, gi_{22}},\ gi_{21} \to \frac{g_{21}\left(-1 + b_1\, gi_{11}\right)\left(-1 + b_2\, gi_{22}\right) t_1}{-1 + b_1\, b_2\, g_{21}\, gi_{12}\, t_1},\right.$$
$$\left. g_{11} \to \frac{gi_{11} - b_2\, g_{21}\, gi_{12}\, t_1}{-1 + b_1\, gi_{11}}\right\} /.\ \{g_{j\_} :\to g_j[b_1, b_2, t_1, t_2],\ gi_{j\_} :\to gi_j[b_1, b_2, t_1, t_2]\} /.\ \text{Rule} \to \text{Set};$$

```
hi[b1, b2, t1, t2] = h[b1, b2, t1, t2]^-1;
```

```
V
```

$$\mathbb{E}\Big[h[b_1, b_2, t_1, t_2],\ \frac{b_1 c_2}{2} + u_2 w_1 g_{21}[b_1, b_2, t_1, t_2] + \left(u_1 w_1 \left(gi_{11}[b_1, b_2, t_1, t_2] - b_2 t_1 g_{21}[b_1, b_2, t_1, t_2] gi_{12}[b_1, b_2, t_1, t_2]\right)\right) / \left(-1 + b_1 gi_{11}[b_1, b_2, t_1, t_2]\right) + \left(t_1 u_1 w_2 gi_{12}[b_1, b_2, t_1, t_2] \left(-1 + b_1 b_2 t_1 g_{21}[b_1, b_2, t_1, t_2] gi_{12}[b_1, b_2, t_1, t_2]\right)\right) / \left(\left(-1 + b_1 gi_{11}[b_1, b_2, t_1, t_2]\right)\left(-1 + b_2 gi_{22}[b_1, b_2, t_1, t_2]\right)\right) + \left(u_2 w_2 \left(-b_1 t_1 g_{21}[b_1, b_2, t_1, t_2] gi_{12}[b_1, b_2, t_1, t_2] + gi_{22}[b_1, b_2, t_1, t_2]\right)\right) / \left(-1 + b_2 gi_{22}[b_1, b_2, t_1, t_2]\right)\Big]$$

```
Eqn1 = (V ** Vi ≡ 𝔼[1, 0])
```

True

```
Φ = Vi^σ[{1,2},3] ** Vi^σ[1,2] ** V^σ[2,3] ** V^σ[1,{2,3}]
```

$$\mathbb{E}\Big[\frac{h[b_1,b_2+b_3,t_1,t_2\,t_3]\,h[b_2,b_3,t_2,t_3]}{h[b_1,b_2,t_1,t_2]\,h[b_1+b_2,b_3,t_1\,t_2,t_3]},$$
$$\left(-u_2 w_1 g_{21}[b_1, b_2, t_1, t_2] + \text{··· 27 789 ···} + b_1^2 b_2^5 b_3^4 t_1 t_2 u_3 w_1 g_{21}[b_1, b_2, t_1, t_2] g_{21}[b_1, b_2 + b_3, t_1, t_2\, t_3]\right.$$
$$g_{21}[b_2, b_3, t_2, t_3] g_{21}[b_1 + b_2, b_3, t_1\, t_2, t_3] gi_{11}[b_1, b_2 + b_3, t_1, t_2\, t_3]$$
$$gi_{11}[b_1 + b_2, b_3, t_1\, t_2, t_3] gi_{12}[b_1, b_2, t_1, t_2] gi_{12}[b_2, b_3, t_2, t_3] gi_{22}[b_1, b_2, t_1, t_2]$$
$$\left.gi_{22}[b_1, b_2 + b_3, t_1, t_2\, t_3] gi_{22}[b_2, b_3, t_2, t_3] gi_{22}[b_1 + b_2, b_3, t_1\, t_2, t_3]\right) /$$
$$\left(1 - b_1 gi_{11}[b_1, b_2 + b_3, t_1, t_2\, t_3] - b_2 gi_{11}\left[b_2, \text{··· 2 ···}, t_3\right] + \text{··· 208 ···} + \text{··· 1 ···} + \right.$$
$$b_1^3 b_2^2 b_3^3 \text{··· 8 ···} gi_{22}[b_1, b_2 + b_3, t_1, t_2\, t_3] gi_{22}[b_2, b_3, t_2, t_3] +$$
$$b_1^2 b_2^3 b_3^3 t_1^2 t_2 g_{21}[b_1, b_2, t_1, t_2] g_{21}[b_1 + b_2, b_3, t_1\, t_2, t_3] gi_{11}[b_1, b_2 + b_3, t_1, t_2\, t_3] gi_{11}[b_2, b_3, t_2, t_3]$$
$$\left.gi_{12}[b_1, b_2, t_1, t_2] gi_{12}[b_1 + b_2, b_3, t_1\, t_2, t_3] gi_{22}[b_1, b_2 + b_3, t_1, t_2\, t_3] gi_{22}[b_2, b_3, t_2, t_3]\right)\Big]$$

large output | show less | show more | show all | set size limit...

```
Table[FactorList@Numerator[∂_{u_i,w_j} (Φ[[2]])], {i, 1}, {j, 1}]
```

... Power: Infinite expression $\frac{1}{0}$ encountered.

{{{{0, 1}}}}

```
-1 + b1 b3 t1 t2 g21[b1 + b2, b3, t1 t2, t3] gi12[b1 + b2, b3, t1 t2, t3] +
  b2 b3 t1 t2 g21[b1 + b2, b3, t1 t2, t3] gi12[b1 + b2, b3, t1 t2, t3] // Simplify
```

0

```
g21[b1_, b2_, t1_, t2_] := 1/(t1 b1 b2); gi12[b1_, b2_, t1_, t2_] := 1
```