

Pensieve header: Finding a  $\$V\$$  satisfying R4, Twist, and Unitarity, with Huan Vo.

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SetDirectory["C:\\drorbn\\AcademicPensieve\\2016-11"];
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CF[expr_] := (expr /. e^x_ -> e^Simplify[x /. b_i -> 2 Log[t_i]]) // Together // ExpandNumerator // ExpandDenominator;
E /: CF[E[ω_, Q_]] := E[CF[ω], CF[Q]];
E /: E[ω1_, Q1_] E[ω2_, Q2_] := CF@E[ω1 ω2, Q1 + Q2];
E[ω1_, Q1_] ≡ E[ω2_, Q2_] := CF[ω1 == ω2 ∧ Q1 == Q2];
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```
Nu_i c_j -> k_ [E[ω_, Q_]] := CF[
  E[ω, e^-y β u_k + γ c_k + (Q /. c_j | u_i -> 0)] /. {γ -> ∂_c_j Q, β -> ∂_u_i Q};
Nw_i c_j -> k_ [E[ω_, Q_]] := CF[
  E[ω, e^y α w_k + γ c_k + (Q /. c_j | w_i -> 0)] /. {γ -> ∂_c_j Q, α -> ∂_w_i Q};
Nw_i u_j -> k_ [E[ω_, Q_]] := CF[
  E[v ω, -b_k v α β + v β u_k + v δ u_k w_k + v α w_k + (Q /. w_i | u_j -> 0)] /. v -> (1 + b_k δ)^-1 /.
  {α -> ∂_w_i Q /. u_j -> 0, β -> ∂_u_j Q /. w_i -> 0, δ -> ∂_w_i u_j Q};
```

```
m_i, j -> k_ [E[ω_, Q_]] := CF[Module[{x},
  (E[ω, Q] /. {b_i | j -> b_k, t_i | j -> t_k} // Nw_i c_j -> x // Nu_i c_x -> x // Nw_k u_j -> x) /. {c_i -> c_k, w_j -> w_k, y_x -> y_k}]]
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```
R_i, j [p_] := E[1, p b_i c_j + b_i^-1 (e^p b_i - 1) u_i w_j] // CF;
R_i, j^+ := R_i, j [1]; R_i, j^- := R_i, j [-1];
```

```
R_{1,2}^+ R_{3,4}^+ R_{5,6}^+ // m_{3,5 -> x} // m_{1,6 -> y} // m_{2,4 -> z}
```

$$E\left[1, \frac{1}{b_x b_y} (b_x^2 b_y c_y + b_x^2 b_y c_z + b_x b_y^2 c_z - b_y u_x w_y + b_y t_x^2 u_x w_y - b_y u_x w_z + b_y t_x^2 u_x w_z - b_x u_y w_z + b_x t_y^2 u_y w_z)\right]$$

```
R_{1,2}^+ R_{3,4}^+ R_{5,6}^+ // m_{1,3 -> x} // m_{2,5 -> y} // m_{4,6 -> z}
```

$$E\left[1, \frac{1}{b_x b_y} (b_x^2 b_y c_y + b_x^2 b_y c_z + b_x b_y^2 c_z - b_y u_x w_y + b_y t_x^2 u_x w_y - b_y u_x w_z + b_y t_x^2 u_x w_z - b_x u_y w_z + b_x t_y^2 u_y w_z)\right]$$

```
z1 = R_{12,1}^- R_{2,7}^- R_{8,3}^- R_{4,11}^- R_{16,5}^+ R_{6,13}^+ R_{14,9}^+ R_{10,15}^+;
```

```
Do[z1 = (z1 // m_{1,n -> 1}) /. b_i -> 2 Log[t_i], {n, 2, 16}];
```

```
{CF@z1, KnotData[{8, 17}, "AlexanderPolynomial"][t]}
```

$$\left\{E\left[-\frac{t_1^6}{1 - 4 t_1^2 + 8 t_1^4 - 11 t_1^6 + 8 t_1^8 - 4 t_1^{10} + t_1^{12}}, 0\right], 11 - \frac{1}{t^3} + \frac{4}{t^2} - \frac{8}{t} - 8 t + 4 t^2 - t^3\right\}$$

```
E /: (e^-E)^σ[s___List] := CF[e /. Flatten@Table[{
  b_k -> Total[b_# & /@ {s}][[k]], c_k -> Total[c_# & /@ {s}][[k]],
  u_k -> Total[u_# & /@ {s}][[k]], w_k -> Total[w_# & /@ {s}][[k]], t_k -> Times@@(t_# & /@ {s}][[k]]
}, {k, 1, Length@{s}}]];
σ[L___, k_Integer, r___] := σ[L, {k}, r];
```

```
(R_{1,2}^+)^σ[{1,2},3]
```

$$E\left[1, \frac{1}{b_1 + b_2} (b_1^2 c_3 + 2 b_1 b_2 c_3 + b_2^2 c_3 - u_1 w_3 + t_1^2 t_2^2 u_1 w_3 - u_2 w_3 + t_1^2 t_2^2 u_2 w_3)\right]$$

```
CF[R_{1,3}^+ /. {t_1 -> t_1 t_2, x_{-1} -> x_1 + x_2}]
```

$$E\left[1, \frac{1}{b_1 + b_2} (b_1^2 c_3 + 2 b_1 b_2 c_3 + b_2^2 c_3 - u_1 w_3 + t_1^2 t_2^2 u_1 w_3 - u_2 w_3 + t_1^2 t_2^2 u_2 w_3)\right]$$

```

E /: e1_E ** e2_E := Module[{λ, v, e},
  λ = (Union@Cases[e1, (b | t | c | u | w)_k_ -> k, ∞]) ∩ (Union@Cases[e2, (b | t | c | u | w)_k_ -> k, ∞]);
  v = Table[Unique[], {Length@λ}];
  e = e1 (e2 /.
    Flatten@Table[{b_λ[[k]] -> b_v[[k]], t_λ[[k]] -> t_v[[k]], c_λ[[k]] -> c_v[[k]], u_λ[[k]] -> u_v[[k]], w_λ[[k]] -> w_v[[k]]}, {k, Length@λ}]);
  Do[e = m_λ[[k], v[[k]] -> λ[[k]] [e], {k, Length@λ}];
  e
]

```

$$R_{1,2}^+ ** R_{1,3}^+ ** R_{2,3}^+$$

$$E\left[1, \frac{1}{b_1 b_2} (b_1^2 b_2 c_2 + b_1^2 b_2 c_3 + b_1 b_2^2 c_3 - b_2 u_1 w_2 + b_2 t_1^2 u_1 w_2 - b_2 u_1 w_3 + b_2 t_1^2 u_1 w_3 - b_1 u_2 w_3 + b_1 t_2^2 u_2 w_3)\right]$$

$$R_{2,3}^+ ** R_{1,3}^+ ** R_{1,2}^+$$

$$E\left[1, \frac{1}{b_1 b_2} (b_1^2 b_2 c_2 + b_1^2 b_2 c_3 + b_1 b_2^2 c_3 - b_2 u_1 w_2 + b_2 t_1^2 u_1 w_2 - b_2 u_1 w_3 + b_2 t_1^2 u_1 w_3 - b_1 u_2 w_3 + b_1 t_2^2 u_2 w_3)\right]$$

$$f_{11} = f_{21} = f_{22} = 0; f_{12} = 1/2;$$

$$V = E[h[b_1, b_2, t_1, t_2], b_1 c_1 f_{11} + b_1 c_2 f_{12} + b_2 c_1 f_{21} + b_2 c_2 f_{22} +$$

$$u_1 w_1 g_{11}[b_1, b_2, t_1, t_2] + u_1 w_2 g_{12}[b_1, b_2, t_1, t_2] + u_2 w_1 g_{21}[b_1, b_2, t_1, t_2] + u_2 w_2 g_{22}[b_1, b_2, t_1, t_2]]$$

$$E[h[b_1, b_2, t_1, t_2],$$

$$\frac{b_1 c_2}{2} + u_1 w_1 g_{11}[b_1, b_2, t_1, t_2] + u_1 w_2 g_{12}[b_1, b_2, t_1, t_2] + u_2 w_1 g_{21}[b_1, b_2, t_1, t_2] + u_2 w_2 g_{22}[b_1, b_2, t_1, t_2]]$$

$$\text{Eqn1} = \text{Simplify}\left[\left(V ** (R_{1,2}^+)^{\sigma[\{1,2\},3]}\right) \equiv (R_{2,3}^+ ** R_{1,3}^+ ** V)\right]$$

$$\frac{1}{b_1 b_2 (b_1 + b_2)} w_3 (b_1^2 t_1 (-1 + t_2^2) u_2 + b_1 b_2 (u_2 + t_1 (-1 + t_2^2) u_2 - b_1 u_1 g_{11}[b_1, b_2, t_1, t_2] - b_1 u_2 g_{21}[b_1, b_2, t_1, t_2] + t_1^2 (u_1 (1 + t_2^2 (-1 + b_1 g_{11}[b_1, b_2, t_1, t_2])) + t_2^2 u_2 (-1 + b_1 g_{21}[b_1, b_2, t_1, t_2]))) + b_2^2 (u_1 (-1 - b_1 g_{12}[b_1, b_2, t_1, t_2] + t_1^2 (1 + b_1 t_2^2 g_{12}[b_1, b_2, t_1, t_2])) + b_1 (-1 + t_1^2 t_2^2) u_2 g_{22}[b_1, b_2, t_1, t_2])) = 0$$

$$uR_{i,j} := (R_{1,2}[1/2] // m_{1,2 \rightarrow 1})^{\sigma[\{i,j\}]} ** (R_{1,2}[-1/2] // m_{1,2 \rightarrow 1})^{\sigma[\{i\}]} ** (R_{1,2}[-1/2] // m_{1,2 \rightarrow 1})^{\sigma[\{j\}]}$$

$$\text{Simplify} / @ uR_{i,j}$$

$$E\left[1, \frac{1}{2 b_i b_j (b_i + b_j) t_i t_j} (b_i^3 b_j c_j t_i t_j - 2 b_j^2 (-1 + t_i) t_i t_j u_i w_i + b_i^2 t_i t_j (b_j^2 (c_i + c_j) - 2 (-1 + t_j) u_j w_j) + b_i b_j (b_j^2 c_i t_i t_j + 2 (t_i^2 t_j u_i w_j - t_j u_j (w_i + w_j) + t_i (u_i ((-1 + t_j) w_i - w_j) + t_j u_j (t_j w_i + w_j))))\right]$$

$$\text{Eqn2} = (V ** uR_{1,2}) \equiv (R_{1,2}^+ ** V^{\sigma[2,1]})$$

$$h[b_1, b_2, t_1, t_2] = h[b_2, b_1, t_2, t_1] \&\&$$

$$\frac{1}{2 b_1^2 b_2 t_1 t_2 + 2 b_1 b_2^2 t_1 t_2} (b_1^2 b_2^2 c_1 t_1 t_2 + b_1 b_2^3 c_1 t_1 t_2 + 2 b_1^3 b_2 c_2 t_1 t_2 + 2 b_1^2 b_2^2 c_2 t_1 t_2 - 2 b_1 b_2 t_1 u_1 w_1 + 2 b_1 b_2 t_1 t_2 u_1 w_1 + 2 b_2^2 t_1 t_2 u_1 w_1 - 2 b_2^2 t_1^2 t_2 u_1 w_1 - 2 b_1 b_2 t_2 u_2 w_1 + 2 b_1 b_2 t_1 t_2^2 u_2 w_1 - 2 b_1 b_2 t_1 u_1 w_2 + 2 b_1 b_2 t_1^2 t_2 u_1 w_2 - 2 b_1 b_2 t_2 u_2 w_2 + 2 b_1^2 t_1 t_2 u_2 w_2 + 2 b_1 b_2 t_1 t_2 u_2 w_2 - 2 b_1^2 t_1 t_2^2 u_2 w_2 + 2 b_1^2 t_1^2 t_2 u_1 w_1 g_{11}[b_1, b_2, t_1, t_2] + 2 b_1 b_2^2 t_1^2 t_2 u_1 w_1 g_{11}[b_1, b_2, t_1, t_2] + 2 b_1^2 b_2 t_1 u_1 w_2 g_{11}[b_1, b_2, t_1, t_2] - 2 b_1^2 b_2 t_1^2 t_2 u_1 w_2 g_{11}[b_1, b_2, t_1, t_2] + 2 b_1 b_2^2 t_1 u_1 w_1 g_{12}[b_1, b_2, t_1, t_2] - 2 b_1 b_2^2 t_1^2 t_2 u_1 w_1 g_{12}[b_1, b_2, t_1, t_2] + 2 b_1 b_2^2 t_1 t_2 u_1 w_2 g_{12}[b_1, b_2, t_1, t_2] + 2 b_1^2 b_2 t_1 t_2 u_1 w_2 g_{12}[b_1, b_2, t_1, t_2] + 2 b_1^2 b_2 t_2 u_2 w_1 g_{21}[b_1, b_2, t_1, t_2] + 2 b_1 b_2^2 t_1 t_2^2 u_2 w_1 g_{21}[b_1, b_2, t_1, t_2] + 2 b_1^2 b_2 t_2 u_2 w_2 g_{21}[b_1, b_2, t_1, t_2] - 2 b_1^2 b_2 t_1 t_2^2 u_2 w_1 g_{21}[b_1, b_2, t_1, t_2] + 2 b_1 b_2^2 t_2 u_2 w_2 g_{21}[b_1, b_2, t_1, t_2] + 2 b_1^2 b_2 t_1 t_2^2 u_2 w_2 g_{21}[b_1, b_2, t_1, t_2] + 2 b_1 b_2^2 t_1 t_2^2 u_2 w_2 g_{21}[b_1, b_2, t_1, t_2] + 2 b_1 b_2^2 t_2 u_2 w_2 g_{22}[b_1, b_2, t_1, t_2] + 2 b_1^2 b_2 t_1 t_2^2 u_2 w_2 g_{22}[b_1, b_2, t_1, t_2]) =$$

$$\frac{1}{2 b_1 t_2} (b_1 b_2 c_1 t_2 + 2 b_1^2 c_2 t_2 - 2 u_1 w_2 + 2 t_1^2 u_1 w_2 + 2 b_2 u_1 w_2 g_{11}[b_2, b_1, t_2, t_1] - 2 b_2 t_1^2 u_1 w_2 g_{11}[b_2, b_1, t_2, t_1] + 2 b_1 t_2 u_2 w_2 g_{11}[b_2, b_1, t_2, t_1] + 2 b_2 u_1 w_1 g_{12}[b_2, b_1, t_2, t_1] - 2 b_2 t_1^2 u_1 w_1 g_{12}[b_2, b_1, t_2, t_1] + 2 b_1 t_2 u_2 w_1 g_{12}[b_2, b_1, t_2, t_1] + 2 b_1 t_2 u_1 w_2 g_{21}[b_2, b_1, t_2, t_1] + 2 b_1 t_2 u_1 w_1 g_{22}[b_2, b_1, t_2, t_1])$$

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A[n_][e_][E] := (e /. {c_n -> -c_n, w_n -> -w_n}) // N_u_n c_n -> n // N_w_n c_n -> n // N_w_n u_n -> n;
A[{}][e_][E] := e;
A[{n_, r___}][e_][E] := e // A[n] // A[{r}];

```

Eqn3 = (V \*\* A[{1, 2}][V] ≡ E[1, 0])

$$h[b_1, b_2, t_1, t_2]^2 / (1 - b_1 g_{11}[b_1, b_2, t_1, t_2] - b_1 b_2 g_{12}[b_1, b_2, t_1, t_2] g_{21}[b_1, b_2, t_1, t_2] - b_2 g_{22}[b_1, b_2, t_1, t_2] + b_1 b_2 g_{11}[b_1, b_2, t_1, t_2] g_{22}[b_1, b_2, t_1, t_2]) = 1$$

$$\left( \text{CF} / @ \left\{ - \frac{1}{(-1 + e^{b_1 + b_2}) b_1 (b_1 + b_2)} e^{-b_1 f_{11} - b_2 f_{12}} \left( \left( e^{b_1 + \frac{b_2}{2}} - e^{\frac{b_2}{2}} + e^{b_1 f_{11} + b_2 f_{12}} - e^{b_1 (1 + f_{11}) + b_2 (1 + f_{12})} \right) b_1 + \left( e^{b_1 + \frac{b_2}{2}} - e^{\frac{b_2}{2}} - e^{(b_1 + b_2) (f_{11} + f_{12})} + e^{(b_1 + b_2) (1 + f_{11} + f_{12})} + e^{b_1 f_{11} + b_2 f_{12}} - e^{b_1 (1 + f_{11}) + b_2 (1 + f_{12})} \right) b_2 \right), \right. \\ \left. - \left( \left( e^{-b_1 f_{11} - b_2 f_{12}} \left( \left( e^{b_1 + \frac{b_2}{2}} - e^{\frac{b_2}{2}} + e^{(b_1 + b_2) (f_{11} + f_{12})} - e^{(b_1 + b_2) (1 + f_{11} + f_{12})} \right) b_1 + e^{\frac{b_2}{2}} (-1 + e^{b_1}) b_2 \right) \right) / \left( (-1 + e^{b_1 + b_2}) b_1 (b_1 + b_2) \right) \right), \right. \\ \left. - \left( \left( e^{-\frac{1}{2} b_2 (1 + 2 f_{11}) - b_1 f_{12}} \left( e^{b_1 + \frac{b_2}{2}} (-1 + e^{b_2}) b_1 + \left( -e^{b_1 + \frac{b_2}{2}} + e^{b_1 + \frac{3 b_2}{2}} + e^{(b_1 + b_2) (f_{11} + f_{12})} - e^{(b_1 + b_2) (1 + f_{11} + f_{12})} \right) b_2 \right) \right) / \left( (-1 + e^{b_1 + b_2}) b_2 (b_1 + b_2) \right) \right), \right. \\ \left. \frac{1}{b_2} \left( 1 - \frac{e^{b_1 - b_2 f_{11} - b_1 f_{12}} (-1 + e^{b_2}) (b_1 + b_2)}{(-1 + e^{b_1 + b_2}) b_2} + \left( e^{-b_2 \left( \frac{1}{2} + f_{11} \right) - b_1 f_{12}} b_1 \left( e^{b_1 + \frac{b_2}{2}} (-1 + e^{b_2}) b_1 + \left( -e^{b_1 + \frac{b_2}{2}} + e^{b_1 + \frac{3 b_2}{2}} + e^{(b_1 + b_2) (f_{11} + f_{12})} - e^{(b_1 + b_2) (1 + f_{11} + f_{12})} \right) b_2 \right) \right) / \left( (-1 + e^{b_1 + b_2}) b_2 (b_1 + b_2) \right) \right) \right\} / . \{b_1 \rightarrow b1, b_2 \rightarrow b2, t_1 \rightarrow t1, t_2 \rightarrow t2\}$$

$$\left\{ \frac{b_2 t_1 - b_1 t_1^2 - b_2 t_1^2 + b_1 t_1^2 t_2^2 + b_2 t_1^2 t_2^2 - b_2 t_1^3 t_2^2}{-b_1^2 - b_1 b_2 + b_1^2 t_1^2 t_2^2 + b_1 b_2 t_1^2 t_2^2}, \frac{b_1 + b_2 - b_1 t_1 - b_1 t_1^2 - b_2 t_1^2 + b_1 t_1^3 t_2^2}{-b_1^2 - b_1 b_2 + b_1^2 t_1^2 t_2^2 + b_1 b_2 t_1^2 t_2^2}, \right. \\ \left. \frac{-b_2 + b_1 t_1 + b_2 t_1 - b_1 t_1 t_2^2 - b_2 t_1 t_2^2 + b_2 t_1^2 t_2^2}{-b_1 b_2 - b_2^2 + b_1 b_2 t_1^2 t_2^2 + b_2^2 t_1^2 t_2^2}, \frac{-b_2 + b_1 t_1 + b_2 t_1 - b_1 t_1 t_2^2 - b_2 t_1 t_2^2 + b_2 t_1^2 t_2^2}{-b_1 b_2 - b_2^2 + b_1 b_2 t_1^2 t_2^2 + b_2^2 t_1^2 t_2^2} \right\}$$

CF / @

$$\left( \{ \text{Eqn1}, \text{Eqn2}, \text{Eqn3} \} / . \text{Thread} \left[ \{ g_{11}[b1_, b2_, t1_, t2_], g_{12}[b1_, b2_, t1_, t2_], g_{21}[b1_, b2_, t1_, t2_], g_{22}[b1_, b2_, t1_, t2_] \} \Rightarrow \left\{ \left( \frac{b_2 t_1 - b_1 t_1^2 - b_2 t_1^2 + b_1 t_1^2 t_2^2 + b_2 t_1^2 t_2^2 - b_2 t_1^3 t_2^2}{(-b_1^2 - b_1 b_2 + b_1^2 t_1^2 t_2^2 + b_1 b_2 t_1^2 t_2^2)}, \frac{b_1 + b_2 - b_1 t_1 - b_1 t_1^2 - b_2 t_1^2 + b_1 t_1^3 t_2^2}{-b_1^2 - b_1 b_2 + b_1^2 t_1^2 t_2^2 + b_1 b_2 t_1^2 t_2^2}, \right. \right. \right. \\ \left. \left. \frac{-b_2 + b_1 t_1 + b_2 t_1 - b_1 t_1 t_2^2 - b_2 t_1 t_2^2 + b_2 t_1^2 t_2^2}{-b_1 b_2 - b_2^2 + b_1 b_2 t_1^2 t_2^2 + b_2^2 t_1^2 t_2^2}, \frac{-b_2 + b_1 t_1 + b_2 t_1 - b_1 t_1 t_2^2 - b_2 t_1 t_2^2 + b_2 t_1^2 t_2^2}{-b_1 b_2 - b_2^2 + b_1 b_2 t_1^2 t_2^2 + b_2^2 t_1^2 t_2^2} \right\} \right] \right)$$

$$\{ \text{True}, h[b_1, b_2, t_1, t_2] = h[b_2, b_1, t_2, t_1], h[b_1, b_2, t_1, t_2]^2 = 1 \}$$

VV =

$$V / . \text{Thread} \left[ \{ h[b1_, b2_, t1_, t2_], g_{11}[b1_, b2_, t1_, t2_], g_{12}[b1_, b2_, t1_, t2_], g_{21}[b1_, b2_, t1_, t2_], g_{22}[b1_, b2_, t1_, t2_] \} \Rightarrow \left\{ 1, \left( \frac{b_2 t_1 - b_1 t_1^2 - b_2 t_1^2 + b_1 t_1^2 t_2^2 + b_2 t_1^2 t_2^2 - b_2 t_1^3 t_2^2}{(-b_1^2 - b_1 b_2 + b_1^2 t_1^2 t_2^2 + b_1 b_2 t_1^2 t_2^2)}, \frac{b_1 + b_2 - b_1 t_1 - b_1 t_1^2 - b_2 t_1^2 + b_1 t_1^3 t_2^2}{-b_1^2 - b_1 b_2 + b_1^2 t_1^2 t_2^2 + b_1 b_2 t_1^2 t_2^2}, \right. \right. \right. \\ \left. \left. \frac{-b_2 + b_1 t_1 + b_2 t_1 - b_1 t_1 t_2^2 - b_2 t_1 t_2^2 + b_2 t_1^2 t_2^2}{-b_1 b_2 - b_2^2 + b_1 b_2 t_1^2 t_2^2 + b_2^2 t_1^2 t_2^2}, \frac{-b_2 + b_1 t_1 + b_2 t_1 - b_1 t_1 t_2^2 - b_2 t_1 t_2^2 + b_2 t_1^2 t_2^2}{-b_1 b_2 - b_2^2 + b_1 b_2 t_1^2 t_2^2 + b_2^2 t_1^2 t_2^2} \right\} \right]$$

E[1,

$$\frac{b_1 c_2}{2} + \frac{(b_2 t_1 - b_1 t_1^2 - b_2 t_1^2 + b_1 t_1^2 t_2^2 + b_2 t_1^2 t_2^2 - b_2 t_1^3 t_2^2) u_1 w_1}{-b_1^2 - b_1 b_2 + b_1^2 t_1^2 t_2^2 + b_1 b_2 t_1^2 t_2^2} + \frac{(-b_2 + b_1 t_1 + b_2 t_1 - b_1 t_1 t_2^2 - b_2 t_1 t_2^2 + b_2 t_1^2 t_2^2) u_2 w_1}{-b_1 b_2 - b_2^2 + b_1 b_2 t_1^2 t_2^2 + b_2^2 t_1^2 t_2^2} + \\ \frac{(b_1 + b_2 - b_1 t_1 - b_1 t_1^2 - b_2 t_1^2 + b_1 t_1^3 t_2^2) u_1 w_2}{-b_1^2 - b_1 b_2 + b_1^2 t_1^2 t_2^2 + b_1 b_2 t_1^2 t_2^2} + \frac{(-b_2 + b_1 t_1 + b_2 t_1 - b_1 t_1 t_2^2 - b_2 t_1 t_2^2 + b_2 t_1^2 t_2^2) u_2 w_2}{-b_1 b_2 - b_2^2 + b_1 b_2 t_1^2 t_2^2 + b_2^2 t_1^2 t_2^2}]$$

$$\mathbf{W}i = \mathbf{A}[\{1, 2\}] [\mathbf{W}]$$

$$\mathbb{E} \left[ 1, \left( b_1^3 b_2 c_2 t_1 + b_1^2 b_2^2 c_2 t_1 - b_1^3 b_2 c_2 t_1^3 t_2^2 - b_1^2 b_2^2 c_2 t_1^3 t_2^2 - 2 b_2^2 t_1 u_1 w_1 + 2 b_1 b_2 t_1^2 u_1 w_1 + 2 b_2^2 t_1^2 u_1 w_1 - 2 b_1 b_2 t_1^2 t_2^2 u_1 w_1 - 2 b_2^2 t_1^2 t_2^2 u_1 w_1 + 2 b_2^2 t_1^3 t_2^2 u_1 w_1 + 2 b_1 b_2 t_1^3 t_2^2 u_1 w_1 - 2 b_1 b_2 t_1^4 t_2^2 u_2 w_1 - 2 b_1 b_2 u_1 w_2 - 2 b_2^2 u_1 w_2 + 2 b_1 b_2 t_1 u_1 w_2 + 2 b_1 b_2 t_1^2 u_1 w_2 + 2 b_2^2 t_1^2 u_1 w_2 - 2 b_1 b_2 t_1^3 t_2^2 u_1 w_2 + 2 b_1 b_2 t_1^2 u_2 w_2 - 2 b_1^2 t_1^3 u_2 w_2 - 2 b_1 b_2 t_1^3 u_2 w_2 + 2 b_1^2 t_1^3 t_2^2 u_2 w_2 + 2 b_1 b_2 t_1^3 t_2^2 u_2 w_2 - 2 b_1 b_2 t_1^4 t_2^2 u_2 w_2 \right) / \left( -2 b_1^2 b_2 t_1 - 2 b_1 b_2^2 t_1 + 2 b_1^2 b_2 t_1^3 t_2^2 + 2 b_1 b_2^2 t_1^3 t_2^2 \right) \right]$$

$$\mathbf{W} \mathbf{W} \mathbf{W}i$$

$$\mathbb{E} [1, \emptyset]$$

$$\mathbf{W} \mathbf{W} = \mathbf{W}i^{\sigma[\{1,2\},3]} \mathbf{W}i^{\sigma[1,2]} \mathbf{W}i^{\sigma[2,3]} \mathbf{W}i^{\sigma[1,\{2,3\}]}$$

$$\mathbb{E} [1,$$

$$\begin{aligned} & \left( -b_1 b_2^3 b_3 t_2 u_1 w_1 - 2 b_1 b_2^2 b_3^2 t_2 u_1 w_1 - b_1 b_2 b_3^3 t_2 u_1 w_1 + b_1 b_2^3 b_3 t_2^2 u_1 w_1 + b_1^4 b_3 t_2^2 u_1 w_1 + 2 b_1 b_2^2 b_3^2 t_2^2 u_1 w_1 + 2 b_2^3 b_3^2 t_2^2 u_1 w_1 + \right. \\ & b_1 b_2 b_3^3 t_2^2 u_1 w_1 + b_2^2 b_3^3 t_2^2 u_1 w_1 - b_1 b_2^3 b_3 t_1^2 t_2^2 u_1 w_1 - b_2^4 b_3 t_1^2 t_2^2 u_1 w_1 - 2 b_1 b_2^2 b_3^2 t_1^2 t_2^2 u_1 w_1 - 2 b_2^3 b_3^2 t_1^2 t_2^2 u_1 w_1 - \\ & b_1 b_2 b_3^3 t_1^2 t_2^2 u_1 w_1 - b_2^2 b_3^3 t_1^2 t_2^2 u_1 w_1 + b_1 b_2^3 b_3 t_1^3 t_2^3 u_1 w_1 + 2 b_1 b_2^2 b_3^2 t_1^3 t_2^3 u_1 w_1 + b_1 b_2 b_3^3 t_1^3 t_2^3 u_1 w_1 + \\ & b_1 b_2^3 b_3 t_2 t_3^2 u_1 w_1 + 2 b_1 b_2^2 b_3^2 t_2 t_3^2 u_1 w_1 + b_1 b_2 b_3^3 t_2 t_3^2 u_1 w_1 - b_1 b_2^3 b_3 t_2^2 t_3^2 u_1 w_1 - b_2^4 b_3 t_2^2 t_3^2 u_1 w_1 - \\ & 2 b_1 b_2^2 b_3^2 t_2^2 t_3^2 u_1 w_1 - 2 b_2^3 b_3^2 t_2^2 t_3^2 u_1 w_1 - b_1 b_2 b_3^3 t_2^2 t_3^2 u_1 w_1 - b_2^2 b_3^3 t_2^2 t_3^2 u_1 w_1 + b_1 b_2^3 b_3 t_1^2 t_2^2 t_3^2 u_1 w_1 + \\ & b_1^4 b_3 t_1^2 t_2^2 t_3^2 u_1 w_1 + 2 b_1 b_2^2 b_3^2 t_1^2 t_2^2 t_3^2 u_1 w_1 + 2 b_2^3 b_3^2 t_1^2 t_2^2 t_3^2 u_1 w_1 + b_1 b_2 b_3^3 t_1^2 t_2^2 t_3^2 u_1 w_1 + b_2^2 b_3^3 t_1^2 t_2^2 t_3^2 u_1 w_1 - \\ & b_1 b_2^3 b_3 t_1^2 t_2^2 t_3^2 u_1 w_1 - 2 b_1 b_2^2 b_3^2 t_1^2 t_2^2 t_3^2 u_1 w_1 - b_1 b_2 b_3^3 t_1^2 t_2^2 t_3^2 u_1 w_1 + b_1^2 b_2 b_3^2 u_2 w_1 + b_1 b_2^2 b_3^2 u_2 w_1 + b_1^2 b_3^3 u_2 w_1 + \\ & b_1 b_2 b_3^3 u_2 w_1 - b_1 b_2^2 b_3^2 u_2 w_1 - 2 b_1 b_2^2 b_3^2 t_2 u_2 w_1 - b_1 b_2 b_3^3 t_2 u_2 w_1 - b_1^2 b_2^2 b_3^2 t_2 u_2 w_1 - b_1 b_2^3 b_3 t_2^2 u_2 w_1 - \\ & 2 b_1^2 b_2 b_3^2 t_2^2 u_2 w_1 - 2 b_1 b_2^2 b_3^2 t_2^2 u_2 w_1 - b_1^2 b_3^3 t_2^2 u_2 w_1 - b_1 b_2 b_3^3 t_2^2 u_2 w_1 + b_1^2 b_2^2 b_3^2 t_1^2 t_2^2 u_2 w_1 + b_1 b_2^3 b_3 t_1^2 t_2^2 u_2 w_1 + \\ & b_1^2 b_2 b_3^2 t_1^2 t_2^2 u_2 w_1 + b_1 b_2^2 b_3^2 t_1^2 t_2^2 u_2 w_1 + b_1 b_2^3 b_3 t_1^3 t_2^3 u_2 w_1 + 2 b_1 b_2^2 b_3^2 t_1^3 t_2^3 u_2 w_1 + b_1 b_2 b_3^3 t_1^3 t_2^3 u_2 w_1 + \\ & b_1 b_2^3 b_3 t_2 t_3^2 u_2 w_1 + 2 b_1 b_2^2 b_3^2 t_2 t_3^2 u_2 w_1 + b_1 b_2 b_3^3 t_2 t_3^2 u_2 w_1 + b_1^2 b_2^2 b_3^2 t_2 t_3^2 u_2 w_1 + b_1 b_2^3 b_3 t_1^2 t_2^2 t_3^2 u_2 w_1 + \\ & b_1^2 b_2 b_3^2 t_1^2 t_2^2 t_3^2 u_2 w_1 - b_1^2 b_2^2 b_3^2 t_1^2 t_2^2 t_3^2 u_2 w_1 - b_1 b_2 b_3^3 t_1^2 t_2^2 t_3^2 u_2 w_1 - b_1 b_2^3 b_3 t_1^2 t_2^2 t_3^2 u_2 w_1 - 2 b_1^2 b_2 b_3^2 t_1^2 t_2^2 t_3^2 u_2 w_1 - \\ & 2 b_1 b_2^2 b_3^2 t_1^2 t_2^2 t_3^2 u_2 w_1 - b_1^2 b_3^3 t_1^2 t_2^2 t_3^2 u_2 w_1 - b_1 b_2 b_3^3 t_1^2 t_2^2 t_3^2 u_2 w_1 - b_1 b_2^3 b_3 t_1^3 t_2^3 t_3^2 u_2 w_1 - 2 b_1 b_2^2 b_3^2 t_1^3 t_2^3 t_3^2 u_2 w_1 - \\ & b_1 b_2 b_3^3 t_1^3 t_2^3 t_3^2 u_2 w_1 + b_1^2 b_2 b_3^2 t_1^3 t_2^3 t_3^2 u_2 w_1 + b_1 b_2^2 b_3^2 t_1^3 t_2^3 t_3^2 u_2 w_1 + b_1 b_2^3 b_3 t_1^4 t_2^4 t_3^2 u_2 w_1 + b_1 b_2 b_3^3 t_1^4 t_2^4 t_3^2 u_2 w_1 - \\ & b_1^2 b_2^2 b_3^2 u_3 w_1 - b_1 b_2^3 b_3 u_3 w_1 - b_1^2 b_2 b_3^2 u_3 w_1 - b_1 b_2^2 b_3^2 u_3 w_1 + b_1^2 b_3^3 t_2 u_3 w_1 + b_1 b_2^4 t_2 u_3 w_1 + 2 b_1^2 b_2^2 b_3^2 t_2 u_3 w_1 + \\ & 2 b_1 b_2^3 b_3 t_2 u_3 w_1 + b_1^2 b_2 b_3^2 t_2 u_3 w_1 + b_1 b_2^2 b_3^2 t_2 u_3 w_1 + b_1^2 b_2^2 b_3^2 t_1^2 t_2^2 u_3 w_1 + b_1 b_2^3 b_3 t_1^2 t_2^2 u_3 w_1 + b_1^2 b_2 b_3^2 t_1^2 t_2^2 u_3 w_1 + \\ & b_1 b_2^2 b_3^2 t_1^2 t_2^2 u_3 w_1 - b_1^2 b_3^3 t_1^2 t_2^2 u_3 w_1 - b_1 b_2 b_3^3 t_1^2 t_2^2 u_3 w_1 - 2 b_1^2 b_2^2 b_3^2 t_1^2 t_2^2 u_3 w_1 - 2 b_1 b_2^3 b_3 t_1^2 t_2^2 u_3 w_1 - b_1^2 b_2 b_3^2 t_1^2 t_2^2 u_3 w_1 - \\ & b_1 b_2^2 b_3^2 t_1^2 t_2^2 u_3 w_1 - b_1^2 b_3^3 t_2 t_3^2 u_3 w_1 - b_1 b_2 b_3^3 t_2 t_3^2 u_3 w_1 - 2 b_1^2 b_2^2 b_3^2 t_2 t_3^2 u_3 w_1 - 2 b_1 b_2^3 b_3 t_2 t_3^2 u_3 w_1 - b_1^2 b_2 b_3^2 t_2 t_3^2 u_3 w_1 + \\ & b_1^2 b_2^2 b_3^2 t_1^2 t_2^2 t_3^2 u_3 w_1 + b_1 b_2^3 b_3 t_1^2 t_2^2 t_3^2 u_3 w_1 + 2 b_1^2 b_2^2 b_3^2 t_1^2 t_2^2 t_3^2 u_3 w_1 + 2 b_1 b_2^3 b_3 t_1^2 t_2^2 t_3^2 u_3 w_1 + b_1^2 b_2 b_3^2 t_1^2 t_2^2 t_3^2 u_3 w_1 + \\ & b_1 b_2^2 b_3^2 t_1^2 t_2^2 t_3^2 u_3 w_1 - b_1^2 b_2^2 b_3^2 t_1^2 t_2^2 t_3^2 u_3 w_1 - b_1 b_2 b_3^3 t_1^2 t_2^2 t_3^2 u_3 w_1 - b_1^2 b_2 b_3^2 t_1^2 t_2^2 t_3^2 u_3 w_1 - b_1 b_2^3 b_3 t_1^4 t_2^4 t_3^2 u_3 w_1 - \\ & b_1 b_2^2 b_3^2 t_1^4 t_2^4 t_3^2 u_3 w_1 - b_1^2 b_2^2 b_3^2 t_1^4 t_2^4 t_3^2 u_3 w_1 - b_1 b_2 b_3^3 t_1^4 t_2^4 t_3^2 u_3 w_1 - b_1^2 b_2 b_3^2 t_1^4 t_2^4 t_3^2 u_3 w_1 - \\ & b_1^2 b_2 b_3^2 u_1 w_2 - b_1 b_2^2 b_3^2 u_1 w_2 - b_1 b_2 b_3^3 u_1 w_2 + b_1^2 b_2^2 b_3^2 t_2 u_1 w_2 + 2 b_1^2 b_2 b_3^2 t_2 u_1 w_2 + 2 b_1 b_2^2 b_3^2 t_2 u_1 w_2 + b_2^3 b_3^2 t_2 u_1 w_2 + \\ & b_1 b_2 b_3^3 t_2 u_1 w_2 + b_2^2 b_3^3 t_2 u_1 w_2 - b_1^2 b_2 b_3^2 t_1^2 t_2 u_1 w_2 - 2 b_1 b_2^2 b_3^2 t_1^2 t_2 u_1 w_2 - b_2^3 b_3^2 t_1^2 t_2 u_1 w_2 - b_1 b_2 b_3^3 t_1^2 t_2 u_1 w_2 - \\ & b_2^2 b_3^3 t_1^2 t_2 u_1 w_2 - b_1^2 b_2^2 b_3^2 t_2^2 u_1 w_2 - b_1 b_2^3 b_3 t_2^2 u_1 w_2 - b_1^2 b_2 b_3^2 t_2^2 u_1 w_2 + b_1^2 b_2^2 b_3^2 t_1^2 t_2^2 u_1 w_2 + \\ & b_1 b_2^3 b_3 t_1^2 t_2^2 u_1 w_2 + 2 b_1^2 b_2 b_3^2 t_1^2 t_2^2 u_1 w_2 + 2 b_1 b_2^2 b_3^2 t_1^2 t_2^2 u_1 w_2 + b_1 b_2 b_3^3 t_1^2 t_2^2 u_1 w_2 - b_1^2 b_2^2 b_3^2 t_1^2 t_2^2 u_1 w_2 - \\ & b_1^2 b_2 b_3^2 t_1^2 t_2^2 u_1 w_2 - b_1 b_2^3 b_3 t_2 t_3^2 u_1 w_2 + b_1^2 b_2^2 b_3^2 t_2 t_3^2 u_1 w_2 + b_1 b_2^3 b_3 t_2 t_3^2 u_1 w_2 + b_1 b_2^2 b_3^2 t_2 t_3^2 u_1 w_2 + \\ & 2 b_1^2 b_2 b_3^2 t_2 t_3^2 u_1 w_2 + 2 b_1 b_2^2 b_3^2 t_2 t_3^2 u_1 w_2 + b_1 b_2 b_3^3 t_2 t_3^2 u_1 w_2 - b_1^2 b_2^2 b_3^2 t_2 t_3^2 u_1 w_2 - b_1 b_2^3 b_3 t_1^2 t_2^2 t_3^2 u_1 w_2 - \\ & b_1^2 b_2 b_3^2 t_1^2 t_2^2 t_3^2 u_1 w_2 - b_1 b_2^3 b_3 t_1^2 t_2^2 t_3^2 u_1 w_2 - 2 b_1 b_2^2 b_3^2 t_1^2 t_2^2 t_3^2 u_1 w_2 - b_2^3 b_3^2 t_1^2 t_2^2 t_3^2 u_1 w_2 - \\ & b_1 b_2 b_3^3 t_1^2 t_2^2 t_3^2 u_1 w_2 - b_2^2 b_3^3 t_1^2 t_2^2 t_3^2 u_1 w_2 + b_1^2 b_2^2 b_3^2 t_1^3 t_2^3 t_3^2 u_1 w_2 + 2 b_1 b_2^2 b_3^2 t_1^3 t_2^3 t_3^2 u_1 w_2 + 2 b_1 b_2^3 b_3 t_1^3 t_2^3 t_3^2 u_1 w_2 + \\ & b_2^2 b_3^3 t_1^3 t_2^3 t_3^2 u_1 w_2 + b_1 b_2 b_3^3 t_1^3 t_2^3 t_3^2 u_1 w_2 + b_2^2 b_3^3 t_1^3 t_2^3 t_3^2 u_1 w_2 - b_1^2 b_2 b_3^2 t_1^3 t_2^3 t_3^2 u_1 w_2 - b_1 b_2^3 b_3 t_1^4 t_2^4 t_3^2 u_1 w_2 - \\ & b_1 b_2^2 b_3^2 t_1^4 t_2^4 t_3^2 u_1 w_2 - b_1^2 b_3^3 t_1^4 t_2^4 t_3^2 u_1 w_2 - b_1 b_2 b_3^3 t_1^4 t_2^4 t_3^2 u_1 w_2 - b_1^2 b_2 b_3^2 t_1^4 t_2^4 t_3^2 u_1 w_2 - b_1 b_2^3 b_3 t_2 u_2 w_2 - \\ & b_1^2 b_2 b_3^2 t_2 u_2 w_2 - b_1 b_2^2 b_3^2 t_2 u_2 w_2 - b_1 b_2 b_3^3 t_2 u_2 w_2 - b_1^2 b_2^2 b_3^2 t_2 u_2 w_2 - b_1 b_2^3 b_3 t_2 u_2 w_2 - b_1^2 b_3^3 t_2 u_2 w_2 - \\ & b_1 b_2 b_3^3 t_2 u_2 w_2 + b_1^2 b_2^2 b_3^2 t_1^2 t_2 u_2 w_2 + 2 b_1^2 b_2 b_3^2 t_1^2 t_2 u_2 w_2 + b_1 b_2^3 b_3 t_1^2 t_2 u_2 w_2 + b_1^2 b_3^3 t_1^2 t_2 u_2 w_2 + b_1 b_2 b_3^3 t_1^2 t_2 u_2 w_2 + \\ & b_1^2 b_2^2 b_3^2 t_1^2 t_2 u_2 w_2 - b_1^2 b_2 b_3^2 t_1^2 t_2 u_2 w_2 - b_1 b_2^3 b_3 t_1^2 t_2 u_2 w_2 - b_1^2 b_2^2 b_3^2 t_1^2 t_2 u_2 w_2 - b_1 b_2^3 b_3 t_2 t_3^2 u_2 w_2 - \\ & b_1^2 b_2 b_3^2 t_2 t_3^2 u_2 w_2 - b_1 b_2^3 b_3 t_2 t_3^2 u_2 w_2 - b_1^2 b_2^2 b_3^2 t_2 t_3^2 u_2 w_2 - b_1 b_2^3 b_3 t_2 t_3^2 u_2 w_2 - b_1^2 b_3^3 t_2 t_3^2 u_2 w_2 - b_1 b_2 b_3^3 t_2 t_3^2 u_2 w_2 + \\ & b_1^2 b_2 b_3^2 t_1^2 t_2^2 t_3^2 u_2 w_2 + b_1^2 b_2^2 b_3^2 t_1^2 t_2^2 t_3^2 u_2 w_2 + b_1 b_2^3 b_3 t_1^2 t_2^2 t_3^2 u_2 w_2 + b_1^2 b_2 b_3^2 t_1^2 t_2^2 t_3^2 u_2 w_2 + b_1^2 b_2^2 b_3^2 t_1^2 t_2^2 t_3^2 u_2 w_2 + \\ & 2 b_1^2 b_2 b_3^2 t_1^2 t_2^2 t_3^2 u_2 w_2 + b_1 b_2^3 b_3 t_1^3 t_2^3 t_3^2 u_2 w_2 + b_1^2 b_2^2 b_3^2 t_1^3 t_2^3 t_3^2 u_2 w_2 + b_1 b_2 b_3^3 t_1^3 t_2^3 t_3^2 u_2 w_2 + b_1^2 b_2^2 b_3^2 t_1^3 t_2^3 t_3^2 u_2 w_2 - \\ & b_1^2 b_2 b_3^2 t_1^3 t_2^3 t_3^2 u_2 w_2 - b_1 b_2^3 b_3 t_1^3 t_2^3 t_3^2 u_2 w_2 - b_1^2 b_3^3 t_1^3 t_2^3 t_3^2 u_2 w_2 - b_1 b_2 b_3^3 t_1^3 t_2^3 t_3^2 u_2 w_2 - b_1^2 b_2^2 b_3^2 t_1^3 t_2^3 t_3^2 u_2 w_2 + \\ & b_1 b_2^3 b_3 t_1^4 t_2^4 t_3^2 u_2 w_2 + b_1^2 b_2 b_3^2 t_1^4 t_2^4 t_3^2 u_2 w_2 - b_1 b_2^3 b_3 t_2 u_3 w_2 - b_1^2 b_2^2 b_3^2 t_2 u_3 w_2 - b_1 b_2 b_3^3 t_2 u_3 w_2 - b_1^2 b_2^2 b_3^2 t_2 u_3 w_2 - \\ & b_1 b_2^3 b_3 t_2 u_3 w_2 + b_1^2 b_2^2 b_3^2 t_1^2 t_2^2 u_3 w_2 + b_1 b_2^3 b_3 t_1^2 t_2^2 u_3 w_2 + b_1^2 b_2 b_3^2 t_1^2 t_2^2 u_3 w_2 + b_1 b_2^3 b_3 t_1^3 t_2^3 u_3 w_2 + b_1^2 b_2^2 b_3^2 t_1^3 t_2^3 u_3 w_2 + \\ & b_1 b_2 b_3^3 t_1^3 t_2^3 u_3 w_2 - b_1^2 b_2 b_3^2 t_1^3 t_2^3 u_3 w_2 - b_1 b_2^3 b_3 t_2 t_3^2 u_3 w_2 - b_1^2 b_2^2 b_3^2 t_2 t_3^2 u_3 w_2 - b_1 b_2 b_3^3 t_2 t_3^2 u_3 w_2 - \\ & b_1^2 b_2 b_3^2 t_2 t_3^2 u_3 w_2 - b_1 b_2^3 b_3 t_2 t_3^2 u_3 w_2 - b_1^2 b_2^2 b_3^2 t_2 t_3^2 u_3 w_2 - b_1 b_2 b_3^3 t_2 t_3^2 u_3 w_2 - b_1^2 b_2 b_3^2 t_2 t_3^2 u_3 w_2 + \\ & b_1^2 b_2^2 b_3^2 t_1^2 t_2^2 t_3^2 u_3 w_2 - b_1 b_2^3 b_3 t_1^2 t_2^2 t_3^2 u_3 w_2 - b_1^2 b_2^2 b_3^2 t_1^2 t_2^2 t_3^2 u_3 w_2 - b_1 b_2 b_3^3 t_1^2 t_2^2 t_3^2 u_3 w_2 + b_1^2 b_2 b_3^2 t_1^2 t_2^2 t_3^2 u_3 w_2 + \\ & b_1 b_2^3 b_3 t_1^2 t_2^2 t_3^2 u_3 w_2 - b_1^2 b_2^2 b_3^2 t_1^2 t_2^2 t_3^2 u_3 w_2 - b_1 b_2 b_3^3 t_1^2 t_2^2 t_3^2 u_3 w_2 - b_1^2 b_2 b_3^2 t_1^2 t_2^2 t_3^2 u_3 w_2 + b_1^2 b_2^2 b_3^2 t_1^4 t_2^4 t_3^2 u_3 w_2 + \\ & b_1 b_2^3 b_3 t_1^4 t_2^4 t_3^2 u_3 w_2 + b_1^2 b_2^2 b_3^2 t_1^4 t_2^4 t_3^2 u_3 w_2 + b_1 b_2 b_3^3 t_1^4 t_2^4 t_3^2 u_3 w_2 + b_1^2 b_2 b_3^2 t_1^4 t_2^4 t_3^2 u_3 w_2 - \\ & 2 b_1 b_2^2 b_3^2 t_2 u_1 w_3 - b_2^4 b_3 t_2 u_1 w_3 + b_1^2 b_2 b_3^2 t_2 u_1 w_3 - b_1 b_2^3 b_3 t_2 u_1 w_3 + b_1^2 b_2^2 b_3^2 t_1^2 t_2 u_1 w_3 + \\ & b_1 b_2^3 b_3 t_1^2 t_2 u_1 w_3 + 2 b_1 b_2^2 b_3^2 t_1^2 t_2 u_1 w_3 + b_1^2 b_3^3 t_1^2 t_2 u_1 w_3 + \end{aligned}$$

$$\begin{aligned}
& b_1^3 b_2^3 t_1^2 t_2 u_1 w_3 - b_1^2 b_2^2 b_3 t_2^2 u_1 w_3 - b_1 b_2^3 b_3 t_2^2 u_1 w_3 - b_1^2 b_2 b_3^2 t_2^2 u_1 w_3 - b_1 b_2^2 b_3^2 t_2^2 u_1 w_3 + b_1^2 b_2 b_3^2 t_1^2 t_2^2 u_1 w_3 - \\
& b_1^2 b_2^2 b_3 t_1^2 t_2^3 u_1 w_3 - b_1^2 b_2 b_3^2 t_1^2 t_2^3 u_1 w_3 - b_1^2 b_2^2 b_3 t_2 t_2^3 u_1 w_3 + b_1^2 b_2 b_3^2 t_2^2 t_2^3 u_1 w_3 - \\
& b_1^2 b_2^2 b_3 t_1^2 t_2^2 t_2^3 u_1 w_3 - b_1 b_2^3 b_3 t_1^2 t_2^2 t_2^3 u_1 w_3 - b_1^2 b_2 b_3^2 t_1^2 t_2^2 t_2^3 u_1 w_3 + b_1^2 b_2^2 b_3 t_2^2 t_2^3 u_1 w_3 + \\
& 2 b_1 b_2^3 b_3 t_2^2 t_2^3 u_1 w_3 + b_1^2 b_2 b_3^2 t_2^2 t_2^3 u_1 w_3 + b_1 b_2^2 b_3^2 t_2^2 t_2^3 u_1 w_3 + b_1^2 b_2^2 b_3 t_2^2 t_2^3 u_1 w_3 - 2 b_1 b_2^3 b_3 t_1^2 t_2^2 t_2^3 u_1 w_3 - \\
& b_1^2 b_2^2 b_3 t_1^2 t_2^2 t_2^3 u_1 w_3 + b_1^2 b_2 b_3^2 t_1^2 t_2^2 t_2^3 u_1 w_3 - b_1^2 b_2^2 b_3 t_1^2 t_2^2 t_2^3 u_1 w_3 - b_1^2 b_2 b_3^2 t_1^2 t_2^2 t_2^3 u_1 w_3 + b_1^2 b_2^2 b_3 t_1^2 t_2^2 t_2^3 u_1 w_3 + \\
& b_1 b_2^3 b_3 t_1^2 t_2^2 t_2^3 u_1 w_3 + b_1 b_2^2 b_3^2 t_1^2 t_2^2 t_2^3 u_1 w_3 - b_1^3 b_2 b_3 u_2 w_3 - b_1^2 b_2^2 b_3 u_2 w_3 - b_1^2 b_2 b_3^2 u_2 w_3 - 2 b_1^2 b_2 b_3^2 u_2 w_3 + \\
& b_1^3 b_2 b_3 t_2 u_2 w_3 + 3 b_1^2 b_2^2 b_3 t_2 u_2 w_3 + b_1 b_2^3 b_3 t_2 u_2 w_3 + 2 b_1^2 b_2 b_3^2 t_2 u_2 w_3 + b_1 b_2^2 b_3^2 t_2 u_2 w_3 - b_1^3 b_2 b_3 t_1^2 t_2 u_2 w_3 - \\
& 2 b_1^2 b_2^2 b_3 t_1^2 t_2 u_2 w_3 - b_1 b_2^3 b_3 t_1^2 t_2 u_2 w_3 - b_1^2 b_2 b_3^2 t_1^2 t_2 u_2 w_3 - b_1 b_2^2 b_3^2 t_1^2 t_2 u_2 w_3 + b_1^2 b_2^2 b_3 t_2^2 u_2 w_3 + b_1^2 b_2^2 b_3 t_2^2 u_2 w_3 + \\
& b_1^3 b_2^2 t_2^2 u_2 w_3 + b_1^2 b_2 b_3^2 t_2^2 u_2 w_3 + b_1^2 b_2 b_3^2 t_1^2 t_2^2 u_2 w_3 - b_1^2 b_2^2 b_3 t_1^2 t_2^2 u_2 w_3 - b_1^2 b_2 b_3^2 t_1^2 t_2^2 u_2 w_3 - b_1^2 b_2^2 b_3 t_2 t_2^3 u_2 w_3 - \\
& b_1^2 b_2 b_3^2 t_2 t_2^3 u_2 w_3 + b_1^2 b_2 b_3^2 t_2^2 t_2^3 u_2 w_3 + b_1^2 b_2 b_3 t_1^2 t_2^2 t_2^3 u_2 w_3 + b_1^2 b_2^2 b_3 t_1^2 t_2^2 t_2^3 u_2 w_3 + b_1^3 b_2^2 t_1^2 t_2^2 t_2^3 u_2 w_3 + \\
& b_1^2 b_2 b_3^2 t_1^2 t_2^2 t_2^3 u_2 w_3 - b_1^3 b_2 b_3 t_2^2 t_2^3 u_2 w_3 - 2 b_1^2 b_2^2 b_3 t_2^2 t_2^3 u_2 w_3 - b_1 b_2^3 b_3 t_2^2 t_2^3 u_2 w_3 - b_1^2 b_2 b_3^2 t_2^2 t_2^3 u_2 w_3 - \\
& b_1 b_2^2 b_3^2 t_2^2 t_2^3 u_2 w_3 + b_1^3 b_2 b_3 t_1^2 t_2^2 t_2^3 u_2 w_3 + 3 b_1^2 b_2^2 b_3 t_1^2 t_2^2 t_2^3 u_2 w_3 + b_1 b_2^3 b_3 t_1^2 t_2^2 t_2^3 u_2 w_3 + 2 b_1^2 b_2 b_3^2 t_1^2 t_2^2 t_2^3 u_2 w_3 + \\
& b_1 b_2^2 b_3^2 t_1^2 t_2^2 t_2^3 u_2 w_3 - b_1^3 b_2 b_3 t_1^2 t_2^2 t_2^3 u_2 w_3 - b_1^2 b_2^2 b_3 t_1^2 t_2^2 t_2^3 u_2 w_3 - b_1^2 b_2 b_3^2 t_1^2 t_2^2 t_2^3 u_2 w_3 - 2 b_1^2 b_2 b_3^2 t_1^2 t_2^2 t_2^3 u_2 w_3 + \\
& b_1^3 b_2 b_3 u_3 w_3 + b_1^2 b_2^2 b_3 u_3 w_3 - b_1 b_2^3 b_3 u_3 w_3 - b_1^2 b_2^2 b_3 u_3 w_3 - b_1^2 b_2 b_3^2 u_3 w_3 - b_1^2 b_2 b_3^2 u_3 w_3 - b_1^3 b_2 b_3 t_1^2 t_2^2 u_3 w_3 - \\
& b_1^2 b_2^2 b_3 t_1^2 t_2^2 u_3 w_3 + b_1^3 b_2^2 t_1^2 t_2^2 u_3 w_3 + b_1^2 b_2^2 t_1^2 t_2^2 u_3 w_3 + b_1^3 b_2 b_3 t_1^2 t_2^2 u_3 w_3 + b_1^2 b_2^2 b_3 t_1^2 t_2^2 u_3 w_3 + b_1^3 b_2^2 t_2 t_2^3 u_3 w_3 + \\
& b_1^2 b_2^2 b_3 t_2 t_2^3 u_3 w_3 + b_1^3 b_2 b_3 t_2 t_2^3 u_3 w_3 + b_1^2 b_2^2 b_3 t_2 t_2^3 u_3 w_3 - b_1^3 b_2 b_3 t_2^2 t_2^3 u_3 w_3 - b_1^2 b_2^2 b_3 t_2^2 t_2^3 u_3 w_3 - b_1^3 b_2^2 t_1^2 t_2^2 t_2^3 u_3 w_3 - \\
& b_1^2 b_2^2 b_3 t_1^2 t_2^2 t_2^3 u_3 w_3 - b_1^3 b_2 b_3 t_1^2 t_2^2 t_2^3 u_3 w_3 - b_1^2 b_2^2 b_3 t_1^2 t_2^2 t_2^3 u_3 w_3 + b_1^3 b_2 b_3 t_1^2 t_2^2 t_2^3 u_3 w_3 + b_1^2 b_2^2 b_3 t_1^2 t_2^2 t_2^3 u_3 w_3) / \\
& (b_1^3 b_2^2 b_3 + 2 b_1^2 b_2^2 b_3 + b_1 b_2^4 b_3 + b_1^2 b_2 b_3^2 + 3 b_1^2 b_2^2 b_3^2 + 2 b_1 b_2^3 b_3^2 + b_1^2 b_2 b_3^3 + b_1 b_2^2 b_3^3 - b_1^3 b_2^2 b_3 t_1^2 t_2^2 - \\
& 2 b_1^2 b_2^2 b_3 t_1^2 t_2^2 - b_1 b_2^4 b_3 t_1^2 t_2^2 - b_1^3 b_2 b_3^2 t_1^2 t_2^2 - 3 b_1^2 b_2^2 b_3^2 t_1^2 t_2^2 - 2 b_1 b_2^3 b_3^2 t_1^2 t_2^2 - b_1^2 b_2 b_3^3 t_1^2 t_2^2 - \\
& b_1 b_2^2 b_3^3 t_1^2 t_2^2 - b_1^3 b_2^2 b_3 t_2^2 t_2^3 - 2 b_1^2 b_2^2 b_3 t_2^2 t_2^3 - b_1 b_2^4 b_3 t_2^2 t_2^3 - b_1^3 b_2 b_3^2 t_2^2 t_2^3 - 3 b_1^2 b_2^2 b_3^2 t_2^2 t_2^3 - \\
& 2 b_1 b_2^3 b_3^2 t_2^2 t_2^3 - b_1^2 b_2 b_3^3 t_2^2 t_2^3 - b_1 b_2^2 b_3^3 t_2^2 t_2^3 + b_1^3 b_2^2 b_3 t_1^2 t_2^4 t_2^3 + 2 b_1^2 b_2^2 b_3 t_1^2 t_2^4 t_2^3 + b_1 b_2^4 b_3 t_1^2 t_2^4 t_2^3 + \\
& b_1^3 b_2 b_3^2 t_1^2 t_2^4 t_2^3 + 3 b_1^2 b_2^2 b_3^2 t_1^2 t_2^4 t_2^3 + 2 b_1 b_2^3 b_3^2 t_1^2 t_2^4 t_2^3 + b_1^2 b_2 b_3^3 t_1^2 t_2^4 t_2^3 + b_1 b_2^2 b_3^3 t_1^2 t_2^4 t_2^3) ]
\end{aligned}$$

```

θ[i_] [e_ℰ] := (e /. {b_i :-> -b_i, c_i :-> -c_i, u_i :-> w_i, w_i :-> u_i, t_i :-> t_i^-1}) // CF // N_w_i u_i -> i;
θ[] [e_ℰ] := e;
θ[n_, r___] [e_ℰ] := e // θ[n] // θ[r];

```

$uR_{1,2}$  //  $\theta[1, 2]$

$E[1,$

$$\frac{1}{2 b_1^2 b_2 t_1 t_2 + 2 b_1 b_2^2 t_1 t_2} (b_1^2 b_2^2 c_1 t_1 t_2 + b_1 b_2^3 c_1 t_1 t_2 + b_1^3 b_2 c_2 t_1 t_2 + b_1^2 b_2^2 c_2 t_1 t_2 - 2 b_1 b_2 t_1 u_1 w_1 + 2 b_1 b_2 t_1 t_2 u_1 w_1 + \\
2 b_2^2 t_1 t_2 u_1 w_1 - 2 b_2^2 t_1^2 t_2 u_1 w_1 - 2 b_1 b_2 t_2 u_2 w_1 + 2 b_1 b_2 t_1 t_2^2 u_2 w_1 - 2 b_1 b_2 t_1 u_1 w_2 + \\
2 b_1 b_2 t_1^2 t_2 u_1 w_2 - 2 b_1 b_2 t_2 u_2 w_2 + 2 b_1^2 t_1 t_2 u_2 w_2 + 2 b_1 b_2 t_1 t_2 u_2 w_2 - 2 b_1^2 t_1 t_2^2 u_2 w_2) ]$$

$uR_{1,2} \equiv (uR_{1,2} // \theta[1, 2])$

True

$R_{1,2}^+ \equiv (R_{1,2}^+ // \theta[1, 2])$

$$\frac{b_1^2 c_2 - u_1 w_2 + t_1^2 u_1 w_2}{b_1} == \frac{b_1^2 c_2 t_1^2 - u_2 w_1 + t_1^2 u_2 w_1}{b_1 t_1^2}$$

$(lhs = uR_{1,2} ** uR_{2,3}) \equiv (lhs // \theta[1, 2, 3])$

True

$(lhs = uR_{1,2} ** uR_{1,3} ** uR_{2,3}) \equiv (lhs // \theta[1, 2, 3])$

True

Simplify[ $V_{\Phi} \equiv (V_{\Phi} // \theta[1, 2, 3])$ ]

$$\begin{aligned}
& (-b_2^2 b_3 (b_2 + b_3) (-1 + t_1^2) t_2 (b_3 ((-1 + t_2^2 t_3^2) u_2 w_1 + u_1 ((-1 + t_2^2 t_3^2 + t_2 (-1 + t_3^2)) w_1 + (-1 + t_2^2 t_3^2) w_2)) + \\
& \quad b_2 (t_2 (-1 + t_3^2) u_1 w_1 + u_3 w_1 + u_1 w_3 - t_2^2 t_3^2 (u_3 w_1 + u_1 w_3))) + \\
& b_1^3 (b_2^2 t_2 (-1 + t_3^2) ((-1 + t_1^2 t_2^2) u_2 w_3 + u_3 ((-1 + t_1^2 t_2^2) w_2 + (1 + t_2) (-1 + t_1^2 t_2) w_3)) + \\
& \quad b_2^3 (-1 + t_2^2) ((-1 + t_1^2 t_2^2 t_3^2) u_3 w_2 + u_2 ((-1 + t_1^2 t_2) (1 + t_2 t_3^2) w_2 + (-1 + t_1^2 t_2^2 t_3^2) w_3)) + \\
& \quad b_2 b_3 (-u_3 w_3 - t_1^2 t_2^4 t_3^2 u_3 w_3 + t_2 ((t_1^2 - t_3^2) u_3 (w_2 + w_3) + u_2 ((-1 + t_3^2) w_2 + (t_1^2 - t_3^2) w_3)) + \\
& \quad t_2^2 (-(-1 + t_1^2) (-1 + t_3^2) u_2 (w_2 + w_3) + u_3 (-(-1 + t_1^2) (-1 + t_3^2) w_2 + (1 + t_1^2 t_3^2) w_3)) + \\
& \quad t_3^2 (t_3^2 (u_2 w_3 + u_3 (w_2 + w_3)) - t_1^2 (u_3 (w_2 + w_3) + u_2 ((-1 + t_3^2) w_2 + w_3)))))) + \\
& b_1 b_2 (b_3^3 (u_1 ((1 + t_1^2 t_2) (-1 + t_2^2) (-1 + t_2 t_3^2) w_1 + t_2 (-t_3^2 + t_2^2 t_3^2 - t_2 (-1 + t_3^2) - t_1^2 (-1 + t_2^2 - t_2 (-1 + t_3^2))) w_2) + \\
& \quad t_2 u_2 (-t_3^2 w_1 + t_2^2 t_3^2 (w_1 - w_2) + w_2 - t_2 (-1 + t_3^2) (w_1 + w_2) + \\
& \quad t_1^2 (-(-1 + t_2^2 - t_2 (-1 + t_3^2)) w_1 + (-1 + t_2^2 t_3^2 + t_2 (-1 + t_3^2)) w_2))) - \\
& \quad b_2^3 t_2 (-1 + t_3^2) ((-1 + t_1^2 t_2^2) u_1 w_3 + u_3 ((-1 + t_1^2 t_2^2) w_1 - (-1 + t_1^2) t_2 w_3)) + \\
& \quad b_2^2 b_3 t_2 ((1 + t_2) u_2 ((-1 + t_1^2 t_2) (-1 + t_3^2) w_1 - (-1 + t_1^2) (-1 + t_2 t_3^2) w_3) + \\
& \quad u_1 ((-1 + t_2) (1 + t_1^2 t_2) (-1 + t_3^2) w_1 + (1 + t_2) ((-1 + t_1^2 t_2) (-1 + t_3^2) w_2 + 2 (-1 + t_2) (t_1^2 - t_3^2) w_3)) + \\
& \quad u_3 (-w_2 - t_2 w_2 - w_3 + t_2 w_3 + t_1^2 (2 (-1 + t_2^2) w_1 + (1 + t_2 - t_2 t_3^2 - t_2^2 t_3^2) w_2 - (-1 + t_2 - t_2 t_3^2 + t_2^2 t_3^2) w_3) + \\
& \quad t_3^2 (-2 (-1 + t_2^2) w_1 + t_2 ((1 + t_2) w_2 + (-1 + t_2) w_3)))) + \\
& b_2 b_2^3 (u_1 ((-1 + t_2) (1 + t_1^2 t_2) (-1 + t_2^2 t_3^2 + 2 t_2 (-1 + t_3^2)) w_1 + t_2 ((1 - 2 t_2^2 + t_2^2 t_3^2 - 2 t_2 (-1 + t_3^2) + \\
& \quad t_1^2 (1 + t_2^2 (-2 + t_3^2) + 2 t_2 (-1 + t_3^2))) w_2 + (1 + (1 - 2 t_2^2) t_3^2 + t_1^2 (-2 + t_2^2 (1 + t_3^2))) w_3)) + \\
& \quad t_2 (u_3 ((1 + (1 - 2 t_2^2) t_3^2 + t_1^2 (-2 + t_2^2 (1 + t_3^2))) w_1 - (-1 + t_1^2) t_2 (-1 + t_3^2) w_2) + \\
& \quad u_2 ((1 - 2 t_3^2 + t_2^2 t_3^2 - 2 t_2 (-1 + t_3^2) + t_1^2 (1 + t_2^2 (-2 + t_3^2) + 2 t_2 (-1 + t_3^2))) w_1 + \\
& \quad (-1 + t_1^2) ((-2 + 2 t_2^2 t_3^2 + t_2 (-1 + t_3^2)) w_2 - t_2 (-1 + t_3^2) w_3)))) + \\
& b_1^2 (-b_3^3 (-1 + t_2^2) ((-1 + t_1^2 t_2^2 t_3^2) u_1 w_2 + u_2 ((-1 + t_1^2 t_2^2 t_3^2) w_1 + (1 + t_1^2 t_2) (-1 + t_2 t_3^2) w_2)) - \\
& \quad b_2^3 t_2 (-1 + t_3^2) ((-1 + t_1^2 t_2^2) (u_1 - u_2) w_3 + u_3 ((-1 + t_1^2 t_2^2) w_1 + (1 - t_1^2 t_2^2) w_2 + (1 - 2 (-1 + t_1^2) t_2 - t_1^2 t_2^2) w_3)) + \\
& \quad b_2 b_2^3 (t_2 t_3^2 u_3 w_1 - t_3^2 t_3^2 u_3 w_1 - u_1 w_2 + 2 t_2^2 u_1 w_2 - t_2^2 t_3^2 u_1 w_2 + u_3 w_2 - 2 t_2^2 u_3 w_2 + t_2^2 t_3^2 u_3 w_2 + t_2 t_3^2 u_1 w_3 - \\
& \quad t_3^2 t_3^2 u_1 w_3 + t_1^2 t_2 (u_3 ((-1 + t_2^2) w_1 + t_2 (1 + (-2 + t_2^2) t_3^2) w_2) - u_1 (t_3^2 t_3^2 w_2 + t_2 (1 - 2 t_3^2) w_2 + w_3 - t_2^2 w_3)) + \\
& \quad u_2 (- (1 + t_1^2 t_2^4 t_3^2 + t_2^2 (-2 + t_3^2 + t_1^2 (1 - 2 t_3^2))) w_1 + 3 t_2 t_3^2 w_2 - 3 t_3^2 t_3^2 w_2 + \\
& \quad w_3 - 2 t_2^2 w_3 + t_2^2 t_3^2 w_3 + t_1^2 t_2 (3 (-1 + t_2^2) w_2 + t_2 (1 + (-2 + t_2^2) t_3^2) w_3))) - \\
& \quad b_2^2 b_3 (u_3 w_3 + t_1^2 t_2^4 t_3^2 u_3 w_3 + t_2^2 (-(-1 + t_1^2) (-1 + t_3^2) u_1 w_2 - (-1 + t_1^2) (-1 + t_3^2) u_2 (w_1 - w_2 - 2 w_3) + \\
& \quad u_3 (2 (-1 + t_1^2) (-1 + t_3^2) w_2 + (-2 + t_3^2 + t_1^2 (1 - 2 t_3^2)) w_3)) + \\
& \quad t_2 ((1 + t_1^2 - 2 t_3^2) u_1 w_3 + u_2 (-2 (-1 + t_3^2) w_2 + (1 - 2 t_1^2 + t_3^2) w_3) + \\
& \quad u_3 ((1 + t_1^2 - 2 t_3^2) w_1 - (-1 + 2 t_1^2 - t_3^2) (w_2 + w_3))) + t_3^2 (t_3^2 ((u_1 - 2 u_2) w_3 + u_3 (w_1 - 2 (w_2 + w_3))) + \\
& \quad t_1^2 ((-2 + t_3^2) u_1 w_3 + u_2 (2 (-1 + t_3^2) w_2 + (1 + t_3^2) w_3) + u_3 ((-2 + t_3^2) w_1 + (1 + t_3^2) (w_2 + w_3)))))) / \\
& (b_1 b_2 (b_1 + b_2) b_3 (b_2 + b_3) (b_1 + b_2 + b_3) (-1 + t_1^2 t_2^2) (-1 + t_2^2 t_3^2)) = \\
& \theta
\end{aligned}$$

The bug: h is not a scalar! It remains a function of the b's and the t's, and the action of strand doubling on those is not trivial. So our conclusion that  $G\Phi$  is independent of h is wrong.

$$\begin{aligned}
\text{sol} = \{ & h[ \_ ] \Rightarrow h, \quad g_{12}[b1_, b2_, t1_, t2_] \Rightarrow \\
& (b2 - b1 t1^2 - b2 t1^2 + b1 t1^2 t2^2 + b1^2 g_{11}[b1, b2, t1, t2] - b1^2 t1^2 t2^2 g_{11}[b1, b2, t1, t2]) / \\
& (b1 b2 (-1 + t1^2 t2^2)), \quad g_{21}[b1_, b2_, t1_, t2_] \Rightarrow \\
& (-b2 + b1 t1 + b2 t1 - b1 t1 t2^2 - b2 t1 t2^2 + b2^2 t1^2 t2^2 + b2^2 g_{22}[b1, b2, t1, t2] - b2^2 t1^2 t2^2 g_{22}[b1, b2, t1, t2]) / \\
& (b1 b2 (-1 + t1^2 t2^2)), \quad g_{22}[b1_, b2_, t1_, t2_] \Rightarrow \frac{1}{b2^2 (b1 + b2) (-1 + t1^2) (-1 + t1^2 t2^2)} \\
& (b2 (b1 + b2) - b2 (b1 + b2) t1 - b2 (b1 + b2) t1^2 + b1 (b1 + b2) t1^3 + b2 (b1 + b2) t1^3 + b2 (b1 + b2) t1 t2^2 - \\
& \quad b2 (b1 + b2) t1^2 t2^2 - 2 b1 (b1 + b2) t1^3 t2^2 - b2 (b1 + b2) t1^3 t2^2 + b2 (b1 + b2) t1^4 t2^2 + b1 (b1 + b2) t1^3 t2^4 - \\
& \quad b1 b2 (-1 + t1^2 t2^2)^2 h[b1, b2, t1, t2]^2 - b1^2 (b1 + b2) t1 (-1 + t2^2) (-1 + t1^2 t2^2) g_{11}[b1, b2, t1, t2]), \\
& g_{11}[b1_, b2_, t1_, t2_] \Rightarrow -((-b2 h^2 t1 + b1 t1^2 + b2 t1^2 - b1 t1^2 t2^2 - b2 t1^2 t2^2 + b2 h^2 t1^3 t2^2) / \\
& \quad (b1 (b1 + b2) (-1 + t1^2 t2^2))) \};
\end{aligned}$$

Flatten[

(CoefficientRules[# // . sol, {u<sub>1</sub>, u<sub>2</sub>, w<sub>1</sub>, w<sub>2</sub>}] /. (p<sub>-</sub> → c<sub>-</sub>) ⇒ Simplify@c) & /@ Simplify[  
 ({Eqn1, Eqn2, Eqn3} /. And → Sequence /. a<sub>-</sub> == b<sub>-</sub> ⇒ Simplify[a - b]) / { $\frac{1}{b_1 b_2 (b_1 + b_2)} w_3, 1,$   
 $\frac{1}{b_1 b_2 (b_1 + b_2) t_1 t_2}, (1 - b_2 g_{22}[b_1, b_2, t_1, t_2] + b_1 (-b_2 g_{12}[b_1, b_2, t_1, t_2] g_{21}[b_1, b_2, t_1, t_2] +$   
 $g_{11}[b_1, b_2, t_1, t_2] (-1 + b_2 g_{22}[b_1, b_2, t_1, t_2]))^{-1}$ } /. {b<sub>1</sub> → b1, b<sub>2</sub> → b2, t<sub>1</sub> → t1, t<sub>2</sub> → t2}

]  
 ] // Column

0  
 0  
 0  
 0  
 0  
 0  
 0

Solve[b1 b2 (-1 + t1<sup>2</sup> t2<sup>2</sup>)<sup>2</sup> h[b1, b2, t1, t2]<sup>2</sup> +  
 (b1 + b2) (-b2 + b2 t1 + b2 t1<sup>2</sup> - b1 t1<sup>3</sup> - b2 t1<sup>3</sup> - b2 t1 t2<sup>2</sup> + b2 t1<sup>2</sup> t2<sup>2</sup> + 2 b1 t1<sup>3</sup> t2<sup>2</sup> +  
 b2 t1<sup>3</sup> t2<sup>2</sup> - b2 t1<sup>4</sup> t2<sup>2</sup> - b1 t1<sup>3</sup> t2<sup>4</sup> + b1<sup>2</sup> t1 (-1 + t2<sup>2</sup>) (-1 + t1<sup>2</sup> t2<sup>2</sup>) g<sub>11</sub>[b1, b2, t1, t2] +  
 b2<sup>2</sup> (-1 + t1<sup>2</sup>) (-1 + t1<sup>2</sup> t2<sup>2</sup>) g<sub>22</sub>[b1, b2, t1, t2] == 0, g<sub>22</sub>[b1, b2, t1, t2]

{ {g<sub>22</sub>[b1, b2, t1, t2] →  $\frac{1}{b_2^2 (b_1 + b_2) (-1 + t_1^2) (-1 + t_1^2 t_2^2)}$   
 (b2 (b1 + b2) - b2 (b1 + b2) t1 - b2 (b1 + b2) t1<sup>2</sup> + b1 (b1 + b2) t1<sup>3</sup> + b2 (b1 + b2) t1<sup>3</sup> + b2 (b1 + b2) t1 t2<sup>2</sup> -  
 b2 (b1 + b2) t1<sup>2</sup> t2<sup>2</sup> - 2 b1 (b1 + b2) t1<sup>3</sup> t2<sup>2</sup> - b2 (b1 + b2) t1<sup>3</sup> t2<sup>2</sup> + b2 (b1 + b2) t1<sup>4</sup> t2<sup>2</sup> + b1 (b1 + b2) t1<sup>3</sup> t2<sup>4</sup> -  
 b1 b2 (-1 + t1<sup>2</sup> t2<sup>2</sup>)<sup>2</sup> h[b1, b2, t1, t2]<sup>2</sup> - b1<sup>2</sup> (b1 + b2) t1 (-1 + t2<sup>2</sup>) (-1 + t1<sup>2</sup> t2<sup>2</sup>) g<sub>11</sub>[b1, b2, t1, t2] ) } }

GV = CF[V // . sol ]

E[h, (-b<sub>1</sub><sup>3</sup> b<sub>2</sub> c<sub>2</sub> - b<sub>1</sub><sup>2</sup> b<sub>2</sub><sup>2</sup> c<sub>2</sub> + b<sub>1</sub><sup>3</sup> b<sub>2</sub> c<sub>2</sub> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>2</sup> + b<sub>1</sub><sup>2</sup> b<sub>2</sub><sup>2</sup> c<sub>2</sub> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>2</sup> + 2 h<sup>2</sup> b<sub>2</sub><sup>2</sup> t<sub>1</sub> u<sub>1</sub> w<sub>1</sub> - 2 b<sub>1</sub> b<sub>2</sub> t<sub>1</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> - 2 b<sub>2</sub><sup>2</sup> t<sub>1</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> + 2 b<sub>1</sub> b<sub>2</sub> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> +  
 2 b<sub>2</sub><sup>2</sup> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> - 2 h<sup>2</sup> b<sub>2</sub><sup>2</sup> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> - 2 h<sup>2</sup> b<sub>1</sub> b<sub>2</sub> u<sub>2</sub> w<sub>1</sub> + 2 b<sub>1</sub><sup>2</sup> t<sub>1</sub> u<sub>2</sub> w<sub>1</sub> + 2 b<sub>1</sub> b<sub>2</sub> t<sub>1</sub> u<sub>2</sub> w<sub>1</sub> - 2 b<sub>1</sub><sup>2</sup> t<sub>1</sub> t<sub>2</sub><sup>2</sup> u<sub>2</sub> w<sub>1</sub> - 2 b<sub>1</sub> b<sub>2</sub> t<sub>1</sub> t<sub>2</sub><sup>2</sup> u<sub>2</sub> w<sub>1</sub> +  
 2 h<sup>2</sup> b<sub>1</sub> b<sub>2</sub> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>2</sup> u<sub>2</sub> w<sub>1</sub> + 2 b<sub>1</sub> b<sub>2</sub> u<sub>1</sub> w<sub>2</sub> + 2 b<sub>2</sub><sup>2</sup> u<sub>1</sub> w<sub>2</sub> - 2 h<sup>2</sup> b<sub>1</sub> b<sub>2</sub> t<sub>1</sub> u<sub>1</sub> w<sub>2</sub> - 2 b<sub>1</sub> b<sub>2</sub> t<sub>1</sub><sup>2</sup> u<sub>1</sub> w<sub>2</sub> - 2 b<sub>2</sub><sup>2</sup> t<sub>1</sub><sup>2</sup> u<sub>1</sub> w<sub>2</sub> + 2 h<sup>2</sup> b<sub>1</sub> b<sub>2</sub> t<sub>1</sub><sup>3</sup> t<sub>2</sub><sup>2</sup> u<sub>1</sub> w<sub>2</sub> -  
 2 b<sub>1</sub><sup>2</sup> u<sub>2</sub> w<sub>2</sub> + 2 h<sup>2</sup> b<sub>1</sub><sup>2</sup> u<sub>2</sub> w<sub>2</sub> - 2 b<sub>1</sub> b<sub>2</sub> u<sub>2</sub> w<sub>2</sub> + 2 b<sub>1</sub><sup>2</sup> t<sub>1</sub> u<sub>2</sub> w<sub>2</sub> + 2 b<sub>1</sub> b<sub>2</sub> t<sub>1</sub> u<sub>2</sub> w<sub>2</sub> - 2 b<sub>1</sub><sup>2</sup> t<sub>1</sub> t<sub>2</sub><sup>2</sup> u<sub>2</sub> w<sub>2</sub> - 2 b<sub>1</sub> b<sub>2</sub> t<sub>1</sub> t<sub>2</sub><sup>2</sup> u<sub>2</sub> w<sub>2</sub> +  
 2 b<sub>1</sub><sup>2</sup> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>2</sup> u<sub>2</sub> w<sub>2</sub> - 2 h<sup>2</sup> b<sub>1</sub><sup>2</sup> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>2</sup> u<sub>2</sub> w<sub>2</sub> + 2 b<sub>1</sub> b<sub>2</sub> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>2</sup> u<sub>2</sub> w<sub>2</sub>) / (-2 b<sub>1</sub><sup>2</sup> b<sub>2</sub> - 2 b<sub>1</sub> b<sub>2</sub><sup>2</sup> + 2 b<sub>1</sub><sup>2</sup> b<sub>2</sub> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>2</sup> + 2 b<sub>1</sub> b<sub>2</sub><sup>2</sup> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>2</sup>) ]

GV<sub>i</sub> = A[{1, 2}] [GV]

E[ $\frac{1}{h}$ , (h<sup>2</sup> b<sub>1</sub><sup>3</sup> b<sub>2</sub> c<sub>2</sub> t<sub>1</sub> + h<sup>2</sup> b<sub>1</sub><sup>2</sup> b<sub>2</sub><sup>2</sup> c<sub>2</sub> t<sub>1</sub> - h<sup>2</sup> b<sub>1</sub><sup>3</sup> b<sub>2</sub> c<sub>2</sub> t<sub>1</sub><sup>3</sup> t<sub>2</sub><sup>2</sup> - h<sup>2</sup> b<sub>1</sub><sup>2</sup> b<sub>2</sub><sup>2</sup> c<sub>2</sub> t<sub>1</sub><sup>3</sup> t<sub>2</sub><sup>2</sup> - 2 h<sup>2</sup> b<sub>2</sub><sup>2</sup> t<sub>1</sub> u<sub>1</sub> w<sub>1</sub> + 2 b<sub>1</sub> b<sub>2</sub> t<sub>1</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> + 2 b<sub>2</sub><sup>2</sup> t<sub>1</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> -  
 2 b<sub>1</sub> b<sub>2</sub> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> - 2 b<sub>2</sub><sup>2</sup> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> + 2 h<sup>2</sup> b<sub>2</sub><sup>2</sup> t<sub>1</sub><sup>3</sup> t<sub>2</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> + 2 h<sup>2</sup> b<sub>1</sub> b<sub>2</sub> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> - 2 b<sub>1</sub><sup>2</sup> t<sub>1</sub><sup>3</sup> u<sub>2</sub> w<sub>1</sub> - 2 b<sub>1</sub> b<sub>2</sub> t<sub>1</sub><sup>3</sup> u<sub>2</sub> w<sub>1</sub> +  
 2 b<sub>1</sub><sup>2</sup> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>2</sup> u<sub>2</sub> w<sub>1</sub> + 2 b<sub>1</sub> b<sub>2</sub> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>2</sup> u<sub>2</sub> w<sub>1</sub> - 2 h<sup>2</sup> b<sub>1</sub> b<sub>2</sub> t<sub>1</sub><sup>4</sup> t<sub>2</sub><sup>2</sup> u<sub>2</sub> w<sub>1</sub> - 2 b<sub>1</sub> b<sub>2</sub> u<sub>1</sub> w<sub>2</sub> - 2 b<sub>2</sub><sup>2</sup> u<sub>1</sub> w<sub>2</sub> + 2 h<sup>2</sup> b<sub>1</sub> b<sub>2</sub> t<sub>1</sub> u<sub>1</sub> w<sub>2</sub> + 2 b<sub>1</sub> b<sub>2</sub> t<sub>1</sub><sup>2</sup> u<sub>1</sub> w<sub>2</sub> +  
 2 b<sub>2</sub><sup>2</sup> t<sub>1</sub><sup>2</sup> u<sub>1</sub> w<sub>2</sub> - 2 h<sup>2</sup> b<sub>1</sub> b<sub>2</sub> t<sub>1</sub><sup>3</sup> t<sub>2</sub><sup>2</sup> u<sub>1</sub> w<sub>2</sub> + 2 b<sub>1</sub><sup>2</sup> t<sub>1</sub> u<sub>2</sub> w<sub>2</sub> - 2 h<sup>2</sup> b<sub>1</sub><sup>2</sup> t<sub>1</sub> u<sub>2</sub> w<sub>2</sub> + 2 b<sub>1</sub> b<sub>2</sub> t<sub>1</sub> u<sub>2</sub> w<sub>2</sub> - 2 h<sup>2</sup> b<sub>1</sub> b<sub>2</sub> t<sub>1</sub> u<sub>2</sub> w<sub>2</sub> +  
 2 h<sup>2</sup> b<sub>1</sub> b<sub>2</sub> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>2</sup> u<sub>2</sub> w<sub>2</sub> - 2 b<sub>1</sub><sup>2</sup> t<sub>1</sub><sup>3</sup> t<sub>2</sub><sup>2</sup> u<sub>2</sub> w<sub>2</sub> + 2 h<sup>2</sup> b<sub>1</sub><sup>2</sup> t<sub>1</sub><sup>3</sup> t<sub>2</sub><sup>2</sup> u<sub>2</sub> w<sub>2</sub> + 2 h<sup>2</sup> b<sub>1</sub> b<sub>2</sub> t<sub>1</sub><sup>3</sup> t<sub>2</sub><sup>2</sup> u<sub>2</sub> w<sub>2</sub> - 2 h<sup>2</sup> b<sub>1</sub> b<sub>2</sub> t<sub>1</sub><sup>4</sup> t<sub>2</sub><sup>2</sup> u<sub>2</sub> w<sub>2</sub>) /  
 (-2 h<sup>2</sup> b<sub>1</sub><sup>2</sup> b<sub>2</sub> t<sub>1</sub> - 2 h<sup>2</sup> b<sub>1</sub> b<sub>2</sub><sup>2</sup> t<sub>1</sub> + 2 h<sup>2</sup> b<sub>1</sub><sup>2</sup> b<sub>2</sub> t<sub>1</sub><sup>3</sup> t<sub>2</sub><sup>2</sup> + 2 h<sup>2</sup> b<sub>1</sub> b<sub>2</sub><sup>2</sup> t<sub>1</sub><sup>3</sup> t<sub>2</sub><sup>2</sup>) ]

GV \*\* GV<sub>i</sub>

E[1, 0]

G<sub>σ</sub> = GV<sub>i</sub><sup>σ</sup>[[{1,2},3]] \*\* GV<sub>i</sub><sup>σ</sup>[1,2] \*\* GV<sup>σ</sup>[2,3] \*\* GV<sup>σ</sup>[1,{2,3}]

E[1,

(-b<sub>1</sub> b<sub>2</sub><sup>3</sup> b<sub>3</sub> t<sub>2</sub> u<sub>1</sub> w<sub>1</sub> - 2 b<sub>1</sub> b<sub>2</sub><sup>2</sup> b<sub>3</sub><sup>2</sup> t<sub>2</sub> u<sub>1</sub> w<sub>1</sub> - b<sub>1</sub> b<sub>2</sub> b<sub>3</sub><sup>3</sup> t<sub>2</sub> u<sub>1</sub> w<sub>1</sub> + b<sub>1</sub> b<sub>2</sub><sup>3</sup> b<sub>3</sub> t<sub>2</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> + b<sub>2</sub><sup>4</sup> b<sub>3</sub> t<sub>2</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> + 2 b<sub>1</sub> b<sub>2</sub><sup>2</sup> b<sub>3</sub><sup>2</sup> t<sub>2</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> + 2 b<sub>2</sub><sup>3</sup> b<sub>3</sub><sup>2</sup> t<sub>2</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> +  
 b<sub>1</sub> b<sub>2</sub> b<sub>3</sub><sup>3</sup> t<sub>2</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> + b<sub>2</sub><sup>2</sup> b<sub>3</sub><sup>3</sup> t<sub>2</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> - b<sub>1</sub> b<sub>2</sub><sup>3</sup> b<sub>3</sub> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> - b<sub>2</sub><sup>4</sup> b<sub>3</sub> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> - 2 b<sub>1</sub> b<sub>2</sub><sup>2</sup> b<sub>3</sub><sup>2</sup> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> - 2 b<sub>2</sub><sup>3</sup> b<sub>3</sub><sup>2</sup> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> -  
 b<sub>1</sub> b<sub>2</sub> b<sub>3</sub><sup>3</sup> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> - b<sub>2</sub><sup>2</sup> b<sub>3</sub><sup>3</sup> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> + b<sub>1</sub> b<sub>2</sub><sup>3</sup> b<sub>3</sub> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>3</sup> u<sub>1</sub> w<sub>1</sub> + 2 b<sub>1</sub> b<sub>2</sub><sup>2</sup> b<sub>3</sub><sup>2</sup> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>3</sup> u<sub>1</sub> w<sub>1</sub> + b<sub>1</sub> b<sub>2</sub> b<sub>3</sub><sup>3</sup> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>3</sup> u<sub>1</sub> w<sub>1</sub> +  
 b<sub>1</sub> b<sub>2</sub><sup>3</sup> b<sub>3</sub> t<sub>2</sub> t<sub>3</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> + 2 b<sub>1</sub> b<sub>2</sub><sup>2</sup> b<sub>3</sub><sup>2</sup> t<sub>2</sub> t<sub>3</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> + b<sub>1</sub> b<sub>2</sub> b<sub>3</sub><sup>3</sup> t<sub>2</sub> t<sub>3</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> - b<sub>1</sub> b<sub>2</sub><sup>3</sup> b<sub>3</sub> t<sub>2</sub><sup>2</sup> t<sub>3</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> - b<sub>2</sub><sup>4</sup> b<sub>3</sub> t<sub>2</sub><sup>2</sup> t<sub>3</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> -  
 2 b<sub>1</sub> b<sub>2</sub><sup>2</sup> b<sub>3</sub><sup>2</sup> t<sub>2</sub><sup>2</sup> t<sub>3</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> - 2 b<sub>2</sub><sup>3</sup> b<sub>3</sub><sup>2</sup> t<sub>2</sub><sup>2</sup> t<sub>3</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> - b<sub>1</sub> b<sub>2</sub> b<sub>3</sub><sup>3</sup> t<sub>2</sub><sup>2</sup> t<sub>3</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> - b<sub>2</sub><sup>2</sup> b<sub>3</sub><sup>3</sup> t<sub>2</sub><sup>2</sup> t<sub>3</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> + b<sub>1</sub> b<sub>2</sub><sup>3</sup> b<sub>3</sub> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>2</sup> t<sub>3</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> +  
 b<sub>2</sub><sup>4</sup> b<sub>3</sub> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>2</sup> t<sub>3</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> + 2 b<sub>1</sub> b<sub>2</sub><sup>2</sup> b<sub>3</sub><sup>2</sup> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>2</sup> t<sub>3</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> + 2 b<sub>2</sub><sup>3</sup> b<sub>3</sub><sup>2</sup> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>2</sup> t<sub>3</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> + b<sub>1</sub> b<sub>2</sub> b<sub>3</sub><sup>3</sup> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>2</sup> t<sub>3</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> + b<sub>2</sub><sup>2</sup> b<sub>3</sub><sup>3</sup> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>2</sup> t<sub>3</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> -  
 b<sub>1</sub> b<sub>2</sub><sup>3</sup> b<sub>3</sub> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>3</sup> t<sub>3</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> - 2 b<sub>1</sub> b<sub>2</sub><sup>2</sup> b<sub>3</sub><sup>2</sup> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>3</sup> t<sub>3</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> - b<sub>1</sub> b<sub>2</sub> b<sub>3</sub><sup>3</sup> t<sub>1</sub><sup>2</sup> t<sub>2</sub><sup>3</sup> t<sub>3</sub><sup>2</sup> u<sub>1</sub> w<sub>1</sub> + b<sub>1</sub><sup>2</sup> b<sub>2</sub> b<sub>3</sub><sup>2</sup> u<sub>2</sub> w<sub>1</sub> + b<sub>1</sub> b<sub>2</sub><sup>2</sup> b<sub>3</sub><sup>2</sup> u<sub>2</sub> w<sub>1</sub> + b<sub>1</sub><sup>2</sup> b<sub>3</sub><sup>3</sup> u<sub>2</sub> w<sub>1</sub> +





$$2 b_1 b_2^3 b_3^2 t_2^2 t_3^2 - b_1^2 b_2 b_3^3 t_2^2 t_3^2 - b_1 b_2^2 b_3^3 t_2^2 t_3^2 + b_1^3 b_2^2 b_3 t_1^2 t_2^4 t_3^2 + 2 b_1^2 b_2^3 b_3 t_1^2 t_2^4 t_3^2 + b_1 b_2^4 b_3 t_1^2 t_2^4 t_3^2 + b_1^3 b_2 b_3^2 t_1^2 t_2^4 t_3^2 + 3 b_1^2 b_2^2 b_3^2 t_1^2 t_2^4 t_3^2 + 2 b_1 b_2^3 b_3^2 t_1^2 t_2^4 t_3^2 + b_1^2 b_2 b_3^3 t_1^2 t_2^4 t_3^2 + b_1 b_2^2 b_3^3 t_1^2 t_2^4 t_3^2 ]$$

**err = Simplify**[**G** ≡ (**G** //  $\theta$ [1, 2, 3])] [**1**]

$$\begin{aligned} & \left( -b_2^2 b_3 (b_2 + b_3) (-1 + t_1^2) t_2 (b_3 ((-1 + t_2^2 t_3^2) u_2 w_1 + u_1 ((-1 + t_2^2 t_3^2 + t_2 (-1 + t_3^2)) w_1 + (-1 + t_2^2 t_3^2) w_2)) \right. \\ & \quad \left. + b_2 (t_2 (-1 + t_3^2) u_1 w_1 + u_3 w_1 + u_1 w_3 - t_2^2 t_3^2 (u_3 w_1 + u_1 w_3)) \right) + \\ & b_1^3 (b_2^2 t_2 (-1 + t_3^2) ((-1 + t_1^2 t_2^2) u_2 w_3 + u_3 ((-1 + t_1^2 t_2^2) w_2 + (1 + t_2) (-1 + t_1^2 t_2) w_3)) + \\ & b_3^2 (-1 + t_2^2) ((-1 + t_1^2 t_2^2 t_3^2) u_3 w_2 + u_2 ((-1 + t_1^2 t_2) (1 + t_2 t_3^2) w_2 + (-1 + t_1^2 t_2^2 t_3^2) w_3)) + \\ & b_2 b_3 (-u_3 w_3 - t_1^2 t_2^4 t_3^2 u_3 w_3 + t_2 ((t_1^2 - t_3^2) u_3 (w_2 + w_3) + u_2 ((-1 + t_3^2) w_2 + (t_1^2 - t_3^2) w_3)) + \\ & \quad t_2^2 (-(-1 + t_1^2) (-1 + t_3^2) u_2 (w_2 + w_3) + u_3 (-(-1 + t_1^2) (-1 + t_3^2) w_2 + (1 + t_1^2 t_3^2) w_3)) + \\ & \quad t_2^3 (t_3^2 (u_2 w_3 + u_3 (w_2 + w_3)) - t_1^2 (u_3 (w_2 + w_3) + u_2 ((-1 + t_3^2) w_2 + w_3))) \Big) + \\ & b_1 b_2 (b_3^3 (u_1 ((1 + t_1^2 t_2) (-1 + t_2^2) (-1 + t_2 t_3^2) w_1 + t_2 (-t_3^2 + t_2^2 t_3^2 - t_2 (-1 + t_3^2) - t_1^2 (-1 + t_2^2 - t_2 (-1 + t_3^2))) w_2) + \\ & \quad t_2 u_2 (-t_3^2 w_1 + t_2^2 t_3^2 (w_1 - w_2) + w_2 - t_2 (-1 + t_3^2) (w_1 + w_2) + \\ & \quad t_1^2 (-(-1 + t_2^2 - t_2 (-1 + t_3^2)) w_1 + (-1 + t_2^2 t_3^2 + t_2 (-1 + t_3^2)) w_2)) \Big) - \\ & b_3^2 t_2 (-1 + t_3^2) ((-1 + t_1^2 t_2^2) u_1 w_3 + u_3 ((-1 + t_1^2 t_2^2) w_1 - (-1 + t_1^2) t_2 w_3)) + \\ & b_2^2 b_3 t_2 ((1 + t_2) u_2 ((-1 + t_1^2 t_2) (-1 + t_3^2) w_1 - (-1 + t_1^2) (-1 + t_2 t_3^2) w_3) + \\ & \quad u_1 ((-1 + t_2) (1 + t_1^2 t_2) (-1 + t_3^2) w_1 + (1 + t_2) ((-1 + t_1^2 t_2) (-1 + t_3^2) w_2 + 2 (-1 + t_2) (t_1^2 - t_3^2) w_3)) + \\ & \quad u_3 (-w_2 - t_2 w_2 - w_3 + t_2 w_3 + t_1^2 (2 (-1 + t_2^2) w_1 + (1 + t_2 - t_2 t_3^2 - t_2^2 t_3^2) w_2 - (-1 + t_2 - t_2 t_3^2 + t_2^2 t_3^2) w_3) + \\ & \quad t_3^2 (-2 (-1 + t_2^2) w_1 + t_2 ((1 + t_2) w_2 + (-1 + t_2) w_3))) \Big) + \\ & b_2 b_3^2 (u_1 ((-1 + t_2) (1 + t_1^2 t_2) (-1 + t_2^2 t_3^2 + 2 t_2 (-1 + t_3^2)) w_1 + t_2 ((1 - 2 t_3^2 + t_2^2 t_3^2 - 2 t_2 (-1 + t_3^2) + \\ & \quad t_1^2 (1 + t_2^2 (-2 + t_3^2) + 2 t_2 (-1 + t_3^2))) w_2 + (1 + (1 - 2 t_2^2) t_3^2 + t_1^2 (-2 + t_2^2 (1 + t_3^2))) w_3)) + \\ & \quad t_2 (u_3 ((1 + (1 - 2 t_2^2) t_3^2 + t_1^2 (-2 + t_2^2 (1 + t_3^2))) w_1 - (-1 + t_1^2) t_2 (-1 + t_3^2) w_2) + \\ & \quad u_2 ((1 - 2 t_3^2 + t_2^2 t_3^2 - 2 t_2 (-1 + t_3^2) + t_1^2 (1 + t_2^2 (-2 + t_3^2) + 2 t_2 (-1 + t_3^2))) w_1 + \\ & \quad (-1 + t_1^2) ((-2 + 2 t_2^2 t_3^2 + t_2 (-1 + t_3^2)) w_2 - t_2 (-1 + t_3^2) w_3))) \Big) + \\ & b_1^2 (-b_3^3 (-1 + t_2^2) ((-1 + t_1^2 t_2^2 t_3^2) u_1 w_2 + u_2 ((-1 + t_1^2 t_2^2 t_3^2) w_1 + (1 + t_1^2 t_2) (-1 + t_2 t_3^2) w_2)) - \\ & b_3^2 t_2 (-1 + t_3^2) ((-1 + t_1^2 t_2^2) (u_1 - u_2) w_3 + u_3 ((-1 + t_1^2 t_2^2) w_1 + (1 - t_1^2 t_2^2) w_2 + (1 - 2 (-1 + t_1^2) t_2 - t_1^2 t_2^2) w_3)) + \\ & b_2 b_3^2 (t_2 t_3^2 u_3 w_1 - t_3^2 t_3^2 u_3 w_1 - u_1 w_2 + 2 t_2^2 u_1 w_2 - t_2^2 t_3^2 u_1 w_2 + u_3 w_2 - 2 t_2^2 u_3 w_2 + t_2^2 t_3^2 u_3 w_2 + t_2 t_3^2 u_1 w_3 - \\ & \quad t_2^3 t_3^2 u_1 w_3 + t_1^2 t_2 (u_3 ((-1 + t_2^2) w_1 + t_2 (1 + (-2 + t_2^2) t_3^2) w_2) - u_1 (t_3^2 t_3^2 w_2 + t_2 (1 - 2 t_3^2) w_2 + w_3 - t_2^2 w_3)) + \\ & \quad u_2 (- (1 + t_1^2 t_2^4 t_3^2 + t_2^2 (-2 + t_3^2 + t_1^2 (1 - 2 t_3^2))) w_1 + 3 t_2 t_3^2 w_2 - 3 t_3^2 t_3^2 w_2 + \\ & \quad w_3 - 2 t_2^2 w_3 + t_2^2 t_3^2 w_3 + t_1^2 t_2 (3 (-1 + t_2^2) w_2 + t_2 (1 + (-2 + t_2^2) t_3^2) w_3)) \Big) - \\ & b_2^2 b_3 (u_3 w_3 + t_1^2 t_2^4 t_3^2 u_3 w_3 + t_2^2 (-(-1 + t_1^2) (-1 + t_3^2) u_1 w_2 - (-1 + t_1^2) (-1 + t_3^2) u_2 (w_1 - w_2 - 2 w_3) + \\ & \quad u_3 (2 (-1 + t_1^2) (-1 + t_3^2) w_2 + (-2 + t_3^2 + t_1^2 (1 - 2 t_3^2)) w_3)) + \\ & \quad t_2 ((1 + t_1^2 - 2 t_3^2) u_1 w_3 + u_2 (-2 (-1 + t_3^2) w_2 + (1 - 2 t_1^2 + t_3^2) w_3) + \\ & \quad u_3 ((1 + t_1^2 - 2 t_3^2) w_1 - (-1 + 2 t_1^2 - t_3^2) (w_2 + w_3)) + t_3^2 (t_3^2 ((u_1 - 2 u_2) w_3 + u_3 (w_1 - 2 (w_2 + w_3))) + \\ & \quad t_1^2 ((-2 + t_3^2) u_1 w_3 + u_2 (2 (-1 + t_3^2) w_2 + (1 + t_3^2) w_3) + u_3 ((-2 + t_3^2) w_1 + (1 + t_3^2) (w_2 + w_3)))) \Big) \Big) / \\ & (b_1 b_2 (b_1 + b_2) b_3 (b_2 + b_3) (b_1 + b_2 + b_3) (-1 + t_1^2 t_2^2) (-1 + t_2^2 t_3^2)) = \\ & \theta \end{aligned}$$

**VV** ≡ (**GV** /. **h** → 1)

True

**GV**

$$\begin{aligned} \mathbb{E} [h, & (-b_1^3 b_2 c_2 - b_1^2 b_2^2 c_2 + b_1^3 b_2 c_2 t_1^2 t_2^2 + b_1^2 b_2^2 c_2 t_1^2 t_2^2 + 2 h^2 b_2^2 t_1 u_1 w_1 - 2 b_1 b_2 t_1^2 u_1 w_1 - 2 b_2^2 t_1^2 u_1 w_1 + 2 b_1 b_2 t_1^2 t_2^2 u_1 w_1 + \\ & 2 b_2^2 t_1^2 t_2^2 u_1 w_1 - 2 h^2 b_2^2 t_1^2 t_2^2 u_1 w_1 - 2 h^2 b_1 b_2 u_2 w_1 + 2 b_1^2 t_1 u_2 w_1 + 2 b_1 b_2 t_1 u_2 w_1 - 2 b_1^2 t_1 t_2^2 u_2 w_1 - 2 b_1 b_2 t_1 t_2^2 u_2 w_1 + \\ & 2 h^2 b_1 b_2 t_1^2 t_2^2 u_2 w_1 + 2 b_1 b_2 u_1 w_2 + 2 b_2^2 u_1 w_2 - 2 h^2 b_1 b_2 t_1 u_1 w_2 - 2 b_1 b_2 t_1^2 u_1 w_2 - 2 b_2^2 t_1^2 u_1 w_2 + 2 h^2 b_1 b_2 t_1^3 t_2^2 u_1 w_2 - \\ & 2 b_1^2 u_2 w_2 + 2 h^2 b_1^2 u_2 w_2 - 2 b_1 b_2 u_2 w_2 + 2 b_2^2 t_1 u_2 w_2 + 2 b_1 b_2 t_1 u_2 w_2 - 2 b_1^2 t_1 t_2^2 u_2 w_2 - 2 b_1 b_2 t_1 t_2^2 u_2 w_2 + \\ & 2 b_1^2 t_1^2 t_2^2 u_2 w_2 - 2 h^2 b_1^2 t_1^2 t_2^2 u_2 w_2 + 2 b_1 b_2 t_1^2 t_2^2 u_2 w_2) / (-2 b_1^2 b_2 - 2 b_1 b_2^2 + 2 b_1^2 b_2 t_1^2 t_2^2 + 2 b_1 b_2^2 t_1^2 t_2^2) ] \end{aligned}$$

**V** ≡ **V** $^{\sigma[3,2,1]}$

$\mathbb{E}[1, \theta]$

(**uR**<sub>1,2</sub>) $^{\sigma[1,2,3]}$  ≡ **V** ≡ **uR**<sub>2,3</sub> ≡ (**V** $^{\sigma[3,2,1]}$ ) $^{\sigma[1,3,2]}$  ≡ **uR**<sub>1,3</sub> ≡ **V** $^{\sigma[3,1,2]}$

True

**lhs** =  $V_{\sigma} \star \star V_{\sigma}^{\sigma[1, \{2, 3\}, 4]} \star \star V_{\sigma}^{\sigma[2, 3, 4]}$

$$\mathbb{E} \left[ 1, \left( \dots 29750 \dots + 2 b_1^2 b_2^2 b_3^2 b_4^2 t_1^4 t_2^6 t_3^6 t_4^4 u_4 w_4 + b_1 b_2^3 b_3^2 b_4^2 t_1^4 t_2^6 t_3^6 t_4^4 u_4 w_4 \right) / \left( b_1^4 b_2^2 b_3^2 b_4 + 3 b_1^3 b_2^3 b_3^2 b_4 + 3 b_1^2 b_2^4 b_3^2 b_4 + \dots 954 \dots + b_1 b_2^3 b_3 b_4^4 t_1^4 t_2^6 t_3^6 t_4^4 + b_1^2 b_2 b_3^2 b_4^4 t_1^4 t_2^6 t_3^6 t_4^4 + b_1 b_2^2 b_3^2 b_4^4 t_1^4 t_2^6 t_3^6 t_4^4 \right) \right]$$

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**rhs** =  $V_{\sigma}^{\sigma[\{1, 2\}, 3, 4]} \star \star V_{\sigma}^{\sigma[1, 2, \{3, 4\}]}$

$$\mathbb{E} \left[ 1, \left( \dots 29750 \dots + 2 b_1^2 b_2^2 b_3^2 b_4^2 t_1^4 t_2^6 t_3^6 t_4^4 u_4 w_4 + b_1 b_2^3 b_3^2 b_4^2 t_1^4 t_2^6 t_3^6 t_4^4 u_4 w_4 \right) / \left( b_1^4 b_2^2 b_3^2 b_4 + 3 b_1^3 b_2^3 b_3^2 b_4 + 3 b_1^2 b_2^4 b_3^2 b_4 + \dots 954 \dots + b_1 b_2^3 b_3 b_4^4 t_1^4 t_2^6 t_3^6 t_4^4 + b_1^2 b_2 b_3^2 b_4^4 t_1^4 t_2^6 t_3^6 t_4^4 + b_1 b_2^2 b_3^2 b_4^4 t_1^4 t_2^6 t_3^6 t_4^4 \right) \right]$$

large output

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**lhs**  $\equiv$  **rhs**

True