Making P manifestly polynomial

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Utilities

```
CF[𝔼[ω_, L_, Q_, P_]] := Expand /@ Together /@
    𝔼[ω /. b_l_ ⧴ Log[t_l], L, Q /. b_l_ ⧴ Log[t_l],
     P /. b_l_ ⧴ Log[t_l]];
𝔼 /: 𝔼[ω1_, L1_, Q1_, P1_] 𝔼[ω2_, L2_, Q2_, P2_] :=
  CF@𝔼[ω1 ω2, L1 + L2, ω2 Q1 + ω1 Q2, ω2^4 P1 + ω1^4 P2];
```

Normal Ordering Operators

```
N_{u_i_ c_j_ →k_}[𝔼[ω_, L_, Q_, P_]] := With[{q = e^-γ β u_k + γ c_k}, CF[
    𝔼[ω, γ c_k + (L /. c_j → 0), ω e^-γ β u_k + (Q /. u_i → 0),
      e^-q DP_{c_j→D_γ, u_i→D_β}[P][e^q]] /. {γ → ∂_{c_j} L, β → ω^-1 ∂_{u_i} Q}]];
N_{w_i_ c_j_ →k_}[𝔼[ω_, L_, Q_, P_]] := With[{q = e^γ α w_k + γ c_k}, CF[
    𝔼[ω, γ c_k + (L /. c_j → 0), ω e^γ α w_k + (Q /. w_i → 0),
      e^-q DP_{c_j→D_γ, w_i→D_α}[P][e^q]] /. {γ → ∂_{c_j} L, α → ω^-1 ∂_{w_i} Q}]];
N_{w_i_ u_j_ →k_}[𝔼[ω_, L_, Q_, P_]] :=
  With[{q = (1 - t_k) μ^-1 α β + μ^-1 β u_k + μ^-1 δ u_k w_k + μ^-1 α w_k}, CF[
    𝔼[μ ω, L, μ ω q + μ (Q /. w_i | u_j → 0),
      μ^4 e^-q DP_{w_i→D_α, u_j→D_β}[P][e^q] + ω^4 Λ[k]] /.
     μ → 1 + (t_k - 1) δ /.
    {α → ω^-1 (∂_{w_i} Q /. u_j → 0), β → ω^-1 (∂_{u_j} Q /. w_i → 0),
     δ → ω^-1 ∂_{w_i,u_j} Q}]];
```

```
Λ[k_] := (1 - t_k) (α^2 β^2 + 4 α β δ μ + 2 δ^2 μ^2) / 2 + 2 μ^2 (α β + δ μ) c_k -
   β (2 μ - 1) (α β + 2 δ μ) u_k + 2 β δ μ^2 c_k u_k - β^2 δ (3 μ - 1) u_k^2 / 2 +
   α (α β + 2 δ μ) w_k + 2 α δ μ^2 c_k w_k - 2 (t_k - 1) δ^2 (α β + δ μ) u_k w_k +
   2 δ^2 μ^2 c_k u_k w_k - β δ^2 (2 μ - 1) u_k^2 w_k + α^2 δ (1 + μ) w_k^2 / 2 +
   α δ^2 u_k w_k^2 - (t_k - 1) δ^4 u_k^2 w_k^2 / 2;
```

The Λόγος

From Projects/OneCo-1606/NOE-1.nb:

```
ϵ /: ϵ^n_ /; n ≥ 1 := 0;
Λ[b_, c_, u_, w_, α_, β_, δ_, ν_] :=
  2 c w α δ ν + 2 c u β δ ν + 2 c u w δ^2 ν + u w^2 α δ^2 ν^3 - 1/2 b u^2 w^2 δ^4 ν^3 + 1/2 w^2 α^2 δ (2 + b δ) ν^3 -
   u^2 w β δ^2 (1 + 2 b δ) ν^3 - 1/2 u^2 β^2 δ (2 + 3 b δ) ν^3 + 2 c (δ + α β ν) - 2 b u w δ^2 ν^2 (δ + α β ν) +
   w α ν^2 (2 δ + α β ν) - u β (1 + 2 b δ) ν^2 (2 δ + α β ν) - 1/2 b ν (2 δ^2 + 4 α β δ ν + α^2 β^2 ν^2);
```

```
NO[u_i_, c_j_, k_][P_. 𝔼[Q_]] := Simp@Module[{q(*,α,β,θ*)},
    q = e^-α β u_k + α c_k + θ;
    e^-q DP[P, c_j → D_α, u_i → D_β][e^q] 𝔼[q] /. {α → Together[∂_{c_j} Q], β → ∂_{u_i} Q, θ → (Q /. c_j | u_i → 0)}
  ];
```

```
NO[w_i_, c_j_, k_][P_. 𝔼[Q_]] := Simp@Module[{q(*,α,β,θ*)},
    q = e^α β w_k + α c_k + θ;
    e^-q DP[P, c_j → D_α, w_i → D_β][e^q] 𝔼[q] /. {α → Together[∂_{c_j} Q], β → ∂_{w_i} Q, θ → (Q /. c_j | w_i → 0)}
  ];
```

```
NO[w_i_, u_j_, k_][P_. 𝔼[Q_]] := Simp@Module[
    {α0 = ∂_{w_i} Q /. u_j → 0, β0 = ∂_{u_j} Q /. w_i → 0, δ0 = ∂_{w_i,u_j} Q, θ0 = Q /. w_i | u_j → 0, q(*,α,β,δ, θ, ν*)},
    q = -b_k ν α β + ν β u_k + ν δ u_k w_k + ν α w_k + θ;
    e^-q DP[P, w_i → D_α, u_j → D_β][ν (1 + ϵ ν Λ[b_k, c_k, u_k, w_k, α, β, δ, ν]) e^q] 𝔼[q] /.
     {α → α0, β → β0, δ → δ0, θ → θ0, ν -> (1 + b_k δ0)^-1}
  ];
```