

Pensieve header: The 1-smidgen invariant on 12-crossing knots.

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SetDirectory["~drorbn/AcademicPensieve/2016-10"]
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/home/drornbn/AcademicPensieve/2016-10
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## NOE-It

$$\Delta[k_] := (1 - t_k) (\alpha^2 \beta^2 + 4 \alpha \beta \delta \mu + 2 \delta^2 \mu^2) / 2 + 2 \mu^2 (\alpha \beta + \delta \mu) c_k - \beta (2 \mu - 1) (\alpha \beta + 2 \delta \mu) u_k + 2 \beta \delta \mu^2 c_k u_k - \beta^2 \delta (3 \mu - 1) * u_k^2 / 2 + \alpha (\alpha \beta + 2 \delta \mu) w_k + 2 \alpha \delta \mu^2 c_k w_k - 2 (t_k - 1) \delta^2 (\alpha \beta + \delta \mu) u_k w_k + 2 \delta^2 \mu^2 c_k u_k w_k - \beta \delta^2 (2 \mu - 1) * u_k^2 w_k + \alpha^2 \delta (1 + \mu) w_k^2 / 2 + \alpha \delta^2 u_k * w_k^2 - (t_k - 1) \delta^4 * u_k^2 * w_k^2 / 2;$$

```
DPx→Dα,y→Dβ[P_][f_] := (* means P[∂α,∂β][f] *)
Total[CoefficientRules[P, {x, y}] /. ({m_, n_} → c_) ⇒ c D[f, {α, m}, {β, n}]]
```

```
CF[IE[ω_, L_, Q_, P_]] := Expand /@ Together /@
IE[ω /. bL ⇒ Log[tL], L, Q /. bL ⇒ Log[tL], P /. bL ⇒ Log[tL]];
IE /: IE[ω1_, L1_, Q1_, P1_] IE[ω2_, L2_, Q2_, P2_] :=
CF@IE[ω1 ω2, L1 + L2, ω2 Q1 + ω1 Q2, ω24 P1 + ω14 P2];
```

```
Nui cj→k[IE[ω_, L_, Q_, P_]] := With[{q = e-γ β uk + γ ck}, CF[
IE[ω, γ ck + (L /. cj → θ), ω e-γ β uk + (Q /. ui → θ), e-q DPcj→Dγ,ui→Dβ[P][eq]] /.
{γ → ∂cj L, β → ω-1 ∂ui Q}]];
Nwi cj→k[IE[ω_, L_, Q_, P_]] := With[{q = eγ α wk + γ ck}, CF[
IE[ω, γ ck + (L /. cj → θ), ω eγ α wk + (Q /. wi → θ), e-q DPcj→Dγ,wi→Dα[P][eq]] /.
{γ → ∂cj L, α → ω-1 ∂wi Q}]];

```

```
Nwi uj→k[IE[ω_, L_, Q_, P_]] := With[{q = (1 - tk) μ-1 α β + μ-1 β uk + μ-1 δ uk wk + μ-1 α wk}, CF[
IE[μ ω, L, μ ω q + μ (Q /. wi | uj → θ), μ4 e-q DPwi→Dα,uj→Dβ[P][eq]] + ω4 Δ[k]] /. μ →
1 + (tk - 1) δ /. {α → ω-1 (∂wi Q /. uj → θ), β → ω-1 (∂uj Q /. wi → θ), δ → ω-1 ∂wi,uj Q}]];

```

```
mi,j→k[Z_] := Module[{x, y, z},
Z // Nwi cj→x // Nwx uj→y // ReplaceAll[{cx|y → cx, wj → wy}] // Nui cx→x //
ReplaceAll[z-i|j|x|y → zk] // CF];
```

```

Ri,j+ := E[1, bi cj, ui wj, -ci (ti - 1)2 / 2 - ci2 (ti - 1)2 / 2 +
  ci cj (tj2 - ti - 2) / 2 - cj ui wi / 2 + ci (1 - ti) ui wi - ui2 wi2 / 2 + ui wj + cj ti ui wj / 2 +
  ci (ti - 2) ti ui wj + ci (1 + tj) uj wj / 2 + (ti - 1) ui2 wi wj - (ti - 2) ti ui2 wj2 / 2];
Ri,j- := E[1, -bi cj, -ti-1 ui wj, ci (ti - 1)2 / 2 + ci2 (ti - 1)2 / 2 + ci cj (2 + ti - tj2) / 2 +
  cj ui wi / 2 + ci (ti - 1) ui wi + ui2 wi2 / 2 + (1 - ti-1) ui wj / 2 +
  ci (2 ti - 5 + 3 ti-1) ui wj / 2 + cj (ti-1 + 1 - ti-1 tj2) ui wj / 2 - ci (tj + 1) uj wj / 2 +
  (2 - 3 ti-1) ui2 wi wj / 2 + (1 + 2 ti-2 - 3 ti-1) ui2 wj2 / 2 - ti-1 (1 + tj) ui uj wj2 / 2];
uri- := E[ti-1/4, 0, 0, ci ti / 4 + ui wi / 8];
nri- := E[ti1/4, 0, 0, -ci ti3 / 4 - ti2 ui wi / 8];
uli- := E[ti1/4, 0, 0, ci ti (4 + ti) / 4 - ti2 ui wi / 8];
nli- := E[ti-1/4, 0, 0, -ci (1 + 4 ti-1) / 4 + ui wi / 8];

```

```

rot[_ , 0] = E[1, 0, 0, 0];
rot[i- , 1] := Module[{y}, nli ury // mi,y→i];
rot[i- , n_Integer] /; n > 1 := Module[{y}, rot[i, n - 1] rot[y, 1] // mi,y→i];
rot[i- , -1] := Module[{y}, nri uly // mi,y→i];
rot[i- , n_Integer] /; n < -1 := Module[{y}, rot[i, n + 1] rot[y, -1] // mi,y→i];

```

## Z

```

Z[K_] := Z[RVK@K];
Z[rvk_RVK] := Z[rvk] = Module[{todo, n, rots, ζ, done, st, x, ζ1, i, j, k, k1, k2, k3},
  {todo, rots} = List@@rvk;
  AppendTo[rots, 0];
  n = Length[todo];
  ζ = E[1, 0, 0, 0];
  done = {0};
  st = Range[0, 2 n + 1];
  While[todo != {},
    {x} = MaximalBy[todo, Length[done ∩ {#[[1]], #[[2]], #[[1]] - 1, #[[2]] - 1}] &, 1];
    Z$todo = todo; Z$x = x;
    {i, j} = List@@x;
    ζ1 = Switch[Head[x],
      Xp, m_{j,k→j} [R_{i,j}^+ (R_{k3,k}^- nr_{k1} ul_{k2} // m_{k,k1→k} // m_{k,k2→k} // m_{k,k3→k})],
      Xm, m_{j,k→j} [R_{i,j}^- (R_{k,k3}^+ nr_{k1} ul_{k2} // m_{k,k1→k} // m_{k,k2→k} // m_{k,k3→k})]
    ];
    ζ1 = rot[k, rots[[i]] ζ1 // m_{k,i→i}; rots[[i]] = 0;
    ζ1 = ζ1 rot[k, rots[[i+1]] // m_{i,k→i}; rots[[i+1]] = 0;
    ζ1 = rot[k, rots[[j]] ζ1 // m_{k,j→j}; rots[[j]] = 0;
    ζ1 = ζ1 rot[k, rots[[j+1]] // m_{j,k→j}; rots[[j+1]] = 0;
    ζ *= ζ1;
    If[MemberQ[done, i], ζ = ζ // m_{i,i+1→i}; st = st /. st[[i+2]] → st[[i+1]];
    If[MemberQ[done, i-1], ζ = ζ // m_{st[[i],i→st[[i]]}; st = st /. st[[i+1]] → st[[i]];
    If[MemberQ[done, j], ζ = ζ // m_{j,j+1→j}; st = st /. st[[j+2]] → st[[j+1]];
    If[MemberQ[done, j-1], ζ = ζ // m_{st[[j],j→st[[j]]}; st = st /. st[[j+1]] → st[[j]];
    done = done ∪ {i-1, i, j-1, j};
    todo = DeleteCases[todo, x]
  ];
  ζ /. {u_0 → u, c_0 → c, w_0 → w}
]

```

```
t_ = t;
```

## Computations

`RVK4K12 = Get["RVK4K12.m"]`

```
{Knot[12, Alternating, 1] →
RVK[{Xp[1, 4], Xm[3, 8], Xp[5, 10], Xm[7, 14], Xm[9, 2], Xp[11, 16],
Xp[13, 20], Xm[15, 6], Xm[17, 22], Xp[19, 12], Xm[21, 24], Xm[23, 18]},
{0, 0, 0, -1, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 1, 0, 0}],
... 2174 ..., Knot[12, NonAlternating, 888] → RVK[{Xp[1, 10], Xp[3, 12],
Xp[20, 5], Xp[16, 7], ... 5 ..., Xp[4, 19], Xp[6, 21], Xp[23, 14]}, ... 1 ...]}
```

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`Dynamic[lastrdone]`



`tab12 = Table[lastrdone = (K → Timing[Z[K /. RVK4K12]]), {K, First /@ RVK4K12}]`

```
{Knot[12, Alternating, 1] →
{55.6995, E[59 + 1/t^4 - 7/t^3 + 23/t^2 - 46/t - 46t + 23t^2 - 7t^3 + t^4, 0, 0, 95835338 + 179754373c/2 + 4/t^16 -
8c/t^16 - 105/t^15 + 208c/t^15 + 1355/t^14 - 5295c/2t^14 - 11436/t^13 + ... 127 ... + 1496898t^10uw - 1043651/2t^11uw +
292065/2t^12uw - 64597/2t^13uw + 5466t^14uw - 1335/2t^15uw + 105/2t^16uw - 2t^17uw]},
... 2174 ..., Knot[12, NonAlternating, 888] → ... 1 ...}
```

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`tab12 >> tab12.m`