

Pensieve header: Tracing Nwu - program, finding the right "in".

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2016-09"];
Once[<< KnotTheory`];
Once[<< ../Projects/Profile/Profile.m]
```

Loading KnotTheory` version of September 6, 2014, 13:37:37.2841.
Read more at <http://katlas.org/wiki/KnotTheory>.

This is Profile.m, Nov 2016 mods of July 1994 version

Rotational Virtual Knots

```
RVK::usage =
  "RVK[xs, rots] represents a Rotational Virtual Knot with a list of n Xp/Xm crossings xs and
  a length 2n list of rotation numbers rots. Crossing sites are indexed 1 through
  2n, and rots[[k]] is the rotation between site k-1 and site k. RVK is also a casting
  operator converting to the RVK presentation from other knot presentations.";
RVK[pd_PD] := Module[{n, xs, x, rots, front, k},
  n = Length[pd];
  xs = List@@pd /. x_X => If[PositiveQ[x], Xp[x[[4]], x[[1]], Xm[x[[2]], x[[1]]];
  rots = Table[0, {2 n}];
  front = {0};
  For[k = 0, k < 2 n, ++k,
  If[k == 0 || FreeQ[front, -k],
  front = Flatten[front /. k -> Catch[xs /. {
    Xp[k + 1, L_] | Xm[L_, k + 1] => Throw[{L, k + 1, 1 - L}],
    Xp[L_, k + 1] | Xm[k + 1, L_] => (++rots[[L]]; Throw[{1 - L, k + 1, L}])
  }]],
  If[MatchQ[front, {___, k, ___, -k, ___}], --rots[[k + 1]]
  ]
  ];
  RVK[xs, rots]
  ];
RVK[K_] := RVK[PD[K]];
```

NOE-It

Logos

$$\Delta[k_-] := (1 - t_k) (\alpha^2 \beta^2 + 4 \alpha \beta \delta \mu + 2 \delta^2 \mu^2) / 2 + 2 \mu^2 (\alpha \beta + \delta \mu) c_k - \beta (2 \mu - 1) (\alpha \beta + 2 \delta \mu) u_k + 2 \beta \delta \mu^2 c_k u_k - \beta^2 \delta (3 \mu - 1) * u_k^2 / 2 + \alpha (\alpha \beta + 2 \delta \mu) w_k + 2 \alpha \delta \mu^2 c_k w_k - 2 (t_k - 1) \delta^2 (\alpha \beta + \delta \mu) u_k w_k + 2 \delta^2 \mu^2 c_k u_k w_k - \beta \delta^2 (2 \mu - 1) * u_k^2 w_k + \alpha^2 \delta (1 + \mu) w_k^2 / 2 + \alpha \delta^2 u_k * w_k^2 - (t_k - 1) \delta^4 * u_k^2 * w_k^2 / 2;$$

```
DP[x->D_alpha, y->D_beta][P_-][f_-] := Profile[DP, (* means P[partial_alpha, partial_beta][f] *)
  Total[CoefficientRules[P, {x, y}] /. ({m_-, n_-} -> c_-) => c D[f, {alpha, m}, {beta, n}]]
  ]
```

```
CF[E[omega_-, L_-, Q_-, P_-]] := Profile[CF,
  Expand /@ Together /@
  E[omega /. b_L_- => Log[t_L], L, Q /. b_L_- => Log[t_L], P /. b_L_- => Log[t_L]]
  ];
E /: E[omega1_-, L1_-, Q1_-, P1_-] E[omega2_-, L2_-, Q2_-, P2_-] := CF@E[omega1 omega2, L1 + L2, omega2 Q1 + omega1 Q2, omega2^4 P1 + omega1^4 P2];
```

```

Nui cj → k- [E [ω-, L-, Q-, P-]] := Profile [Nuc,
  With [ {q = e-γ β uk + γ ck }, CF [
    E [ω, γ ck + (L / . cj → θ), ω e-γ β uk + (Q / . ui → θ), e-q DPcj → Dγ, ui → Dβ [P] [eq]] /. {γ → ∂cj L, β → ω-1 ∂ui Q}]]
];
Nwi cj → k- [E [ω-, L-, Q-, P-]] := Profile [Nwc,
  With [ {q = eγ α wk + γ ck }, CF [
    E [ω, γ ck + (L / . cj → θ), ω eγ α wk + (Q / . wi → θ), e-q DPcj → Dγ, wi → Dα [P] [eq]] /. {γ → ∂cj L, α → ω-1 ∂wi Q}]]
];

```

```

Nwi uj → k- [E [ω-, L-, Q-, P-]] := Profile [Nwu,
  in = E [ω, L, Q, P];
  out = Timing [With [ {q = (1 - tk) μ-1 α β + μ-1 β uk + μ-1 δ uk wk + μ-1 α wk }, CF [
    E [μ ω, L, μ ω q + μ (Q / . wi | uj → θ), μ4 e-q DPwi → Dα, uj → Dβ [P] [eq] + ω4 Δ [k]] /. μ → 1 + (tk - 1) δ /.
    {α → ω-1 (∂wi Q / . uj → θ), β → ω-1 (∂uj Q / . wi → θ), δ → ω-1 ∂wi, uj Q}]]];
  AppendTo [NwuTrace, {in, out, i, j, k}];
  out [[2]]
];

```

```

mi-, j- → k- [Z-] := Profile [m,
  Module [ {x, y, z},
    Z // Nwi cj → x // Nwx uj → y // ReplaceAll [ {cx|y → cx, wj → wy } ] // Nui cx → x //
    ReplaceAll [z-i|j|x|y → zk] // CF
  ]
];

```

```

Ri-, j-+ := E [1, bi cj, ui wj, -ci (ti - 1)2 / 2 - ci2 (ti - 1)2 / 2 + ci cj (tj2 - ti - 2) / 2 - cj ui wi / 2 + ci (1 - ti) ui wi -
  ui2 wi2 / 2 + ui wj + cj ti ui wj / 2 + ci (ti - 2) ti ui wj + ci (1 + tj) uj wj / 2 + (ti - 1) ui2 wi wj - (ti - 2) ti ui2 wj2 / 2];
Ri-, j-- := E [1, -bi cj, -ti-1 ui wj, ci (ti - 1)2 / 2 + ci2 (ti - 1)2 / 2 + ci cj (2 + ti - tj2) / 2 + cj ui wi / 2 +
  ci (ti - 1) ui wi + ui2 wi2 / 2 + (1 - ti-1) ui wj / 2 + ci (2 ti - 5 + 3 ti-1) ui wj / 2 + cj (ti-1 + 1 - ti-1 tj2) ui wj / 2 -
  ci (tj + 1) uj wj / 2 + (2 - 3 ti-1) ui2 wi wj / 2 + (1 + 2 ti-2 - 3 ti-1) ui2 wj2 / 2 - ti-1 (1 + tj) ui uj wj2 / 2];
uri- := E [ti-1/4, 0, 0, ci ti / 4 + ui wi / 8];
nri- := E [ti1/4, 0, 0, -ci ti3 / 4 - ti2 ui wi / 8];
uli- := E [ti1/4, 0, 0, ci ti (4 + ti) / 4 - ti2 ui wi / 8];
nli- := E [ti-1/4, 0, 0, -ci (1 + 4 ti-1) / 4 + ui wi / 8];

```

```

rot [_, 0] = E [1, 0, 0, 0];
rot [i-, 1] := Module [ {y}, nli ury // mi, y → i ];
rot [i-, n_Integer] /; n > 1 := Module [ {y}, rot [i, n - 1] rot [y, 1] // mi, y → i ];
rot [i-, -1] := Module [ {y}, nri uly // mi, y → i ];
rot [i-, n_Integer] /; n < -1 := Module [ {y}, rot [i, n + 1] rot [y, -1] // mi, y → i ];

```

Z

```

t_ = t;
Z[K_] := Z[RVK@K];
Z[rvk_RVK] := Profile[Z,
  NwuTrace = {};
  Module[{todo, n, rots,  $\xi$ , done, st, x,  $\xi$ 1, i, j, k, k1, k2, k3},
    {todo, rots} = List@@rvk;
    AppendTo[rots, 0];
    n = Length[todo];
     $\xi$  =  $\mathbb{E}$ [1, 0, 0, 0];
    done = {0};
    st = Range[0, 2 n + 1];
    While[todo != {},
      {x} = MaximalBy[todo, Length[done  $\cap$  {#[[1]], #[[2]], #[[1]] - 1, #[[2]] - 1}] &, 1];
      Z$todo = todo; Z$x = x;
      {i, j} = List@@x;
       $\xi$ 1 = Switch[Head[x],
        Xp, mj,k→j[R+i,j (R-k3,k nrk1 ulk2 // mk,k1→k // mk,k2→k // mk,k3→k)],
        Xm, mj,k→j[R-i,j (R+k,k3 nrk1 ulk2 // mk,k1→k // mk,k2→k // mk,k3→k)],
      ];
       $\xi$ 1 = rot[k, rots[[i]]]  $\xi$ 1 // mk,i→i; rots[[i]] = 0;
       $\xi$ 1 =  $\xi$ 1 rot[k, rots[[i + 1]]] // mi,k→i; rots[[i + 1]] = 0;
       $\xi$ 1 = rot[k, rots[[j]]]  $\xi$ 1 // mk,j→j; rots[[j]] = 0;
       $\xi$ 1 =  $\xi$ 1 rot[k, rots[[j + 1]]] // mj,k→j; rots[[j + 1]] = 0;
       $\xi$  *=  $\xi$ 1;
      If[MemberQ[done, i],  $\xi$  =  $\xi$  // mi,i+1→i; st = st /. st[[i + 2]] → st[[i + 1]]];
      If[MemberQ[done, i - 1],  $\xi$  =  $\xi$  // mst[[i],i→st[[i]]; st = st /. st[[i + 1]] → st[[i]]];
      If[MemberQ[done, j],  $\xi$  =  $\xi$  // mj,j+1→j; st = st /. st[[j + 2]] → st[[j + 1]]];
      If[MemberQ[done, j - 1],  $\xi$  =  $\xi$  // mst[[j],j→st[[j]]; st = st /. st[[j + 1]] → st[[j]]];
      done = done  $\cup$  {i - 1, i, j - 1, j};
      todo = DeleteCases[todo, x]
    ];
     $\xi$  /. {u0 → u, c0 → c, w0 → w}
  ]
]

```

```
BeginProfile[];
Timing[Z[Knot[10, 100]]]
EndProfile[];
```

 KnotTheory: Loading precomputed data in PD4Knots` 

$$\left\{ 20.0625, \mathbb{E} \left[13 + \frac{1}{t^4} - \frac{4}{t^3} + \frac{9}{t^2} - \frac{12}{t} - 12t + 9t^2 - 4t^3 + t^4, 0, 0, \right. \right.$$

$$2563146 + 667500c + \frac{6}{t^{16}} - \frac{8c}{t^{16}} - \frac{92}{t^{15}} + \frac{118c}{t^{15}} + \frac{723}{t^{14}} - \frac{892c}{t^{14}} - \frac{3818}{t^{13}} + \frac{4523c}{t^{13}} + \frac{15133}{t^{12}} - \frac{17161c}{t^{12}} - \frac{47848}{t^{11}} +$$

$$\frac{51709c}{t^{11}} + \frac{125539}{t^{10}} - \frac{128505c}{t^{10}} - \frac{281054}{t^9} + \frac{270279c}{t^9} + \frac{548129}{t^8} - \frac{489715c}{t^8} - \frac{945756}{t^7} + \frac{772841c}{t^7} + \frac{1460263}{t^6} -$$

$$\frac{1066829c}{t^6} - \frac{2034106}{t^5} + \frac{1282861c}{t^5} + \frac{2570432}{t^4} - \frac{1320331c}{t^4} - \frac{2956518}{t^3} + \frac{1107336c}{t^3} + \frac{3099338}{t^2} - \frac{640054c}{t^2} -$$

$$\frac{2958726}{t} - \frac{540c}{t} - 2000454t - 1197840ct + 1387610t^2 + 1472160ct^2 - 832998t^3 - 1456020ct^3 + 407256t^4 +$$

$$1204364ct^4 - 132546t^5 - 829886ct^5 - 9557t^6 + 453636ct^6 + 59220t^7 - 162131ct^7 - 58859t^8 - 11711ct^8 +$$

$$40498t^9 + 81439ct^9 - 22001t^{10} - 84595ct^{10} + 9704t^{11} + 59721ct^{11} - 3455t^{12} - 32685ct^{12} + 966t^{13} +$$

$$14251ct^{13} - 201t^{14} - 4919ct^{14} + 28t^{15} + 1307ct^{15} - 2t^{16} - 253ct^{16} + 32ct^{17} - 2ct^{18} - 493132uw + \frac{8uw}{t^{16}} -$$

$$\frac{110uw}{t^{15}} + \frac{782uw}{t^{14}} - \frac{3741uw}{t^{13}} + \frac{13420uw}{t^{12}} - \frac{38289uw}{t^{11}} + \frac{90216uw}{t^{10}} - \frac{180063uw}{t^9} + \frac{309652uw}{t^8} - \frac{463189uw}{t^7} +$$

$$\frac{603640uw}{t^6} - \frac{679221uw}{t^5} + \frac{641110uw}{t^4} - \frac{466226uw}{t^3} + \frac{173828uw}{t^2} + \frac{174368uw}{t} + 704708t uw - 767452t^2 uw +$$

$$688568t^3 uw - 515796t^4 uw + 314090t^5 uw - 139546t^6 uw + 22585t^7 uw + 34296t^8 uw - 47143t^9 uw +$$

$$37452t^{10} uw - 22269t^{11} uw + 10416t^{12} uw - 3835t^{13} uw + 1084t^{14} uw - 223t^{15} uw + 30t^{16} uw - 2t^{17} uw \left. \right\}$$

```
PrintProfile[];
```

CF: called 490 times, time in 18.265/18.265

Parents:

- (102) 1.127/ 1.127 under m
- (102) 2.717/ 2.717 under Nuc
- (102) 2.218/ 2.218 under Nwc
- (102) 11.660/ 11.660 under Nwu
- (82) 0.547/ 0.547 under Z

DP: called 306 times, time in 1.016/1.016

Parents:

- (102) 0.392/ 0.392 under Nuc
- (102) 0.298/ 0.298 under Nwc
- (102) 0.326/ 0.326 under Nwu

m: called 102 times, time in 0.284/19.377

Parents:

- (102) 0.284/ 19.380 under Z

Children:

- (102) 1.127/ 1.127 above CF
- (102) 0.109/ 3.218 above Nuc
- (102) 0.109/ 2.625 above Nwc
- (102) 0.141/ 12.120 above Nwu

Nwu: called 102 times, time in 0.141/12.123

Parents:

(102) 0.141/ 12.120 under m

Children:

(102) 11.660/ 11.660 above CF

(102) 0.326/ 0.326 above DP

Nwc: called 102 times, time in 0.109/2.625

Parents:

(102) 0.109/ 2.625 under m

Children:

(102) 2.218/ 2.218 above CF

(102) 0.298/ 0.298 above DP

Nuc: called 102 times, time in 0.109/3.218

Parents:

(102) 0.109/ 3.218 under m

Children:

(102) 2.717/ 2.717 above CF

(102) 0.392/ 0.392 above DP

Z: called 1 times, time in 0.076/20.

Parents:

(1) 0.076/ 20.000 under ProfileRoot

Children:

(82) 0.547/ 0.547 above CF

(102) 0.284/ 19.380 above m

ProfileRoot: called 0 times, time in 0./0.

Children:

(1) 0.076/ 20.000 above Z

```
{in, {time, out}, i, j, k} = SortBy[NwuTrace, -#[[2, 1]] &] [[3]];
{time, LeafCount@in, LeafCount@out, i, j, k}
{2.53125, 18338, 31227, x$1313, 13, y$1313}
```

$k = y;$ $in = in /. i \rightarrow (i = x)$

$$\mathbb{E} \left[3 + \frac{1}{t^2} - \frac{2}{t} - 4t + 5t^2 - 3t^3 + t^4, \dots 8 \dots + b_{13} c_x, \dots 1 \dots, \right. \\ \left. 7217 + \frac{4}{t^8} - \frac{30}{t^7} + \frac{124}{t^6} - \frac{374}{t^5} + \frac{914}{t^4} - \frac{1889}{t^3} + \frac{3372}{t^2} - \frac{5263}{t} - 8666t + 8987t^2 - 7778t^3 + 5103t^4 - \right. \\ \left. 1564t^5 - 1887t^6 + 4339t^7 - 5307t^8 + \dots 1850 \dots + \frac{68 u_{13}^2 w_x^2}{t^8} - \frac{343 u_{13}^2 w_x^2}{2 t^7} + \frac{715 u_{13}^2 w_x^2}{2 t^6} - \frac{1283 u_{13}^2 w_x^2}{2 t^5} + \right. \\ \left. \frac{2037 u_{13}^2 w_x^2}{2 t^4} - \frac{2919 u_{13}^2 w_x^2}{2 t^3} + \frac{1898 u_{13}^2 w_x^2}{t^2} - \frac{4451 u_{13}^2 w_x^2}{2 t} - 2110t u_{13}^2 w_x^2 + 1663t^2 u_{13}^2 w_x^2 - \frac{2247}{2} t^3 u_{13}^2 w_x^2 + \right. \\ \left. 641t^4 u_{13}^2 w_x^2 - 301t^5 u_{13}^2 w_x^2 + \frac{223}{2} t^6 u_{13}^2 w_x^2 - \frac{61}{2} t^7 u_{13}^2 w_x^2 + \frac{11}{2} t^8 u_{13}^2 w_x^2 - \frac{1}{2} t^9 u_{13}^2 w_x^2 \right]$$

large output

show less

show more

show all

set size limit...

Factor[in[[1]]]

$$\frac{1 - 2t + 3t^2 - 4t^3 + 5t^4 - 3t^5 + t^6}{t^2}$$

vars = Union@Cases[in, (c | u | w)_, ∞]

{C0, C20, Cx, U0, U13, U20, W13, W20, Wx}

ωs = CoefficientRules[in[[4]], vars] /.

(p_ → c_) ⇒ (Factor[in[[1]] c] /. a_. Numerator[Factor[in[[1]]]]^m_ ⇒ ω^{m-1}) Times@@(vars^p)

$$\left\{ \omega^4 C_0^2, \omega^4 C_0 C_x, \omega^2 C_0 U_0 W_{13}, \omega^2 C_0 U_0 W_x, \omega^2 C_0 U_{13} W_{13}, \omega^4 C_0 U_{13} W_{20}, \omega^2 C_0 U_{13} W_x, \omega^3 C_0, \omega^4 C_{20}^2, \omega^4 C_{20} C_x, \right. \\ \omega^3 C_{20} U_0 W_x, \omega^3 C_{20} U_{13} W_{13}, \omega^4 C_{20} U_{13} W_{20}, \omega^3 C_{20} U_{13} W_x, \omega^4 C_{20} U_{20} W_{20}, \omega^4 C_{20}, \omega^4 C_x^2, \omega^2 C_x U_0 W_{13}, \omega^2 C_x U_0 W_x, \\ \omega^2 C_x U_{13} W_{13}, \omega^4 C_x U_{13} W_{20}, \omega^2 C_x U_{13} W_x, \omega^4 C_x U_{20} W_{20}, \omega^3 C_x, U_0^2 W_{13}^2, U_0^2 W_{13} W_x, U_0^2 W_x^2, U_0 U_{13} W_{13}^2, \omega^2 U_0 U_{13} W_{13} W_{20}, \\ U_0 U_{13} W_{13} W_x, \omega^2 U_0 U_{13} W_{20} W_x, U_0 U_{13} W_x^2, \omega^3 U_0 U_{20} W_{20} W_x, \omega U_0 W_{13}, \omega U_0 W_x, U_{13}^2 W_{13}^2, \omega^2 U_{13}^2 W_{13} W_{20}, U_{13}^2 W_{13} W_x, \\ \omega^4 U_{13}^2 W_{20}^2, \omega^2 U_{13}^2 W_{20} W_x, U_{13}^2 W_x^2, \omega^4 U_{13} U_{20} W_{20}^2, \omega^3 U_{13} U_{20} W_{20} W_x, \omega U_{13} W_{13}, \omega^3 U_{13} W_{20}, \omega U_{13} W_x, \omega^4 U_{20} W_{20}, \omega^2 \left. \right\}$$

ωs /. {c_ → 1, ω | u_ | w_ → 2}

{16, 16, 16, 16, 16, 64, 16, 8, 16, 16, 32, 32, 64, 32, 64, 16, 16, 16, 16, 16, 64, 16, 64, 8, 16, 16, 16, 16, 64, 16, 64, 16, 128, 8, 8, 16, 64, 16, 256, 64, 16, 256, 128, 8, 32, 8, 64, 4}

ωs1 = CoefficientRules[in[[4]], vars] /.

(p_ → c_) ⇒ (Factor[c] /. a_. Numerator[Factor[in[[1]]]]^m_ ⇒ ω^m) Times@@(vars^p)

$$\left\{ \frac{C_0^2}{\omega}, \frac{C_0 C_x}{\omega}, \frac{C_0 U_0 W_{13}}{\omega^3}, \frac{C_0 U_0 W_x}{\omega^3}, \frac{C_0 U_{13} W_{13}}{\omega^3}, \frac{C_0 U_{13} W_{20}}{\omega}, \frac{C_0 U_{13} W_x}{\omega^3}, \frac{C_0}{\omega^2}, \frac{C_{20}^2}{\omega}, \frac{C_{20} C_x}{\omega}, \frac{C_{20} U_0 W_x}{\omega^2}, \frac{C_{20} U_{13} W_{13}}{\omega^2}, \frac{C_{20} U_{13} W_{20}}{\omega}, \right. \\ \frac{C_{20} U_{13} W_x}{\omega^2}, \frac{C_{20} U_{20} W_{20}}{\omega}, \frac{C_{20}}{\omega}, \frac{C_x^2}{\omega}, \frac{C_x U_0 W_{13}}{\omega^3}, \frac{C_x U_0 W_x}{\omega^3}, \frac{C_x U_{13} W_{13}}{\omega^3}, \frac{C_x U_{13} W_{20}}{\omega}, \frac{C_x U_{13} W_x}{\omega^3}, \frac{C_x U_{20} W_{20}}{\omega}, \frac{C_x}{\omega^2}, \frac{U_0^2 W_{13}^2}{\omega^5}, \\ \frac{U_0^2 W_{13} W_x}{\omega^5}, \frac{U_0^2 W_x^2}{\omega^5}, \frac{U_0 U_{13} W_{13}^2}{\omega^5}, \frac{U_0 U_{13} W_{13} W_{20}}{\omega^3}, \frac{U_0 U_{13} W_{13} W_x}{\omega^5}, \frac{U_0 U_{13} W_{20} W_x}{\omega^3}, \frac{U_0 U_{13} W_x^2}{\omega^5}, \frac{U_0 U_{20} W_{20} W_x}{\omega^2}, \frac{U_0 W_{13}}{\omega^4}, \frac{U_0 W_x}{\omega^4}, \frac{U_{13}^2 W_{13}^2}{\omega^5}, \\ \frac{U_{13}^2 W_{13} W_{20}}{\omega^3}, \frac{U_{13}^2 W_{13} W_x}{\omega^5}, \frac{U_{13}^2 W_{20}^2}{\omega}, \frac{U_{13}^2 W_{20} W_x}{\omega^3}, \frac{U_{13}^2 W_x^2}{\omega^5}, \frac{U_{13} U_{20} W_{20}^2}{\omega}, \frac{U_{13} U_{20} W_{20} W_x}{\omega^2}, \frac{U_{13} W_{13}}{\omega^4}, \frac{U_{13} W_{20}}{\omega^2}, \frac{U_{13} W_x}{\omega^4}, \frac{U_{20} W_{20}}{\omega}, \frac{1}{\omega^3} \left. \right\}$$

ωs1 /. {c_ → 1/2, ω | u_ | w_ → 2}

$$\left\{ \frac{1}{8}, \frac{1}{8}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, 1, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{2}, \frac{1}{2}, 1, \frac{1}{2}, 1, \frac{1}{4}, \frac{1}{8}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, 1, \frac{1}{4}, \right. \\ \left. 1, \frac{1}{8}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 2, \frac{1}{2}, 2, \frac{1}{2}, 4, \frac{1}{4}, \frac{1}{4}, \frac{1}{2}, 2, \frac{1}{2}, 8, 2, \frac{1}{2}, 8, 4, \frac{1}{4}, 1, \frac{1}{4}, 2, \frac{1}{8} \right\}$$