

Pensieve header: Optimizing the 1-smidgen program: a better CF.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2016-09"];
Once[<< KnotTheory`];
Once[<< ../Projects/Profile/Profile.m]
```

Loading KnotTheory` version of September 6, 2014, 13:37:37.2841.

Read more at <http://katlas.org/wiki/KnotTheory>.

This is Profile.m, Sep 2016 mods of July 1994 version

Rotational Virtual Knots

```
RVK::usage =
  "RVK[xs, rots] represents a Rotational Virtual Knot with a list of n Xp/Xm crossings xs and
  a length 2n list of rotation numbers rots. Crossing sites are indexed 1 through
  2n, and rots[[k]] is the rotation between site k-1 and site k. RVK is also a casting
  operator converting to the RVK presentation from other knot presentations.";
RVK[pd_PD] := Module[{n, xs, x, rots, front, k},
  n = Length[pd];
  xs = List@@pd /. x_X => If[PositiveQ[x], Xp[x[[4]], x[[1]], Xm[x[[2]], x[[1]]];
  rots = Table[0, {2 n}];
  front = {0};
  For[k = 0, k < 2 n, ++k,
  If[k == 0 || FreeQ[front, -k],
  front = Flatten[front /. k -> Catch[xs /. {
    Xp[k + 1, L_] | Xm[L_, k + 1] => Throw[{L, k + 1, 1 - L}],
    Xp[L_, k + 1] | Xm[k + 1, L_] => (++rots[[L]]; Throw[{1 - L, k + 1, L}])
  }]],
  If[MatchQ[front, {_, k, _, -k, _}], --rots[[k + 1]]
  ];
  ];
  RVK[xs, rots]
];
RVK[K_] := RVK[PD[K]];
```

NOE-It

$$\Lambda[k_-] := (1 - t_k) (\alpha^2 \beta^2 + 4 \alpha \beta \delta \mu + 2 \delta^2 \mu^2) / 2 + 2 \mu^2 (\alpha \beta + \delta \mu) c_k - \beta (2 \mu - 1) (\alpha \beta + 2 \delta \mu) u_k + 2 \beta \delta \mu^2 c_k u_k - \beta^2 \delta (3 \mu - 1) * u_k^2 / 2 + \alpha (\alpha \beta + 2 \delta \mu) w_k + 2 \alpha \delta \mu^2 c_k w_k - 2 (t_k - 1) \delta^2 (\alpha \beta + \delta \mu) u_k w_k + 2 \delta^2 \mu^2 c_k u_k w_k - \beta \delta^2 (2 \mu - 1) * u_k^2 w_k + \alpha^2 \delta (1 + \mu) w_k^2 / 2 + \alpha \delta^2 u_k * w_k^2 - (t_k - 1) \delta^4 * u_k^2 * w_k^2 / 2;$$

```
DP_{x \to \partial_\alpha, y \to \partial_\beta}[P_-][f_-] := Profile[DP, (* means P[\partial_\alpha, \partial_\beta][f] *)
  Total[CoefficientRules[P, {x, y}] /. ({m_-, n_-} \to c_-) => c D[f, {\alpha, m}, {\beta, n}]]
]
```

```
CF[ $\mathbb{E}[\omega\_ , L\_ , Q\_ , P\_ ] := Profile[CF, \mathbb{E}[$ 
  Expand[Together[ $\omega / . b_{L\_} \Rightarrow \text{Log}[t_L]$ ]],
  Expand[Together[L]],
  Expand[Together[ $Q / . b_{L\_} \Rightarrow \text{Log}[t_L]$ ]],
  Profile["CF@P", Expand[Together[
    tp =  $P / . b_{L\_} \Rightarrow \text{Log}[t_L]$ ;
    AppendTo[CF$P, tp];
    tp
  ]]]
];
```

```
 $\mathbb{E} /: \mathbb{E}[\omega1\_ , L1\_ , Q1\_ , P1\_ ] \mathbb{E}[\omega2\_ , L2\_ , Q2\_ , P2\_ ] := CF@\mathbb{E}[\omega1 \omega2, L1 + L2, \omega2 Q1 + \omega1 Q2, \omega2^4 P1 + \omega1^4 P2];$ 
```

```
Nu $_{i, c_j \rightarrow k}$ [ $\mathbb{E}[\omega\_ , L\_ , Q\_ , P\_ ] := Profile[Nuc,$ 
  With[{ $q = e^{-\gamma} \beta u_k + \gamma c_k$ }, CF[
     $\mathbb{E}[\omega, \gamma c_k + (L / . c_j \rightarrow \theta), \omega e^{-\gamma} \beta u_k + (Q / . u_i \rightarrow \theta),$ 
    Together[ $e^{-q} DP_{c_j \rightarrow D_\gamma, u_i \rightarrow D_\beta}[P][e^q]$ ]] /. { $\gamma \rightarrow \partial_{c_j} L, \beta \rightarrow \omega^{-1} \partial_{u_i} Q$ }}]
];
```

```
Nw $_{i, c_j \rightarrow k}$ [ $\mathbb{E}[\omega\_ , L\_ , Q\_ , P\_ ] := Profile[Nwc,$ 
  With[{ $q = e^\gamma \alpha w_k + \gamma c_k$ }, CF[
     $\mathbb{E}[\omega, \gamma c_k + (L / . c_j \rightarrow \theta), \omega e^\gamma \alpha w_k + (Q / . w_i \rightarrow \theta),$ 
    Together[ $e^{-q} DP_{c_j \rightarrow D_\gamma, w_i \rightarrow D_\alpha}[P][e^q]$ ]] /. { $\gamma \rightarrow \partial_{c_j} L, \alpha \rightarrow \omega^{-1} \partial_{w_i} Q$ }}]
];
```

```
Nw $_{i, u_j \rightarrow k}$ [ $\mathbb{E}[\omega\_ , L\_ , Q\_ , P\_ ] := Profile[Nwu,$ 
  With[{ $q = (1 - t_k) \mu^{-1} \alpha \beta + \mu^{-1} \beta u_k + \mu^{-1} \delta u_k w_k + \mu^{-1} \alpha w_k$ }, CF[
     $\mathbb{E}[\mu \omega, L, \mu \omega q + \mu (Q / . w_i | u_j \rightarrow \theta), \mu^4 \text{Together}[e^{-q} DP_{w_i \rightarrow D_\alpha, u_j \rightarrow D_\beta}[P][e^q] + \omega^4 \Lambda[k]] / .$ 
     $\mu \rightarrow 1 + (t_k - 1) \delta / . \{\alpha \rightarrow \omega^{-1} (\partial_{w_i} Q / . u_j \rightarrow \theta), \beta \rightarrow \omega^{-1} (\partial_{u_j} Q / . w_i \rightarrow \theta), \delta \rightarrow \omega^{-1} \partial_{w_i, u_j} Q\}$ ]]
];
```

```
m $_{i, j \rightarrow k}$ [ $Z\_ ] := Profile[m,$ 
  Module[{ $x, y, z$ },
     $Z // N_{w_i c_j \rightarrow x} // N_{w_x u_j \rightarrow y} // \text{ReplaceAll}[\{c_{x|y} \rightarrow c_x, w_j \rightarrow w_y\}] // N_{u_i c_x \rightarrow x} //$ 
     $\text{ReplaceAll}[Z_{-i|j|x|y} \rightarrow z_k] // CF$ 
];
```

```
R $_{i, j}^+$  :=  $\mathbb{E}[1, b_i c_j, u_i w_j, -c_i (t_i - 1)^2 / 2 - c_i^2 (t_i - 1)^2 / 2 + c_i c_j (t_j^2 - t_i - 2) / 2 - c_j u_i w_i / 2 + c_i (1 - t_i) u_i w_i -$ 
 $u_i^2 w_i^2 / 2 + u_i w_j + c_j t_i u_i w_j / 2 + c_i (t_i - 2) t_i u_i w_j + c_i (1 + t_j) u_j w_j / 2 + (t_i - 1) u_i^2 w_i w_j - (t_i - 2) t_i u_i^2 w_j^2 / 2];$ 
R $_{i, j}^- := \mathbb{E}[1, -b_i c_j, -t_i^{-1} u_i w_j, c_i (t_i - 1)^2 / 2 + c_i^2 (t_i - 1)^2 / 2 + c_i c_j (2 + t_i - t_j^2) / 2 + c_j u_i w_i / 2 +$ 
 $c_i (t_i - 1) u_i w_i + u_i^2 w_i^2 / 2 + (1 - t_i^{-1}) u_i w_j / 2 + c_i (2 t_i - 5 + 3 t_i^{-1}) u_i w_j / 2 + c_j (t_i^{-1} + 1 - t_i^{-1} t_j^2) u_i w_j / 2 -$ 
 $c_i (t_j + 1) u_j w_j / 2 + (2 - 3 t_i^{-1}) u_i^2 w_i w_j / 2 + (1 + 2 t_i^{-2} - 3 t_i^{-1}) u_i^2 w_j^2 / 2 - t_i^{-1} (1 + t_j) u_i u_j w_j^2 / 2];$ 
ur $_i := \mathbb{E}[t_i^{-1/4}, \theta, \theta, c_i t_i / 4 + u_i w_i / 8];$ 
nr $_i := \mathbb{E}[t_i^{1/4}, \theta, \theta, -c_i t_i^3 / 4 - t_i^2 u_i w_i / 8];$ 
ul $_i := \mathbb{E}[t_i^{1/4}, \theta, \theta, c_i t_i (4 + t_i) / 4 - t_i^2 u_i w_i / 8];$ 
nl $_i := \mathbb{E}[t_i^{-1/4}, \theta, \theta, -c_i (1 + 4 t_i^{-1}) / 4 + u_i w_i / 8];$ 
```

```
rot[ $\_ , \theta$ ] =  $\mathbb{E}[1, \theta, \theta, \theta];$ 
rot[ $i, 1$ ] := Module[{ $y$ }, nl $_i$  ur $_y // m_{i, y \rightarrow i}$ ];
rot[ $i, n\_Integer$ ] /;  $n > 1$  := Module[{ $y$ }, rot[ $i, n - 1$ ] rot[ $y, 1$ ] //  $m_{i, y \rightarrow i}$ ];
rot[ $i, -1$ ] := Module[{ $y$ }, nr $_i$  ul $_y // m_{i, y \rightarrow i}$ ];
rot[ $i, n\_Integer$ ] /;  $n < -1$  := Module[{ $y$ }, rot[ $i, n + 1$ ] rot[ $y, -1$ ] //  $m_{i, y \rightarrow i}$ ];
```

Z

```

t_ = t;
Z[K_] := Z[RVK@K];
Z[rvk_RVK] := Profile[Z,
  Module[{todo, n, rots,  $\xi$ , done, st, x,  $\xi$ 1, i, j, k, k1, k2, k3},
    {todo, rots} = List@@rvk;
    AppendTo[rots, 0];
    n = Length[todo];
     $\xi$  =  $\mathbb{E}$ [1, 0, 0, 0];
    done = {0};
    st = Range[0, 2 n + 1];
    Z$ $\xi$ s = CF$P = {};
    While[todo != {},
      {x} = MaximalBy[todo, Length[done  $\cap$  {#[[1]], #[[2]], #[[1]] - 1, #[[2]] - 1}] &, 1];
      Z$todo = todo; Z$x = x;
      {i, j} = List@@x;
       $\xi$ 1 = Switch[Head[x],
        Xp, mj,k→j[R+i,j (R-k3,k nrk1 ulk2 // mk,k1→k // mk,k2→k // mk,k3→k)],
        Xm, mj,k→j[R-i,j (R+k,k3 nrk1 ulk2 // mk,k1→k // mk,k2→k // mk,k3→k)],
      ];
       $\xi$ 1 = rot[k, rots[[i]]]  $\xi$ 1 // mk,i→i; rots[[i]] = 0;
       $\xi$ 1 =  $\xi$ 1 rot[k, rots[[i + 1]]] // mi,k→i; rots[[i + 1]] = 0;
       $\xi$ 1 = rot[k, rots[[j]]]  $\xi$ 1 // mk,j→j; rots[[j]] = 0;
       $\xi$ 1 =  $\xi$ 1 rot[k, rots[[j + 1]]] // mj,k→j; rots[[j + 1]] = 0;
       $\xi$  *=  $\xi$ 1;
      If[MemberQ[done, i],  $\xi$  =  $\xi$  // mi,i+1→i; st = st /. st[[i + 2]] → st[[i + 1]]];
      If[MemberQ[done, i - 1],  $\xi$  =  $\xi$  // mst[[i],i→st[[i]]; st = st /. st[[i + 1]] → st[[i]]];
      If[MemberQ[done, j],  $\xi$  =  $\xi$  // mj,j+1→j; st = st /. st[[j + 2]] → st[[j + 1]]];
      If[MemberQ[done, j - 1],  $\xi$  =  $\xi$  // mst[[j],j→st[[j]]; st = st /. st[[j + 1]] → st[[j]]];
      AppendTo[Z$ $\xi$ s,  $\xi$ ];
      done = done  $\cup$  {i - 1, i, j - 1, j};
      todo = DeleteCases[todo, x]
    ];
     $\xi$  /. {u0 → u, c0 → c, w0 → w}
  ]
]

```

```
BeginProfile[];
Timing[Z[Knot[10, 100]]]
EndProfile[];
```

$$\left\{ 383.172, \mathbb{E} \left[13 + \frac{1}{t^4} - \frac{4}{t^3} + \frac{9}{t^2} - \frac{12}{t} - 12t + 9t^2 - 4t^3 + t^4, 0, 0, \right. \right.$$

$$2563146 + 667500c + \frac{6}{t^{16}} - \frac{8c}{t^{16}} - \frac{92}{t^{15}} + \frac{118c}{t^{15}} + \frac{723}{t^{14}} - \frac{892c}{t^{14}} - \frac{3818}{t^{13}} + \frac{4523c}{t^{13}} + \frac{15133}{t^{12}} - \frac{17161c}{t^{12}} - \frac{47848}{t^{11}} +$$

$$\frac{51709c}{t^{11}} + \frac{125539}{t^{10}} - \frac{128505c}{t^{10}} - \frac{281054}{t^9} + \frac{270279c}{t^9} + \frac{548129}{t^8} - \frac{489715c}{t^8} - \frac{945756}{t^7} + \frac{772841c}{t^7} + \frac{1460263}{t^6} -$$

$$\frac{1066829c}{t^6} - \frac{2034106}{t^5} + \frac{1282861c}{t^5} + \frac{2570432}{t^4} - \frac{1320331c}{t^4} - \frac{2956518}{t^3} + \frac{1107336c}{t^3} + \frac{3099338}{t^2} - \frac{640054c}{t^2} -$$

$$\frac{2958726}{t} - \frac{540c}{t} - 2000454t - 1197840ct + 1387610t^2 + 1472160ct^2 - 832998t^3 - 1456020ct^3 + 407256t^4 +$$

$$1204364ct^4 - 132546t^5 - 829886ct^5 - 9557t^6 + 453636ct^6 + 59220t^7 - 162131ct^7 - 58859t^8 - 11711ct^8 +$$

$$40498t^9 + 81439ct^9 - 22001t^{10} - 84595ct^{10} + 9704t^{11} + 59721ct^{11} - 3455t^{12} - 32685ct^{12} + 966t^{13} +$$

$$14251ct^{13} - 201t^{14} - 4919ct^{14} + 28t^{15} + 1307ct^{15} - 2t^{16} - 253ct^{16} + 32ct^{17} - 2ct^{18} - 493132uw + \frac{8uw}{t^{16}} -$$

$$\frac{110uw}{t^{15}} + \frac{782uw}{t^{14}} - \frac{3741uw}{t^{13}} + \frac{13420uw}{t^{12}} - \frac{38289uw}{t^{11}} + \frac{90216uw}{t^{10}} - \frac{180063uw}{t^9} + \frac{309652uw}{t^8} - \frac{463189uw}{t^7} +$$

$$\frac{603640uw}{t^6} - \frac{679221uw}{t^5} + \frac{641110uw}{t^4} - \frac{466226uw}{t^3} + \frac{173828uw}{t^2} + \frac{174368uw}{t} + 704708t uw - 767452t^2 uw +$$

$$688568t^3 uw - 515796t^4 uw + 314090t^5 uw - 139546t^6 uw + 22585t^7 uw + 34296t^8 uw - 47143t^9 uw +$$

$$37452t^{10} uw - 22269t^{11} uw + 10416t^{12} uw - 3835t^{13} uw + 1084t^{14} uw - 223t^{15} uw + 30t^{16} uw - 2t^{17} uw \left. \right\}$$

```
PrintProfile[]
```

CF@P: called 490 times, time in 369.689/369.689

Parents:

(490) 369.689/ 369.689 under CF

Nwu: called 102 times, time in 5.545/358.979

Parents:

(102) 5.545/ 358.979 under m

Children:

(102) 0.155/ 353.092 above CF

(102) 0.342/ 0.342 above DP

Nuc: called 102 times, time in 3.454/12.955

Parents:

(102) 3.454/ 12.955 under m

Children:

(102) 0.111/ 9.080 above CF

(102) 0.421/ 0.421 above DP

Nwc: called 102 times, time in 2.426/8.873

Parents:

(102) 2.426/ 8.873 under m

Children:

(102) 0.108/ 6.090 above CF

(102) 0.357/ 0.357 above DP

DP: called 306 times, time in 1.12/1.12

Parents:

(102) 0.421/ 0.421 under Nuc

(102) 0.357/ 0.357 under Nwc

(102) 0.342/ 0.342 under Nwu

CF: called 490 times, time in 0.532/370.221

Parents:

(102) 0.094/ 1.282 under m

(102) 0.111/ 9.080 under Nuc

(102) 0.108/ 6.090 under Nwc

(102) 0.155/ 353.092 under Nwu

(82) 0.064/ 0.677 under Z

Children:

(490) 369.689/ 369.689 above CF@P

m: called 102 times, time in 0.296/382.385

Parents:

(102) 0.296/ 382.385 under Z

Children:

(102) 0.094/ 1.282 above CF

(102) 3.454/ 12.955 above Nuc

(102) 2.426/ 8.873 above Nwc

(102) 5.545/ 358.979 above Nwu

Z: called 1 times, time in 0.109/383.171

Parents:

(1) 0.109/ 383.171 under ProfileRoot

Children:

(82) 0.064/ 0.677 above CF

(102) 0.296/ 382.385 above m

ProfileRoot: called 0 times, time in 0./0.

Children:

(1) 0.109/ 383.171 above Z

{Null, Null, Null, Null, Null, Null, Null, Null, Null}