

Pensieve header: Optimizing the 1-smidgen program using RuntimeTools`Profile (fail).

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2016-09"];
Once[<< KnotTheory`]
```

Loading KnotTheory` version of September 6, 2014, 13:37:37.2841.

Read more at <http://katlas.org/wiki/KnotTheory>.

## Rotational Virtual Knots

```
RVK::usage =
  "RVK[xs, rots] represents a Rotational Virtual Knot with a list of n Xp/Xm crossings xs and
  a length 2n list of rotation numbers rots. Crossing sites are indexed 1 through
  2n, and rots[[k]] is the rotation between site k-1 and site k. RVK is also a casting
  operator converting to the RVK presentation from other knot presentations.";
RVK[pd_PD] := Module[{n, xs, x, rots, front, k},
  n = Length[pd];
  xs = List@@pd /. x_X => If[PositiveQ[x], Xp[x[[4]], x[[1]], Xm[x[[2]], x[[1]]];
  rots = Table[0, {2 n]];
  front = {0};
  For[k = 0, k < 2 n, ++k,
  If[k == 0 ∨ FreeQ[front, -k],
  front = Flatten[front /. k → Catch[xs /. {
    Xp[k + 1, L_] | Xm[L_, k + 1] => Throw[{L, k + 1, 1 - L]],
    Xp[L_, k + 1] | Xm[k + 1, L_] => (++rots[[L]]; Throw[{1 - L, k + 1, L])
  }]],
  If[MatchQ[front, {___, k, ___, -k, ___}], --rots[[k + 1]]
  ]
  ];
  RVK[xs, rots]
  ];
RVK[K_] := RVK[PD[K]];
```

## NOE-It

Logos

$$\Delta[k_-] := (1 - t_k) (\alpha^2 \beta^2 + 4 \alpha \beta \delta \mu + 2 \delta^2 \mu^2) / 2 + 2 \mu^2 (\alpha \beta + \delta \mu) c_k - \beta (2 \mu - 1) (\alpha \beta + 2 \delta \mu) u_k + 2 \beta \delta \mu^2 c_k u_k - \beta^2 \delta (3 \mu - 1) * u_k^2 / 2 + \alpha (\alpha \beta + 2 \delta \mu) w_k + 2 \alpha \delta \mu^2 c_k w_k - 2 (t_k - 1) \delta^2 (\alpha \beta + \delta \mu) u_k w_k + 2 \delta^2 \mu^2 c_k u_k w_k - \beta \delta^2 (2 \mu - 1) * u_k^2 w_k + \alpha^2 \delta (1 + \mu) w_k^2 / 2 + \alpha \delta^2 u_k * w_k^2 - (t_k - 1) \delta^4 * u_k^2 * w_k^2 / 2;$$

```
DPx→Dα, y→Dβ[P_][f_] := (* means P[∂α, ∂β][f] *)
Total[CoefficientRules[P, {x, y}] /. ({m_, n_} → c_) => c D[f, {α, m}, {β, n}]
```

```
CF[IE[ω_, L_, Q_, P_]] := Expand /@ Together /@
  IE[ω /. bL_ => Log[tL], L, Q /. bL_ => Log[tL], P /. bL_ => Log[tL]];
IE /: IE[ω1_, L1_, Q1_, P1_] IE[ω2_, L2_, Q2_, P2_] := CF@IE[ω1 ω2, L1 + L2, ω2 Q1 + ω1 Q2, ω24 P1 + ω14 P2];
```

```
Nui cj→k_[IE[ω_, L_, Q_, P_]] := With[{q = e-γ β uk + γ ck}, CF[
  IE[ω, γ ck + (L /. cj → 0), ω e-γ β uk + (Q /. ui → 0), e-q DPcj→Dγ, ui→Dβ[P][eq]] /. {γ → ∂cj L, β → ω-1 ∂ui Q}}];
Nwi cj→k_[IE[ω_, L_, Q_, P_]] := With[{q = eγ α wk + γ ck}, CF[
  IE[ω, γ ck + (L /. cj → 0), ω eγ α wk + (Q /. wi → 0), e-q DPcj→Dγ, wi→Dα[P][eq]] /. {γ → ∂cj L, α → ω-1 ∂wi Q}}];
```

```

N_{w_i, u_j \to k} [\mathbb{E}[\omega, L, Q, P]] := With[{q = (1 - t_k) \mu^{-1} \alpha \beta + \mu^{-1} \beta u_k + \mu^{-1} \delta u_k w_k + \mu^{-1} \alpha w_k}, CF[
  \mathbb{E}[\mu \omega, L, \mu \omega q + \mu (Q /. w_i | u_j \to \theta), \mu^4 e^{-q} DP_{w_i \to D_\alpha, u_j \to D_\beta}[P][e^q] + \omega^4 \Delta[k]] /. \mu \to 1 + (t_k - 1) \delta /.
  {\alpha \to \omega^{-1} (\partial_{w_i} Q /. u_j \to \theta), \beta \to \omega^{-1} (\partial_{u_j} Q /. w_i \to \theta), \delta \to \omega^{-1} \partial_{w_i, u_j} Q}]];

```

```

m_{i, j \to k} [Z_] := Module[{x, y, z},
  Z // N_{w_i, c_j \to x} // N_{w_i, u_j \to y} // ReplaceAll[{c_x | y \to c_x, w_j \to w_y}] // N_{u_i, c_x \to x} // ReplaceAll[z_{-i | j | x | y \to z_k}] // CF];

```

```

R_{i, j}^+ := \mathbb{E}[1, b_i c_j, u_i w_j, -c_i (t_i - 1)^2 / 2 - c_i^2 (t_i - 1)^2 / 2 + c_i c_j (t_j^2 - t_i - 2) / 2 - c_j u_i w_i / 2 + c_i (1 - t_i) u_i w_i -
  u_i^2 w_i^2 / 2 + u_i w_j + c_j t_i u_i w_j / 2 + c_i (t_i - 2) t_i u_i w_j + c_i (1 + t_j) u_j w_j / 2 + (t_i - 1) u_i^2 w_i w_j - (t_i - 2) t_i u_i^2 w_j^2 / 2];
R_{i, j}^- := \mathbb{E}[1, -b_i c_j, -t_i^{-1} u_i w_j, c_i (t_i - 1)^2 / 2 + c_i^2 (t_i - 1)^2 / 2 + c_i c_j (2 + t_i - t_j^2) / 2 + c_j u_i w_i / 2 +
  c_i (t_i - 1) u_i w_i + u_i^2 w_i^2 / 2 + (1 - t_i^{-1}) u_i w_j / 2 + c_i (2 t_i - 5 + 3 t_i^{-1}) u_i w_j / 2 + c_j (t_i^{-1} + 1 - t_i^{-1} t_j^2) u_i w_j / 2 -
  c_i (t_j + 1) u_j w_j / 2 + (2 - 3 t_i^{-1}) u_i^2 w_i w_j / 2 + (1 + 2 t_i^{-2} - 3 t_i^{-1}) u_i^2 w_j^2 / 2 - t_i^{-1} (1 + t_j) u_i u_j w_j^2 / 2];
ur_{i, j} := \mathbb{E}[t_i^{-1/4}, 0, 0, c_i t_i / 4 + u_i w_i / 8];
nr_{i, j} := \mathbb{E}[t_i^{1/4}, 0, 0, -c_i t_i^3 / 4 - t_i^2 u_i w_i / 8];
ul_{i, j} := \mathbb{E}[t_i^{1/4}, 0, 0, c_i t_i (4 + t_i) / 4 - t_i^2 u_i w_i / 8];
nl_{i, j} := \mathbb{E}[t_i^{-1/4}, 0, 0, -c_i (1 + 4 t_i^{-1}) / 4 + u_i w_i / 8];

```

```

rot[_ , 0] = \mathbb{E}[1, 0, 0, 0];
rot[i_, 1] := Module[{y}, nl_i ur_y // m_{i, y \to i}];
rot[i_, n_Integer] /; n > 1 := Module[{y}, rot[i, n - 1] rot[y, 1] // m_{i, y \to i}];
rot[i_, -1] := Module[{y}, nr_i ul_y // m_{i, y \to i}];
rot[i_, n_Integer] /; n < -1 := Module[{y}, rot[i, n + 1] rot[y, -1] // m_{i, y \to i}];

```

## Z

```

t_ = t;
Z[K_] := Z[RVK@K];
Z[rvk_RVK] := Module[{todo, n, rots, ζ, done, st, x, ζ1, i, j, k, k1, k2, k3},
  {todo, rots} = List@@rvk;
  AppendTo[rots, 0];
  n = Length[todo];
  ζ = E[1, 0, 0, 0];
  done = {0};
  st = Range[0, 2 n + 1];
  While[todo != {},
    {x} = MaximalBy[todo, Length[done ∩ {#[[1]], #[[2]], #[[1]] - 1, #[[2]] - 1}] &, 1];
    Z$tudo = ttodo; Z$x = x;
    {i, j} = List@@x;
    ζ1 = Switch[Head[x],
      Xp, mj,k→j [R+i,j (R-k3,k nrk1 ulk2 // mk,k1→k // mk,k2→k // mk,k3→k) ],
      Xm, mj,k→j [R-i,j (R+k,k3 nrk1 ulk2 // mk,k1→k // mk,k2→k // mk,k3→k) ]
    ];
    ζ1 = rot[k, rots[[i]] ζ1 // mk,i→i; rots[[i]] = 0;
    ζ1 = ζ1 rot[k, rots[[i + 1]] // mi,k→i; rots[[i + 1]] = 0;
    ζ1 = rot[k, rots[[j]] ζ1 // mk,j→j; rots[[j]] = 0;
    ζ1 = ζ1 rot[k, rots[[j + 1]] // mj,k→j; rots[[j + 1]] = 0;
    ζ *= ζ1;
    If[MemberQ[done, i], ζ = ζ // mi,i+1→i; st = st /. st[[i + 2]] → st[[i + 1]];
    If[MemberQ[done, i - 1], ζ = ζ // mst[[i],i→st[[i]]; st = st /. st[[i + 1]] → st[[i]];
    If[MemberQ[done, j], ζ = ζ // mj,j+1→j; st = st /. st[[j + 2]] → st[[j + 1]];
    If[MemberQ[done, j - 1], ζ = ζ // mst[[j],j→st[[j]]; st = st /. st[[j + 1]] → st[[j]];
    done = done ∪ {i - 1, i, j - 1, j};
    ttodo = DeleteCases[todo, x]
  ];
  ζ /. {u0 → u, c0 → c, w0 → w}
]

```

## Timing[Z[Knot[10, 100]]]

KnotTheory: Loading precomputed data in PD4Knots`.



$$\left\{ 20.125, \mathbb{E} \left[ 13 + \frac{1}{t^4} - \frac{4}{t^3} + \frac{9}{t^2} - \frac{12}{t} - 12t + 9t^2 - 4t^3 + t^4, 0, 0, \right. \right.$$

$$\left. \begin{aligned} & 2563146 + 667500c + \frac{6}{t^{16}} - \frac{8c}{t^{16}} - \frac{92}{t^{15}} + \frac{118c}{t^{15}} + \frac{723}{t^{14}} - \frac{892c}{t^{14}} - \frac{3818}{t^{13}} + \frac{4523c}{t^{13}} + \frac{15133}{t^{12}} - \frac{17161c}{t^{12}} - \frac{47848}{t^{11}} + \\ & \frac{51709c}{t^{11}} + \frac{125539}{t^{10}} - \frac{128505c}{t^{10}} - \frac{281054}{t^9} + \frac{270279c}{t^9} + \frac{548129}{t^8} - \frac{489715c}{t^8} - \frac{945756}{t^7} + \frac{772841c}{t^7} + \frac{1460263}{t^6} - \\ & \frac{1066829c}{t^6} - \frac{2034106}{t^5} + \frac{1282861c}{t^5} + \frac{2570432}{t^4} - \frac{1320331c}{t^4} - \frac{2956518}{t^3} + \frac{1107336c}{t^3} + \frac{3099338}{t^2} - \frac{640054c}{t^2} - \\ & \frac{2958726}{t} - \frac{540c}{t} - 2000454t - 1197840ct + 1387610t^2 + 1472160ct^2 - 832998t^3 - 1456020ct^3 + 407256t^4 + \\ & 1204364ct^4 - 132546t^5 - 829886ct^5 - 9557t^6 + 453636ct^6 + 59220t^7 - 162131ct^7 - 58859t^8 - 11711ct^8 + \\ & 40498t^9 + 81439ct^9 - 22001t^{10} - 84595ct^{10} + 9704t^{11} + 59721ct^{11} - 3455t^{12} - 32685ct^{12} + 966t^{13} + \\ & 14251ct^{13} - 201t^{14} - 4919ct^{14} + 28t^{15} + 1307ct^{15} - 2t^{16} - 253ct^{16} + 32ct^{17} - 2ct^{18} - 493132uw + \frac{8uw}{t^{16}} - \\ & \frac{110uw}{t^{15}} + \frac{782uw}{t^{14}} - \frac{3741uw}{t^{13}} + \frac{13420uw}{t^{12}} - \frac{38289uw}{t^{11}} + \frac{90216uw}{t^{10}} - \frac{180063uw}{t^9} + \frac{309652uw}{t^8} - \frac{463189uw}{t^7} + \\ & \frac{603640uw}{t^6} - \frac{679221uw}{t^5} + \frac{641110uw}{t^4} - \frac{466226uw}{t^3} + \frac{173828uw}{t^2} + \frac{174368uw}{t} + 704708t uw - 767452t^2 uw + \\ & 688568t^3 uw - 515796t^4 uw + 314090t^5 uw - 139546t^6 uw + 22585t^7 uw + 34296t^8 uw - 47143t^9 uw + \\ & 37452t^{10} uw - 22269t^{11} uw + 10416t^{12} uw - 3835t^{13} uw + 1084t^{14} uw - 223t^{15} uw + 30t^{16} uw - 2t^{17} uw \left. \right\} \end{aligned}$$

## Timing[Z[Knot[10, 100]]] // RuntimeTools`Profile

$$\left\{ 21.8906, \mathbb{E} \left[ 13 + \frac{1}{t^4} - \frac{4}{t^3} + \frac{9}{t^2} - \frac{12}{t} - 12t + 9t^2 - 4t^3 + t^4, 0, 0, \right. \right.$$

$$\left. \begin{aligned} & 2563146 + 667500c + \frac{6}{t^{16}} - \frac{8c}{t^{16}} - \frac{92}{t^{15}} + \frac{118c}{t^{15}} + \frac{723}{t^{14}} - \frac{892c}{t^{14}} - \frac{3818}{t^{13}} + \frac{4523c}{t^{13}} + \frac{15133}{t^{12}} - \frac{17161c}{t^{12}} - \frac{47848}{t^{11}} + \\ & \frac{51709c}{t^{11}} + \frac{125539}{t^{10}} - \frac{128505c}{t^{10}} - \frac{281054}{t^9} + \frac{270279c}{t^9} + \frac{548129}{t^8} - \frac{489715c}{t^8} - \frac{945756}{t^7} + \frac{772841c}{t^7} + \frac{1460263}{t^6} - \\ & \frac{1066829c}{t^6} - \frac{2034106}{t^5} + \frac{1282861c}{t^5} + \frac{2570432}{t^4} - \frac{1320331c}{t^4} - \frac{2956518}{t^3} + \frac{1107336c}{t^3} + \frac{3099338}{t^2} - \frac{640054c}{t^2} - \\ & \frac{2958726}{t} - \frac{540c}{t} - 2000454t - 1197840ct + 1387610t^2 + 1472160ct^2 - 832998t^3 - 1456020ct^3 + 407256t^4 + \\ & 1204364ct^4 - 132546t^5 - 829886ct^5 - 9557t^6 + 453636ct^6 + 59220t^7 - 162131ct^7 - 58859t^8 - 11711ct^8 + \\ & 40498t^9 + 81439ct^9 - 22001t^{10} - 84595ct^{10} + 9704t^{11} + 59721ct^{11} - 3455t^{12} - 32685ct^{12} + 966t^{13} + \\ & 14251ct^{13} - 201t^{14} - 4919ct^{14} + 28t^{15} + 1307ct^{15} - 2t^{16} - 253ct^{16} + 32ct^{17} - 2ct^{18} - 493132uw + \frac{8uw}{t^{16}} - \\ & \frac{110uw}{t^{15}} + \frac{782uw}{t^{14}} - \frac{3741uw}{t^{13}} + \frac{13420uw}{t^{12}} - \frac{38289uw}{t^{11}} + \frac{90216uw}{t^{10}} - \frac{180063uw}{t^9} + \frac{309652uw}{t^8} - \frac{463189uw}{t^7} + \\ & \frac{603640uw}{t^6} - \frac{679221uw}{t^5} + \frac{641110uw}{t^4} - \frac{466226uw}{t^3} + \frac{173828uw}{t^2} + \frac{174368uw}{t} + 704708t uw - 767452t^2 uw + \\ & 688568t^3 uw - 515796t^4 uw + 314090t^5 uw - 139546t^6 uw + 22585t^7 uw + 34296t^8 uw - 47143t^9 uw + \\ & 37452t^{10} uw - 22269t^{11} uw + 10416t^{12} uw - 3835t^{13} uw + 1084t^{14} uw - 223t^{15} uw + 30t^{16} uw - 2t^{17} uw \left. \right\} \end{aligned}$$

## ? RuntimeTools`Profile

Info323684128353-7960612

RuntimeTools`Profile

Info323684128353-7960612

Attributes[RuntimeTools`Profile] = {HoldAllComplete, Protected}

Options[RuntimeTools`Profile] = {RuntimeTools`TimeLimit -&gt; Automatic}