

Pensieve header: The 1-co invariant for all ribbon knots with up to 10 crossings.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2016-09"];
```

```
<< OneSmidgen.m
```

```
tab = Get["tab.m"]
```

Loading KnotTheory` version of September 6, 2014, 13:37:37.2841.

Read more at <http://katlas.org/wiki/KnotTheory>.

```
{Knot[3, 1] -> {1.28125,
  E[-1 + 1/t + t, 0, 0, 16 + 9c/2 + 2/t^4 - 2c/t^4 - 7/t^3 + 11c/2t^3 + 14/t^2 - 8c/t^2 - 18/t + 4c/t - 10t - 10ct + 4t^2 + 8ct^2 - t^3 - 3ct^3/2 - 2ct^4 +
  2ct^5 - ct^6/2 - 4uw + 2uw/t^4 - 7uw/2t^3 + 9uw/2t^2 + uw/2t + 6tuw - 2t^2uw - 1/2t^3uw + 3/2t^4uw - 1/2t^5uw]}, ... 247 ... ,
Knot[10, 165] -> {19.2656, E[-15 - 2/t^2 + 10/t + 10t - 2t^2, 0, 0, -747172 + 122705c + 12/t^8 - 64c/t^8 - 120/t^7 + 1104c/t^7 + 116 -
  8344c/t^6 + 4092/t^5 + 36360c/t^5 - 30717/t^4 - 100668c/t^4 + 118992 + ... 43 ... + 71612uw/t^4 - 109796uw/t^3 + 89902uw/t^2 + 7192uw/t + 144712tuw -
  85883t^2uw + 10652t^3uw + 23594t^4uw - 20548t^5uw + 8644t^6uw - 2096t^7uw + 280t^8uw - 16t^9uw]}]}
```

large output

show less

show more

show all

set size limit...

```
z[K_Knot] := (K /. tab) [[2]];
za[K_Knot] := z[K] [[1]];
zp1[K_Knot] := Factor[z[K] [[4]] /. c | u | w -> 0];
zp2[K_Knot] := Factor[Coefficient[z[K] [[4]], c] / (t - 1/t) (-4 - t + t^2) za[K]^3];
```

```
Solve[-4 - t + t^2 == 0]
```

```
{t -> 1/2 (1 - sqrt(17))}, {t -> 1/2 (1 + sqrt(17))}
```

```
Ribbons = {61, 88, 89, 820, 927, 941, 946, 103, 1022, 1035, 1042,
  1048, 1075, 1087, 1099, 10123, 10129, 10137, 10140, 10153, 10155} /. n_k -> Knot[n, k]
```

```
Length@Ribbons
```

```
21
```

```
Factor[Alexander[#][t]] & /@ Ribbons
```

```
KnotTheory: Loading precomputed data in PD4Knots`.
```

```
{Knot[6, 1], Knot[8, 8], Knot[8, 9], Knot[8, 20], Knot[9, 27], Knot[9, 41], Knot[9, 46], Knot[10, 3],
  Knot[10, 22], Knot[10, 35], Knot[10, 42], Knot[10, 48], Knot[10, 75], Knot[10, 87], Knot[10, 99],
  Knot[10, 123], Knot[10, 129], Knot[10, 137], Knot[10, 140], Knot[10, 153], Knot[10, 155]}
```

```
Length@Ribbons
```

```
21
```

```
Factor[Alexander[#][t]] & /@ Ribbons
```

```
KnotTheory: Loading precomputed data in PD4Knots`.
```

```
{ - ( -2 + t ) ( -1 + 2 t ) / t , ( 2 - 2 t + t^2 ) ( 1 - 2 t + 2 t^2 ) / t^2 , - ( -1 + t - 2 t^2 + t^3 ) ( -1 + 2 t - t^2 + t^3 ) / t^3 ,
  ( 1 - t + t^2 )^2 / t^2 , - ( -1 + 2 t - 3 t^2 + t^3 ) ( -1 + 3 t - 2 t^2 + t^3 ) / t^3 , ( 3 - 3 t + t^2 ) ( 1 - 3 t + 3 t^2 ) / t^2 ,
  - ( -2 + t ) ( -1 + 2 t ) / t , - ( -3 + 2 t ) ( -2 + 3 t ) / t , - ( -2 + 2 t - 2 t^2 + t^3 ) ( -1 + 2 t - 2 t^2 + 2 t^3 ) / t^3 ,
  ( 2 - 4 t + t^2 ) ( 1 - 4 t + 2 t^2 ) / t^2 , - ( -1 + 3 t - 4 t^2 + t^3 ) ( -1 + 4 t - 3 t^2 + t^3 ) / t^3 ,
  ( 1 - t + 2 t^2 - 2 t^3 + t^4 ) ( 1 - 2 t + 2 t^2 - t^3 + t^4 ) / t^4 , - ( -1 + 3 t - 4 t^2 + t^3 ) ( -1 + 4 t - 3 t^2 + t^3 ) / t^3 ,
  - ( -2 + t ) ( -1 + 2 t ) ( 1 - t + t^2 )^2 / t^3 , ( 1 - t + t^2 )^4 / t^4 , ( 1 - 3 t + 3 t^2 - 3 t^3 + t^4 )^2 / t^4 , ( 2 - 2 t + t^2 ) ( 1 - 2 t + 2 t^2 ) / t^2 ,
  ( 1 - 3 t + t^2 )^2 / t^2 , ( 1 - t + t^2 )^2 / t^2 , ( 1 - t + t^3 ) ( 1 - t^2 + t^3 ) / t^3 , - ( -1 + t - 2 t^2 + t^3 ) ( -1 + 2 t - t^2 + t^3 ) / t^3 }
```

MatrixForm@

Table[{K[[1]]K[[2]], Factor@za[K], zp1[K], zp2[K], Vassiliev[2][K], Vassiliev[3][K]}, {K, Ribbons}]

KnotTheory: Loading precomputed data in Jones4Knots`.

6 ₁	$-\frac{(-2+t)(-1+2t)}{t}$	$-\frac{5-11t-t^2+3t^3}{t^2}$	
8 ₈	$\frac{(2-2t+t^2)(1-2t+2t^2)}{t^2}$	$-\frac{7-23t+28t^2+6t^3-50t^4+60t^5-33t^6+9t^7}{t^4}$	-
8 ₉	$-\frac{(-1+t-2t^2+t^3)(-1+2t-t^2+t^3)}{t^3}$	$-\frac{(1+t)(-1+t-2t^2+t^3)(-1+2t-t^2+t^3)(3-6t+8t^2-6t^3+3t^4)}{t^6}$	$3-6t+$
8 ₂₀	$\frac{(1-t+t^2)^2}{t^2}$	$-\frac{2(1-t+t^2)(1-t+3t^2-t^3-t^4+t^5)}{t^4}$	-
9 ₂₇	$-\frac{(-1+2t-3t^2+t^3)(-1+3t-2t^2+t^3)}{t^3}$	$-\frac{(-1+t)(-3+19t-54t^2+93t^3-94t^4+32t^5+46t^6-77t^7+52t^8-19t^9+3t^{10})}{t^6}$	$(-1+t)$
9 ₄₁	$\frac{(3-3t+t^2)(1-3t+3t^2)}{t^2}$	$-\frac{(-1+t)(-21+92t-166t^2+108t^3+26t^4-52t^5+15t^6)}{t^4}$	-
9 ₄₆	$-\frac{(-2+t)(-1+2t)}{t}$	$-\frac{7-21t+9t^2+t^3}{t^2}$	
10 ₃	$-\frac{(-3+2t)(-2+3t)}{t}$	$-\frac{47-81t-3t^2+25t^3}{t^2}$	
10 ₂₂	$-\frac{(-2+2t-2t^2+t^3)(-1+2t-2t^2+2t^3)}{t^3}$	$-\frac{11-43t+90t^2-120t^3+81t^4+21t^5-141t^6+203t^7-188t^8+118t^9-53t^{10}+13t^{11}}{t^6}$	$3-6t+$
10 ₃₅	$\frac{(2-4t+t^2)(1-4t+2t^2)}{t^2}$	$-\frac{7-51t+105t^2+11t^3-235t^4+223t^5-77t^6+9t^7}{t^4}$	$-\frac{2}{t^4}$
10 ₄₂	$-\frac{(-1+3t-4t^2+t^3)(-1+4t-3t^2+t^3)}{t^3}$	$-\frac{(-1+t)(-3+29t-115t^2+233t^3-225t^4+12t^5+203t^6-217t^7+111t^8-29t^9+3t^{10})}{t^6}$	$(-1+t)$
10 ₄₈	$\frac{(1-t+2t^2-2t^3+t^4)(1-2t+2t^2-t^3+t^4)}{t^4}$	$-\frac{(1+t)(1+t^2)(4-21t+64t^2-139t^3+235t^4-315t^5+342t^6-303t^7+221t^8-131t^9+62t^{10}-21t^{11}+4t^{12})}{t^8}$	$-\frac{(1+t^2)(4-21t+64t^2-139t^3+235t^4-315t^5+342t^6-303t^7+221t^8-131t^9+62t^{10}-21t^{11}+4t^{12})}{t^8}$
10 ₇₅	$-\frac{(-1+3t-4t^2+t^3)(-1+4t-3t^2+t^3)}{t^3}$	$-\frac{(-1+t)(-3+29t-109t^2+189t^3-97t^4-172t^5+331t^6-261t^7+117t^8-29t^9+3t^{10})}{t^6}$	$(-1+t)$
10 ₈₇	$-\frac{(-2+t)(-1+2t)(1-t+t^2)^2}{t^3}$	$-\frac{(-1+t)(1-t+t^2)(-11+49t-79t^2+40t^3+64t^4-128t^5+113t^6-59t^7+13t^8)}{t^6}$	$3(-1+t)$
10 ₉₉	$\frac{(1-t+t^2)^4}{t^4}$	$-\frac{4(1+t)(1-t+t^2)^7}{t^8}$	$-\frac{2}{t^8}$
10 ₁₂₃	$\frac{(1-3t+3t^2-3t^3+t^4)^2}{t^4}$	$-\frac{2(1+t)(2-3t+2t^2)(1-3t+3t^2-3t^3+t^4)^3}{t^8}$	$-\frac{(2-3t+2t^2)^3}{t^8}$
10 ₁₂₉	$\frac{(2-2t+t^2)(1-2t+2t^2)}{t^2}$	$-\frac{7-29t+60t^2-56t^3+12t^4+28t^5-27t^6+9t^7}{t^4}$	-
10 ₁₃₇	$\frac{(1-3t+t^2)^2}{t^2}$	$-\frac{2(1-3t+t^2)^2(1-4t+t^3)}{t^4}$	-
10 ₁₄₀	$\frac{(1-t+t^2)^2}{t^2}$	$-\frac{2(1-t+t^2)(1-t+5t^2-3t^3-t^4+t^5)}{t^4}$	-
10 ₁₅₃	$\frac{(1-t+t^3)(1-t^2+t^3)}{t^3}$	$-\frac{4-5t-t^2+9t^3-2t^4-t^5-3t^6+4t^7+7t^8-7t^9+t^{10}+2t^{11}}{t^6}$	$-\frac{3-2t+t^2}{t^6}$
10 ₁₅₅	$-\frac{(-1+t-2t^2+t^3)(-1+2t-t^2+t^3)}{t^3}$	$-\frac{3-12t+24t^2-26t^3+3t^4+34t^5-66t^6+71t^7-54t^8+28t^9-12t^{10}+3t^{11}}{t^6}$	$3-6t+$

MatrixForm@Table[{K[[1]]_{K[[2]]}, Factor@za[K], zp1[K], Factor[zp1[K] - (zp1[K] /. t → 1/t)], zp2[K], Vassiliev[2][K], Vassiliev[3][K]}, {K, AllKnots[{3, 8}]}

3 ₁	$\frac{1-t+t^2}{t}$	$-\frac{2-t+t^2}{t^2}$	$\frac{(-1+t)(1+t)(2-2+t^2)}{t^2}$
4 ₁	$-\frac{1-3t+t^2}{t}$	$-\frac{(1+t)(1-3t+t^2)}{t^2}$	$\frac{(-1+t)(1+t)(1-3t+t^2)}{t^2}$
5 ₁	$\frac{1-t+t^2-t^3+t^4}{t^2}$	$-\frac{4-3t+5t^2-3t^3+3t^4-t^5+t^6}{t^4}$	$\frac{(-1+t)(1+t)(4-3t+8t^2-5t^3+8t^4-3t^5+t^6)}{t^4}$
5 ₂	$\frac{2-3t+2t^2}{t}$	$\frac{-9+11t-7t^2+t^3}{t^2}$	$\frac{(-1+t)(1+t)(9-10t+9t^2)}{t^2}$
6 ₁	$-\frac{(-2+t)(-1+2t)}{t}$	$-\frac{5-11t-t^2+3t^3}{t^2}$	$\frac{(-1+t)(1+t)(5-14t+5t^2)}{t^2}$
6 ₂	$-\frac{1-3t+3t^2-3t^3+t^4}{t^2}$	$-\frac{3-12t+16t^2-12t^3+4t^4-2t^6+t^7}{t^4}$	$\frac{(-1+t)(1+t)(3-13t+21t^2-25t^3+21t^4-13t^5+t^6)}{t^4}$
6 ₃	$\frac{1-3t+5t^2-3t^3+t^4}{t^2}$	$-\frac{(1+t)(2-3t+2t^2)(1-3t+5t^2-3t^3+t^4)}{t^4}$	$\frac{(-1+t)(1+t)(2-3t+2t^2)(1-3t+5t^2-3t^3+t^4)}{t^4}$
7 ₁	$\frac{1-t+t^2-t^3+t^4-t^5+t^6}{t^3}$	$-\frac{6-5t+9t^2-7t^3+9t^4-6t^5+6t^6-3t^7+3t^8-t^9+t^{10}}{t^6}$	$\frac{(-1+t)(1+t)(1-t+t^2)(6t+9t^2-3t^3+8t^4-3t^5+t^6)}{t^6}$
7 ₂	$\frac{3-5t+3t^2}{t}$	$-\frac{23+36t-24t^2+5t^3}{t^2}$	$\frac{(-1+t)(1+t)(23-31t+23t^2)}{t^2}$
7 ₃	$\frac{2-3t+3t^2-3t^3+2t^4}{t^2}$	$-\frac{-1+7t-13t^2+24t^3-32t^4+35t^5-27t^6+17t^7}{t^4}$	$-\frac{(-1+t)(1+t)(1+10t-13t^2+21t^3-13t^4+1t^5)}{t^4}$
7 ₄	$\frac{4-7t+4t^2}{t}$	$-\frac{4(-2+11t-17t^2+10t^3)}{t^2}$	$-\frac{4(-1+t)(1+t)(2-t+2t^2)}{t^2}$
7 ₅	$\frac{2-4t+5t^2-4t^3+2t^4}{t^2}$	$-\frac{-17+41t-65t^2+65t^3-49t^4+25t^5-9t^6+t^7}{t^4}$	$\frac{(-1+t)(1+t)(1+t^2)(17-40t+56t^2-40t^3+t^4)}{t^4}$
7 ₆	$-\frac{1-5t+7t^2-5t^3+t^4}{t^2}$	$-\frac{3-22t+53t^2-53t^3+25t^4-t^5-4t^6+t^7}{t^4}$	$\frac{(-1+t)(1+t)(3-23t+60t^2-75t^3+60t^4-23t^5+t^6)}{t^4}$
7 ₇	$\frac{1-5t+9t^2-5t^3+t^4}{t^2}$	$-\frac{2-13t+27t^2-9t^3-31t^4+33t^5-13t^6+2t^7}{t^4}$	$\frac{(-1+t)(1+t)(1-3t+t^2)^2(2-3t+2t^2)}{t^4}$
8 ₁	$-\frac{3-7t+3t^2}{t}$	$-\frac{14-33t+9t^2+4t^3}{t^2}$	$\frac{(-1+t)(1+t)(14-37t+14t^2)}{t^2}$
8 ₂	$-\frac{1-3t+3t^2-3t^3+3t^4-3t^5+t^6}{t^3}$	$-\frac{(-1+t)(-5+17t-19t^2+21t^3-19t^4+12t^5-7t^6+3t^7-t^8-t^9+t^{10})}{t^6}$	$\frac{(-1+t)(1+t)(1-t+t^2)(5-18t+20t^2-25t^3+34t^4-25t^5+t^6)}{t^6}$
8 ₃	$-\frac{4-9t+4t^2}{t}$	$-\frac{4(1+t)(4-9t+4t^2)}{t^2}$	$\frac{4(-1+t)(1+t)(4-9t+4t^2)}{t^2}$
8 ₄	$-\frac{2-5t+5t^2-5t^3+2t^4}{t^2}$	$-\frac{11-33t+37t^2-22t^3-2t^4+9t^5-11t^6+5t^7}{t^4}$	$\frac{(-1+t)(1+t)(11-38t+59t^2-69t^3+59t^4-38t^5+t^6)}{t^4}$
8 ₅	$-\frac{(1-t+t^2)(1-2t+t^2-2t^3+t^4)}{t^3}$	$-\frac{1-2t+t^2+4t^3-17t^4+36t^5-56t^6+67t^7-62t^8+43t^9-22t^{10}+5t^{11}}{t^6}$	$\frac{(-1+t)(1+t)(1-7t+24t^2-46t^3+69t^4-77t^5+69t^6-46t^7+t^8)}{t^6}$
8 ₆	$-\frac{2-6t+7t^2-6t^3+2t^4}{t^2}$	$-\frac{13-53t+84t^2-78t^3+42t^4-12t^5-3t^6+3t^7}{t^4}$	$\frac{(-1+t)(1+t)(13-56t+100t^2-122t^3+100t^4-51t^5+t^6)}{t^4}$
8 ₇	$\frac{1-3t+5t^2-5t^3+5t^4-3t^5+t^6}{t^3}$	$-\frac{2-7t+12t^2-12t^3+6t^4+13t^5-37t^6+56t^7-56t^8+40t^9-17t^{10}+4t^{11}}{t^6}$	$\frac{(-1+t)(1+t)(1-3t+3t^2-3t^3+t^4)(2-5t+10t^2-12t^3+t^4)}{t^6}$
8 ₈	$\frac{(2-2t+t^2)(1-2t+2t^2)}{t^2}$	$-\frac{7-23t+28t^2+6t^3-50t^4+60t^5-33t^6+9t^7}{t^4}$	$\frac{(-1+t)^3(1+t)(1-t+t^2)(7-11t+7t^2)}{t^4}$
8 ₉	$-\frac{(-1+t-2t^2+t^3)(-1+2t-t^2+t^3)}{t^3}$	$-\frac{(1+t)(-1+t-2t^2+t^3)(-1+2t-t^2+t^3)(3-6t+8t^2-6t^3+3t^4)}{t^6}$	$\frac{(-1+t)(1+t)(-1+t-2t^2+t^3)(-1+2t-t^2+t^3)(3-6t+t^2)}{t^6}$
8 ₁₀	$\frac{(1-t+t^2)^3}{t^3}$	$-\frac{(1-t+t^2)^2(2-3t+3t^2+t^3-7t^4+15t^5-9t^6+4t^7)}{t^6}$	$\frac{(-1+t)(1+t)(1-t+t^2)^2(2-7t+14t^2-21t^3+14t^4-t^5)}{t^6}$
8 ₁₁	$-\frac{(-2+t)(-1+2t)(1-t+t^2)}{t^2}$	$-\frac{13-63t+114t^2-109t^3+57t^4-12t^5-5t^6+3t^7}{t^4}$	$\frac{(-1+t)(1+t)(13-66t+132t^2-163t^3+132t^4-61t^5+t^6)}{t^4}$
8 ₁₂	$\frac{1-7t+13t^2-7t^3+t^4}{t^2}$	$-\frac{(1+t)(2-7t+2t^2)(1-7t+13t^2-7t^3+t^4)}{t^4}$	$\frac{(-1+t)(1+t)(2-7t+2t^2)(1-7t+13t^2-7t^3+t^4)}{t^4}$
8 ₁₃	$\frac{2-7t+11t^2-7t^3+2t^4}{t^2}$	$-\frac{7-29t+41t^2+2t^3-66t^4+77t^5-39t^6+9t^7}{t^4}$	$\frac{(-1+t)(1+t)(7-38t+87t^2-113t^3+87t^4-38t^5+t^6)}{t^4}$
8 ₁₄	$-\frac{2-8t+11t^2-8t^3+2t^4}{t^2}$	$-\frac{(-1+t)^2(13-47t+46t^2-22t^3-t^4+3t^5)}{t^4}$	$\frac{(-1+t)^3(1+t)(13-50t+60t^2-50t^3+13t^4-t^5)}{t^4}$
8 ₁₅	$\frac{(1-t+t^2)(3-5t+3t^2)}{t^2}$	$\frac{(3-4t+3t^2)(-13+29t-35t^2+23t^3-9t^4+t^5)}{t^4}$	$\frac{(-1+t)(1+t)(3-4t+3t^2)(13-28t+39t^2-28t^3+t^4)}{t^4}$
8 ₁₆	$\frac{1-4t+8t^2-9t^3+8t^4-4t^5+t^6}{t^3}$	$-\frac{(1-t+t^2)(4-20t+46t^2-55t^3+38t^4-10t^5-9t^6+14t^7-8t^8+2t^9)}{t^6}$	$\frac{(-1+t)(1+t)(1-t+t^2)(2-3t+2t^2)(2-8t+15t^2-15t^3+t^4)}{t^6}$
8 ₁₇	$-\frac{1-4t+8t^2-11t^3+8t^4-4t^5+t^6}{t^3}$	$-\frac{(1+t)(1-t+t^2)(3-5t+3t^2)(1-4t+8t^2-11t^3+8t^4-4t^5+t^6)}{t^6}$	$\frac{(-1+t)(1+t)(1-t+t^2)(3-5t+3t^2)(1-4t+8t^2-11t^3+8t^4-4t^5+t^6)}{t^6}$
8 ₁₈	$-\frac{(1-3t+t^2)(1-t+t^2)^2}{t^3}$	$-\frac{(1+t)(1-3t+t^2)(1-t+t^2)^3(3-7t+3t^2)}{t^6}$	$\frac{(-1+t)(1+t)(1-3t+t^2)(1-t+t^2)^3(3-7t+t^2)}{t^6}$
8 ₁₉	$\frac{(1-t+t^2)(1-t^2+t^4)}{t^3}$	$-\frac{1-t^2+2t^3+3t^4-3t^5+7t^7-5t^9+6t^{10}}{t^5}$	$-\frac{(-1+t)(1+t)(5-5t+6t^2+3t^4+6t^6-5t^7+t^8)}{t^5}$
8 ₂₀	$\frac{(1-t+t^2)^2}{t^2}$	$-\frac{2(1-t+t^2)(1-t+3t^2-t^3-t^4+t^5)}{t^4}$	$\frac{2(-1+t)(1+t)(1-t+t^2)(1-2t+5t^2-2t^3+t^4)}{t^4}$
8 ₂₁	$-\frac{(1-3t+t^2)(1-t+t^2)}{t^2}$	$-\frac{(-1+t)(-3+16t-24t^2+20t^3-8t^4+t^6)}{t^4}$	$\frac{(-1+t)(1+t)(3-20t+44t^2-56t^3+44t^4-20t^5+t^6)}{t^4}$

MatrixForm@Table[{K[[1]]_{K[[2]]}, Factor@za[K], zp1[K], zp2[K], Vassiliev[2][K], Vassiliev[3][K]}, {K, Select[AllKnots[{3, 10}], Vassiliev[3][#] == 0 &]}]

4 ₁	$-\frac{1-3t+t^2}{t}$	$-\frac{(1+t)(1-3t+t^2)}{t^2}$
6 ₃	$\frac{1-3t+5t^2-3t^3+t^4}{t^2}$	$-\frac{(1+t)(2-3t+2t^2)(1-3t+5t^2-3t^3+t^4)}{t^4}$
o	$\frac{4-9t+4t^2}{t}$	$\frac{4(1+t)(4-9t+4t^2)}{t^2}$

$$\begin{array}{l}
\begin{array}{l}
8_9 \\
8_{12} \\
8_{14} \\
8_{17} \\
8_{18} \\
9_{17} \\
9_{28} \\
9_{34} \\
9_{42} \\
10_{17} \\
10_{19} \\
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10_{108} \\
10_{109} \\
10_{111} \\
10_{115} \\
10_{116} \\
10_{118} \\
10_{119} \\
10_{123} \\
10_{125} \\
10_{133} \\
10_{146} \\
10_{147} \\
10_{164}
\end{array}
\end{array}$$

$$\begin{array}{l}
- \frac{(-1+t-2t^2+t^3)(-1+2t-t^2+t^3)}{t^3} \\
\frac{1-7t+13t^2-7t^3+t^4}{t^2} \\
- \frac{2-8t+11t^2-8t^3+2t^4}{t^2} \\
- \frac{1-4t+8t^2-11t^3+8t^4-4t^5+t^6}{t^3} \\
- \frac{(1-3t+t^2)(1-t+t^2)^2}{t^3} \\
\frac{1-5t+9t^2-9t^3+9t^4-5t^5+t^6}{t^3} \\
\frac{(1-t+t^2)(1-4t+7t^2-4t^3+t^4)}{t^3} \\
- \frac{1-6t+16t^2-23t^3+16t^4-6t^5+t^6}{t^3} \\
- \frac{1-2t+t^2-2t^3+t^4}{t^2} \\
\frac{1-3t+5t^2-7t^3+9t^4-7t^5+5t^6-3t^7+t^8}{t^4} \\
\frac{2-7t+11t^2-11t^3+11t^4-7t^5+2t^6}{t^3} \\
- \frac{(-2+t)(-1+2t)(1-t+t^2-t^3+t^4)}{t^3} \\
- \frac{(1-t+t^2)(2-6t+7t^2-6t^3+2t^4)}{t^3} \\
\frac{4-16t+25t^2-16t^3+4t^4}{t^2} \\
\frac{4-13t+19t^2-13t^3+4t^4}{t^2} \\
- \frac{1-7t+17t^2-23t^3+17t^4-7t^5+t^6}{t^3} \\
- \frac{1-7t+21t^2-31t^3+21t^4-7t^5+t^6}{t^3} \\
\frac{(1-t+2t^2-2t^3+t^4)(1-2t+2t^2-t^3+t^4)}{t^4} \\
- \frac{(-2+t)(-1+2t)(2-3t+2t^2)}{t^2} \\
- \frac{1-7t+18t^2-25t^3+18t^4-7t^5+t^6}{t^3} \\
\frac{1-3t+7t^2-12t^3+15t^4-12t^5+7t^6-3t^7+t^8}{t^4} \\
- \frac{1-8t+20t^2-27t^3+20t^4-8t^5+t^6}{t^3} \\
- \frac{(1-t+t^2)^2(1-2t+t^2-2t^3+t^4)}{t^4} \\
- \frac{1-8t+24t^2-35t^3+24t^4-8t^5+t^6}{t^3} \\
\frac{1-4t+9t^2-14t^3+17t^4-14t^5+9t^6-4t^7+t^8}{t^4} \\
\frac{(1-t+t^2)^4}{t^4} \\
\frac{1-4t+9t^2-15t^3+19t^4-15t^5+9t^6-4t^7+t^8}{t^4} \\
\frac{1-8t+22t^2-29t^3+22t^4-8t^5+t^6}{t^3} \\
\frac{2-8t+14t^2-15t^3+14t^4-8t^5+2t^6}{t^3} \\
\frac{1-4t+10t^2-17t^3+21t^4-17t^5+10t^6-4t^7+t^8}{t^4} \\
- \frac{(2-3t+2t^2)(1-3t+3t^2-3t^3+t^4)}{t^3} \\
- \frac{1-9t+26t^2-37t^3+26t^4-9t^5+t^6}{t^3} \\
- \frac{1-5t+12t^2-19t^3+21t^4-19t^5+12t^6-5t^7+t^8}{t^4} \\
\frac{1-5t+12t^2-19t^3+23t^4-19t^5+12t^6-5t^7+t^8}{t^4} \\
- \frac{2-10t+23t^2-31t^3+23t^4-10t^5+2t^6}{t^3} \\
\frac{(1-3t+3t^2-3t^3+t^4)^2}{t^4} \\
\frac{1-2t+2t^2-t^3+2t^4-2t^5+t^6}{t^3} \\
- \frac{1-5t+7t^2-5t^3+t^4}{t^2} \\
\frac{2-8t+13t^2-8t^3+2t^4}{t^2} \\
- \frac{(-2+t)(-1+2t)(1-t+t^2)}{t^2} \\
\frac{3-11t+17t^2-11t^3+3t^4}{t^2}
\end{array}$$

$$\begin{array}{l}
- \frac{t^2}{t^6} \\
- \frac{(1+t)(-1+2t-t^2+t^3)(-1+2t-t^2+t^3)(3-6t+8t^2-6t^3+2t^4)}{t^6} \\
- \frac{(1+t)(2-7t+2t^2)(1-7t+13t^2-7t^3+t^4)}{t^4} \\
- \frac{(-1+t)^2(13-47t+46t^2-22t^3-t^4+3t^5)}{t^4} \\
- \frac{(1+t)(1-t+t^2)(3-5t+3t^2)(1-4t+8t^2-11t^3+8t^4-4t^5+t^6)}{t^6} \\
- \frac{(1+t)(1-3t+t^2)(1-t+t^2)^3(3-7t+3t^2)}{t^6} \\
- \frac{4-31t+95t^2-153t^3+151t^4-92t^5+12t^6+31t^7-43t^8+33t^9-13t^{10}+2t^{11}}{t^6} \\
- \frac{(1-4t+7t^2-4t^3+t^4)(4-15t+26t^2-22t^3+8t^4+4t^5-5t^6+2t^7)}{t^6} \\
- \frac{3-27t+112t^2-269t^3+377t^4-253t^5-55t^6+255t^7-227t^8+106t^9-27t^{10}+3t^{11}}{t^6} \\
- \frac{(1+t)(1-2t+3t^2-10t^3+13t^4-10t^5+3t^6)}{t^4} \\
- \frac{(1+t)(4-9t+14t^2-16t^3+14t^4-9t^5+4t^6)(1-3t+5t^2-7t^3+9t^4-7t^5+5t^6-3t^7+t^8)}{t^8} \\
- \frac{15-77t+179t^2-244t^3+220t^4-116t^5-8t^6+80t^7-94t^8+77t^9-39t^{10}+9t^{11}}{t^6} \\
- \frac{21-111t+236t^2-315t^3+337t^4-291t^5+199t^6-113t^7+53t^8-12t^9-5t^{10}+3t^{11}}{t^6} \\
- \frac{13-73t+197t^2-347t^3+413t^4-300t^5+48t^6+177t^7-241t^8+163t^9-63t^{10}+11t^{11}}{t^6} \\
- \frac{8(-1+t)^2(1+t)(4-16t+25t^2-16t^3+4t^4)}{t^4} \\
- \frac{(1+t)(8-13t+8t^2)(4-13t+19t^2-13t^3+4t^4)}{t^4} \\
- \frac{(1+t)(3-14t+20t^2-14t^3+3t^4)(1-7t+17t^2-23t^3+17t^4-7t^5+t^6)}{t^6} \\
- \frac{(1+t)(3-14t+24t^2-14t^3+3t^4)(1-7t+21t^2-31t^3+21t^4-7t^5+t^6)}{t^6} \\
- \frac{(1+t)(1+t^2)(4-21t+64t^2-139t^3+235t^4-315t^5+342t^6-303t^7+221t^8-131t^9+62t^{10}-21t^{11}+4t^{12})}{t^8} \\
- \frac{4(-1+t)^2(14-53t+63t^2-35t^3+5t^4+2t^5)}{t^4} \\
- \frac{(1+t)(3-35t+174t^2-492t^3+888t^4-1071t^5+878t^6-484t^7+172t^8-35t^9+3t^{10})}{t^6} \\
- \frac{(1+t)(4-9t+18t^2-21t^3+18t^4-9t^5+4t^6)(1-3t+7t^2-12t^3+15t^4-12t^5+7t^6-3t^7+t^8)}{t^8} \\
- \frac{(1+t)(3-16t+23t^2-16t^3+3t^4)(1-8t+20t^2-27t^3+20t^4-8t^5+t^6)}{t^6} \\
- \frac{(-1+t)^2(1+t^2)(1-t+t^2)(5-16t+24t^2-27t^3+8t^4-5t^6+8t^7-8t^8+3t^9)}{t^8} \\
- \frac{(1+t)(1-3t+t^2)(3-7t+3t^2)(1-8t+24t^2-35t^3+24t^4-8t^5+t^6)}{t^6} \\
- \frac{(1+t)(1-t+t^2)(4-24t+79t^2-179t^3+310t^4-423t^5+466t^6-417t^7+302t^8-173t^9+77t^{10}-24t^{11}+4t^{12})}{t^8} \\
- \frac{4(1+t)(1-t+t^2)^7}{t^8} \\
- \frac{(1+t)(4-28t+107t^2-287t^3+592t^4-971t^5+1296t^6-1421t^7+1282t^8-951t^9+576t^{10}-279t^{11}+105t^{12}-28t^{13}+4t^{14})}{t^8} \\
- \frac{2-22t+94t^2-181t^3+80t^4+358t^5-890t^6+1032t^7-691t^8+264t^9-52t^{10}+4t^{11}}{t^6} \\
- \frac{(-1+t)^2(9-29t+46t^2-42t^3+18t^4+50t^5-90t^6+98t^7-59t^8+15t^9)}{t^6} \\
- \frac{(1+t)(4-12t+24t^2-29t^3+24t^4-12t^5+4t^6)(1-4t+10t^2-17t^3+21t^4-17t^5+10t^6-4t^7+t^8)}{t^8} \\
- \frac{3-9t-11t^2+122t^3-382t^4+758t^5-1074t^6+1122t^7-852t^8+451t^9-147t^{10}+21t^{11}}{t^6} \\
- \frac{(1+t)(3-18t+29t^2-18t^3+3t^4)(1-9t+26t^2-37t^3+26t^4-9t^5+t^6)}{t^6} \\
- \frac{(-1+t)^2(5-30t+89t^2-175t^3+240t^4-244t^5+169t^6-59t^7-38t^8+78t^9-73t^{10}+43t^{11}-16t^{12}+3t^{13})}{t^8} \\
- \frac{(-1+t)^2(1+t)(4-7t+10t^2-7t^3+4t^4)(1-5t+12t^2-19t^3+23t^4-19t^5+12t^6-5t^7+t^8)}{t^8} \\
- \frac{11-81t+264t^2-468t^3+400t^4+93t^5-697t^6+922t^7-692t^8+328t^9-95t^{10}+13t^{11}}{t^6} \\
- \frac{2(1+t)(2-3t+2t^2)(1-3t+3t^2-3t^3+t^4)^3}{t^8} \\
- \frac{(1+t)(2-6t+11t^2-15t^3+24t^4-33t^5+38t^6-35t^7+27t^8-14t^9+4t^{10})}{t^6} \\
- \frac{3-28t+77t^2-99t^3+71t^4-25t^5+2t^6+t^7}{t^4} \\
- \frac{(-1+t)^2(9-31t+34t^2+6t^3-17t^4+7t^5)}{t^4} \\
- \frac{5-19t+24t^2+t^3-53t^4+78t^5-49t^6+11t^7}{t^4} \\
- \frac{19-92t+181t^2-147t^3-9t^4+103t^5-70t^6+17t^7}{t^4}
\end{array}$$

PolynomialGCD[$(-1 + 2 t - t^2 - t^3 + 2 t^4 - t^5 + t^8)$, $(-1 + t^3 - 2 t^4 + t^5 + t^6 - 2 t^7 + t^8)$]

1

MatrixForm@Sort@Table[**Factor@za**[K], **zp1**[K], **zp2**[K], **K[[1]]_{K[[2]]}**], {K, **AllKnots** [{3, 10}]}

(... 1 ...)

large output

show less

show more

show all

set size limit...

Length@Union@Table[**Factor@za**[K] → **zp2**[K], {K, **AllKnots** [{3, 10}]}

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Length@Union@Table[**Factor@za**[K], {K, **AllKnots** [{3, 10}]}

211

Union@Table[**Expand@za**[K] → **Expand**[-2 **zp2**[K]], {K, **AllKnots** [{3, 7}]}

 // Column

$$5 - \frac{2}{t} - 2 t \rightarrow -2$$

$$3 - \frac{1}{t} - t \rightarrow -1$$

$$-1 + \frac{1}{t} + t \rightarrow 1$$

$$-3 + \frac{2}{t} + 2 t \rightarrow 2$$

$$-5 + \frac{3}{t} + 3 t \rightarrow 3$$

$$-7 + \frac{4}{t} + 4 t \rightarrow 4$$

$$-3 - \frac{1}{t^2} + \frac{3}{t} + 3 t - t^2 \rightarrow 3 - \frac{2}{t} - 2 t$$

$$-7 - \frac{1}{t^2} + \frac{5}{t} + 5 t - t^2 \rightarrow 5 - \frac{2}{t} - 2 t$$

$$9 + \frac{1}{t^2} - \frac{5}{t} - 5 t + t^2 \rightarrow -5 + \frac{2}{t} + 2 t$$

$$5 + \frac{1}{t^2} - \frac{3}{t} - 3 t + t^2 \rightarrow -3 + \frac{2}{t} + 2 t$$

$$1 + \frac{1}{t^2} - \frac{1}{t} - t + t^2 \rightarrow -1 + \frac{2}{t} + 2 t$$

$$5 + \frac{2}{t^2} - \frac{4}{t} - 4 t + 2 t^2 \rightarrow -4 + \frac{4}{t} + 4 t$$

$$3 + \frac{2}{t^2} - \frac{3}{t} - 3 t + 2 t^2 \rightarrow -3 + \frac{4}{t} + 4 t$$

$$-1 + \frac{1}{t^3} - \frac{1}{t^2} + \frac{1}{t} + t - t^2 + t^3 \rightarrow 4 + \frac{3}{t^2} - \frac{2}{t} - 2 t + 3 t^2$$

$$\text{Table} \left[\left\{ K[[1]]_{K[[2]]}, \text{Factor}@za[K], \text{Factor}@ \left(\frac{za[K] t^2 D[za[K], t] + t (t-1) zp1[K]}{(t-1)^2} \right) \right\}, \right.$$

$$2 \text{ Exponent}[za[K], t, \text{Min}] == \text{Exponent}[zp1[K], t, \text{Min}] \wedge$$

$$2 \text{ Exponent}[za[K], t, \text{Max}] == \text{Exponent}[t zp1[K], t, \text{Max}], \{K, \text{Ribbons}\} // \text{MatrixForm}$$

6 ₁	$-\frac{(-2+t)(-1+2t)}{t}$	$\frac{1-4t+t^2}{t}$	True	1
8 ₈	$\frac{(2-2t+t^2)(1-2t+2t^2)}{t^2}$	$-\frac{1-4t+12t^2-16t^3+12t^4-4t^5+t^6}{t^3}$	True	2
8 ₉	$-\frac{(-1+t-2t^2+t^3)(-1+2t-t^2+t^3)}{t^3}$	0	True	3
8 ₂₀	$\frac{(1-t+t^2)^2}{t^2}$	$\frac{4(1-t+t^2)}{t}$	True	2
9 ₂₇	$-\frac{(-1+2t-3t^2+t^3)(-1+3t-2t^2+t^3)}{t^3}$	$\frac{1-8t+24t^2-32t^3+24t^4-8t^5+t^6}{t^3}$	True	3
9 ₄₁	$\frac{(3-3t+t^2)(1-3t+3t^2)}{t^2}$	$\frac{3-20t+70t^2-108t^3+70t^4-20t^5+3t^6}{t^3}$	True	2
9 ₄₆	$-\frac{(-2+t)(-1+2t)}{t}$	$\frac{3(1-4t+t^2)}{t}$	True	1
10 ₃	$-\frac{(-3+2t)(-2+3t)}{t}$	$\frac{11-28t+11t^2}{t}$	True	1
10 ₂₂	$-\frac{(-2+2t-2t^2+t^3)(-1+2t-2t^2+2t^3)}{t^3}$	$-\frac{1-4t+10t^2-24t^3+37t^4-44t^5+37t^6-24t^7+10t^8-4t^9+t^{10}}{t^5}$	True	3
10 ₃₅	$\frac{(2-4t+t^2)(1-4t+2t^2)}{t^2}$	$-\frac{(-1+5t-7t^2+t^3)(-1+7t-5t^2+t^3)}{t^3}$	True	2
10 ₄₂	$-\frac{(-1+3t-4t^2+t^3)(-1+4t-3t^2+t^3)}{t^3}$	$\frac{2-8t+11t^2-12t^3+11t^4-8t^5+2t^6}{t^3}$	True	3
10 ₄₈	$\frac{(1-t+2t^2-2t^3+t^4)(1-2t+2t^2-t^3+t^4)}{t^4}$	$\frac{(-1+t)^2(1+t)^2(1+t^2)(1-t+t^2)^2}{t^5}$	True	4
10 ₇₅	$-\frac{(-1+3t-4t^2+t^3)(-1+4t-3t^2+t^3)}{t^3}$	$-\frac{(-2+t)^2(-1+2t)^2(1-4t+t^2)}{t^3}$	True	3
10 ₈₇	$-\frac{(-2+t)(-1+2t)(1-t+t^2)^2}{t^3}$	$-\frac{(1-t+t^2)(1-5t+17t^2-44t^3+64t^4-44t^5+17t^6-5t^7+t^8)}{t^5}$	True	3
10 ₉₉	$\frac{(1-t+t^2)^4}{t^4}$	0	True	4
10 ₁₂₃	$\frac{(1-3t+3t^2-3t^3+t^4)^2}{t^4}$	0	True	4
10 ₁₂₉	$\frac{(2-2t+t^2)(1-2t+2t^2)}{t^2}$	$-\frac{1+2t-14t^2+20t^3-14t^4+2t^5+t^6}{t^3}$	True	2
10 ₁₃₇	$\frac{(1-3t+t^2)^2}{t^2}$	$-\frac{4(1-3t+t^2)^2}{t^2}$	True	2
10 ₁₄₀	$\frac{(1-t+t^2)^2}{t^2}$	$\frac{8(1-t+t^2)}{t}$	True	2
10 ₁₅₃	$\frac{(1-t+t^3)(1-t^2+t^3)}{t^3}$	$\frac{1-2t+t^2+2t^3-t^4-t^6+2t^7+t^8-2t^9+t^{10}}{t^5}$	True	3
10 ₁₅₅	$-\frac{(-1+t-2t^2+t^3)(-1+2t-t^2+t^3)}{t^3}$	$-\frac{2(1-6t+11t^2-14t^3+11t^4-6t^5+t^6)}{t^3}$	True	3