Rescaling u for health and beauty

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$\mathfrak{g}_0$ **bi-local exponentiation relations.** In $\mathfrak{g}_0 = \mathcal{D}\mathbb{I} = \mathcal{D}\text{ii} = \mathcal{D}\text{II} = \mathcal{D}\text{II} := \langle b, c, u, w\rangle/([b,\cdot] = 0, [c,u] = u, [c,w] = -w, [u,w] = b)$ with $\deg(b,c,u,w) = (1,0,1,0)$ and $a_{12} = b_1c_2 + u_1w_2$: (verifications in `G0.nb`)

$u = \frac{b\bar{u}}{e^b-1} = \zeta\bar{u}$ $\zeta = \frac{b}{e^b-1}$ $\zeta^{-1} = \frac{e^b-1}{b}$

$[\bar{u}, w] = e^b - 1$

1. The Yang-Baxter element,
$$\exp_{\mathcal{U}}(a_{12}) = \exp\left(b_1c_2 + \tfrac{e^{b_1}-1}{b_1}u_1w_2\right) /\!\!/ m_1^{u_1} /\!\!/ m_2^{c_2w_2} \to b_1c_2 + \bar{u}_1w_2$$

2. ✓ $[c, uw] = 0$ ✓
   ✓ $e^{\beta u}e^{\alpha c} = e^{\alpha c}e^{e^{-\alpha}\beta u}$ and $e^{\beta w}e^{\alpha c} = e^{\alpha c}e^{e^{\alpha}\beta w}$ ✓

3. ✓ $[w, e^{\gamma u}] = -b\gamma e^{\gamma u}$ and $[u, e^{\gamma w}] = b\gamma e^{\gamma w}$

4. With $M_{uw} = M_{uw}(\gamma) := e^{\gamma uw} /\!\!/ m^{uw} = \sum_{k\geq 0}\frac{\gamma^k}{k!}u^kw^k$,
   ✓ $[u, M_{uw}] = b\gamma u M_{uw}$ and $[w, M_{uw}] = -b\gamma M_{uw}w$
   ✓ $M_{uw}^{-1}(\gamma(\alpha))\partial_\alpha M_{uw}(\gamma(\alpha)) = \frac{\partial_\alpha\gamma(\alpha)}{1-b\gamma(\alpha)}uw$

$b\nu\beta =$

5. With $M_{wu} = M_{wu}(\delta) := e^{\delta uw} /\!\!/ m^{wu} = \sum_{k\geq 0}\frac{\delta^k}{k!}w^ku^k$,
   ✓ $[u, M_{wu}] = b\delta M_{wu}u$ and $[w, M_{wu}] = -b\delta w M_{wu}$
   ✓ $M_{wu}^{-1}(\alpha\delta)\partial_\alpha M_{wu}(\alpha\delta) = \frac{\delta}{1+b\alpha\delta}wu = \frac{\delta}{1+b\alpha\delta}(uw - b)$

$\frac{1}{1+b\zeta^{-1}\delta} = \frac{1}{1+(e^b-1)\delta} = \frac{1}{\quad}$

6. ✓ $M_{wu}(\delta) = \frac{1}{1+b\delta}M_{uw}\left(\frac{\delta}{1+b\delta}\right)$

$M_{w\bar{u}}(\delta) = M_{wu}(\zeta^{-1}\delta) = \frac{1}{1+(e^b-1)\delta}M_{uw}\left(\frac{\zeta\delta}{1+(e^b-1)\delta}\right)$

$= \frac{1}{1+(e^b-1)\delta}M_{\bar{u}w}\left(\frac{\delta}{1+(e^b-1)\delta}\right)$

7. ✓ The hard core uw relation. $e^{\alpha w}e^{\beta u} = e^{-b\alpha\beta}e^{\beta u}e^{\alpha w}$
   with $\nu = (1+b\delta)^{-1}$, $e^{\alpha w}M_{wu}(\delta)e^{\beta u} = \nu e^{-b\nu\alpha\beta}e^{\nu\beta u}M_{uw}(\nu\delta)e^{\nu\alpha w}$

$\to \bar{\nu} = (1+(e^b-1)\delta)^{-1}$ $\bar{\beta} = \zeta\beta = \frac{b}{e^b-1}\beta$

$e^{\alpha w}M_{w\bar{u}}(\delta)e^{\bar{\beta}\bar{u}} = \bar{\nu}e^{(1-e^b)}$

✓ **1-Smidgen** sl_2 / $\mathfrak{g}_1$ **bi-local exponentiation relations.** With $\epsilon^2 = 0$, in $\mathfrak{g}_1 := \mathbb{Q}[\epsilon]\langle b, c, u, w\rangle/([b,\cdot] = 0, [c,u] = u, [c,w] = -w, [u,w] = b - 2\epsilon c)$ with $\deg(b,c,u,w,\epsilon) = (1,0,1,0,1)$ and $a_{12} = (b_1 - \epsilon c_1)c_2 + u_1w_2$: (verifications in `G1.nb`)

1. c relations: $e^{\beta u}e^{\alpha c} = e^{\alpha c}e^{e^{-\alpha}\beta u}$ and $e^{\beta w}e^{\alpha c} = e^{\alpha c}e^{e^{\alpha}\beta w}$

2. With $\nu = (1+b\delta)^{-1}$ and Λ as below,
$$\mathbb{O}\left(e^{\alpha w+\beta u+\delta uw} \mid wu\right) = \mathbb{O}\left(\nu(1+\epsilon\nu\Lambda)e^{\nu(-b\alpha\beta+\alpha w+\beta u+\delta uw)} \mid cuw\right)$$

(160805) Λ for Λόγος, "a principle of order and knowledge": $\Lambda = -\frac{1}{2}b\nu\left(\alpha^2\beta^2\nu^2 + 4\alpha\beta\delta\nu + 2\delta^2\right) - \frac{1}{2}\beta^2\delta\nu^3u^2(3b\delta+2) - \frac{1}{2}b\delta^4\nu^3u^2w^2 - \beta\delta^2\nu^3u^2w(2b\delta+1) - \beta\nu^2u(2b\delta+1)(\alpha\beta\nu+2\delta) - 2b\delta^2\nu^2uw(\alpha\beta\nu+\delta) + \frac{1}{2}\alpha^2\delta\nu^3w^2(b\delta+2) + 2c(\alpha\beta\nu+\delta) + 2\beta c\delta\nu u + 2c\delta^2\nu uw + 2\alpha c\delta\nu w + \alpha\delta^2\nu^3uw^2 + \alpha\nu^2w(\alpha\beta\nu+2\delta)$.

(160801a) Roland's $sm_qsl(2)$ formulas, BBS:VanDerVeen-160731, with $q = e^\epsilon$, $t = e^b$: b central, $[w,c] = w$, $[c,u] = u$, $wu - quw = 1 - te^{2\epsilon c}$ (at $\epsilon^2 = 0$: $[w,u] = \epsilon uw + 1 - t - 2\epsilon tc$), $R = \sum_{m,n}\frac{u^n(b+\epsilon c)^m\otimes c^mw^n}{m![n]_q!} \to \sum_{m,n}\frac{u^n(b+\epsilon c)^m\otimes c^mw^n}{m!n!}\left(1 - \frac{\epsilon}{2}\binom{n}{2}\right)$. Also, $\Delta(b,c,u,w) = (b_1 + b_2, c_1 + c_2, t_2e^{\epsilon c_2}u_1 + u_2, e^{\epsilon c_2}w_1 + w_2)$ and $S(b,c,u,w) = (-b, -c, -t^{-1}ue^{-\epsilon c}, -we^{-\epsilon c})$. Verified `VdVAlgebraAt1-Testing.nb`.