Manolescu on Homology Cobordism Invariants from Floer Theory

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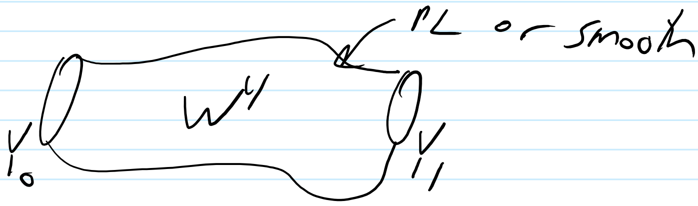
$$\Theta^3_{\mathbb{Z}} = \{\text{oriented } \overset{PL}{\mathbb{Z}HS^3}\}/\sim \xrightarrow[\text{homo } \mu]{\text{Rokhlin}} \mathbb{Z}/2$$

$\sim$:



Motivation: Triangulation of manifolds dim ≥ 5

M^n $(n \geq 5)$ is triangulable

$\iff$ obsr $\in H^5(M, \ker \mu) = 0$

and then the triangulation are in a bijection

w/ $H^4(M, \ker \mu)$

<u>Thm</u> (Furuta, Stern) $\Theta^3_{\mathbb{Z}}$ has a $\mathbb{Z}^\infty$ subgroup.

<u>Thm</u> (Frøyshov) $\Theta^3_{\mathbb{Z}}$ has $\mathbb{Z}$ summand.

<u>Open problems</u> 1. Does $\Theta^3_{\mathbb{Z}}$ have a $\mathbb{Z}^\infty$ summand?

2. Does $\Theta^3_{\mathbb{Z}}$ have torsion?

2' (maybe within reach) is there torsion w/ $\mu = k$

3. Is $\Theta^3_{\mathbb{Z}}$ generated by Seifert fibrations?

4. Do the following vanish in $\Theta^3_{\mathbb{Z}}$?

$\Sigma(2,3,Rn+1)$ $\Sigma(3,5,8)$ $\Sigma(3,5,8,241)$

5. Can we have $[\Sigma(a_1 \ldots a_k)] = 0$ for $k \geq 4$?