Adam Levine: Khovanov Homology & Knot Floer Homology for Pointed Links

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w/ J. Baldwin and S. Sarkar.

$L \subset S^3$ a link. Khovanov: reduced.

$$\widetilde{Kh}(L) = \bigoplus_{i,j} \widetilde{Kh}^{i,j}(L) \quad \text{over } \mathbb{Z}/2$$

Knot Floer homology:

$$\widehat{HFK}(L) = \bigoplus_{a,n} \widehat{HFK}_n(L, a)$$

<u>Conjecture</u> (Rasmussen) For any knot K

$$\operatorname{rank} \widetilde{Kh}(K) \geq \operatorname{rank} \widehat{HFK}(K)$$

$\Rightarrow$ if $\operatorname{rk} \widetilde{Kh}(K) = 1$ then K = unknot.

(Later proven by Kronheimer Mrowka)

$\overset{\text{also}}{\Rightarrow}$ If $\operatorname{rk} \widetilde{Kh}(K) = 3$ then K = trefoil

(not proven)

For an l-component link, conjecture: $2^{l-1} \operatorname{rk} \widetilde{Kh}(L) \geq \operatorname{rk} \widehat{HFK}(L)$

Relations between $\widetilde{Kh}$ and other invts:

<u>Thm</u> $\exists$ a spectral seq w/ $E_2 = \widetilde{Kh}(L)$

and $E_\infty = \widehat{HF}(-\Sigma(L))$

branched double cover.

$\Rightarrow \operatorname{rk} E_2 \geq \operatorname{rk} E_\infty$

Similar stories: 1. Monopole Floer homology of $\Sigma(L)$

2. Instanton knot homology [which by Kron-Mrow, detects the unknot]

How do these spectral sequences arise?

A: One of these theories.

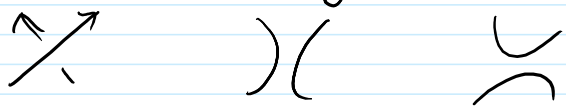
$$A(L) = H_*(C_A(L))$$

chain cmplx

s.t. 1. $A(\text{unlink}) \cong Kh(\text{unlink})$, $\operatorname{rk} = 2^{l-1}$.

2. Skein exact sequence:

$L \qquad L_0 \qquad L_1$



exact:

$$\begin{tikzcd} A(L) \arrow[r] & A(L_0) \arrow[r] & A(L_1) \arrow[ll, bend left] \end{tikzcd}$$

or, $C_A(L) \simeq MC(C_A(L_0) \to C_A(L_1))$

chain homotopy

Idea: use this for all (n) crossings at the same time

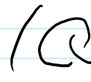
resolution of L

$$X_A(L) = \bigoplus C_A(L_v) \qquad D = \sum \ldots$$

$v \in \{0,1\}^n$

then $H_*(X_A, D) \cong A(L)$

Filter $X_A(L)$ by $|v|$ & get the said spectral sequence...

What with $\widehat{HFK}$? rk $\widehat{HFK}$() $= 4$

but resolving a crossing in any way gives the unknot, and

rk $\widehat{HFK}(O) = 1$

So $\widehat{HFK}$ does not fit in a skein exact sequence.

It is true that rk $\widehat{HFK}(O^l) = 2^{l-1}$.

Rationale: $\widehat{HFK}$ is really an invariant of pointed links:

$$(L, \underbrace{\{p_1, \dots, p_n\}}_{\underline{p}}) \quad \underline{p} \subset L$$

such each component contains at least one point.

$\widehat{HFK}^{\text{of before}} = \widehat{HFK}\left(\begin{smallmatrix}\text{one point}\\ \text{on each component}\end{smallmatrix}\right)$

Adding a pt to component that already has

one $\otimes$ the HFK w/ V, where $\dim V = 2$.
If every component is pointed, then there is an appropriate skein exact sequence.

Baldwin-Levine: with enough points sprinkled, there is a "cube of resolutions" computation of HFK; unfortunately, E_2 is not invariant.

Baldwin-Levine, Dec 2015: Kh can be tweaked to have similar properties.

Conjecture With the extra grading on HFK coming from the Alexander, it's gr is that extended Kh.