

Pensieve header: Attempting to find the Gassner+ representation. Continued pensieve://2016-06/.

The terms to understand:

$$a[f[b_1, b_2], 1, h\infty] \rightarrow a[f[b_1, b_2], 1, h\infty]$$

$$a[f[b_1, b_2], 2, h\infty] \rightarrow a[e^{b_1} f[b_1, b_2], 2, h\infty] + a\left[-\frac{(-1+e^{b_1}) f[b_1, b_2] b_2}{b_1}, 1, h\infty\right] +$$

$$ao\left[\frac{1}{b_1} (f[b_1, b_2] (-1 + e^{b_1} - e^{b_1} b_2) + (-1 + e^{b_1}) b_2 (f^{(0,1)}[b_1, b_2] - f^{(1,0)}[b_1, b_2]))\right], 1, h\infty]$$

$$\sigma_{i,j} := \text{ReplaceAll}\left[\left\{f_{-} \cdot v_j \Rightarrow f(1 - t_i) v_i + f t_i v_j + (t_i \partial_{t_i} f - t_j \partial_{t_j} f) \alpha_{i,j} w_i + f \beta_{i,j} w_i, w_j \Rightarrow (1 - t_i) w_i + t_i w_j\right\}\right]$$

lhs =

$$(\{v_1, v_2, v_3, w_1, w_2, w_3, t_1 v_1, t_2 v_1, t_3 v_1, t_1 v_2, t_2 v_2, t_3 v_2, t_1 v_3, t_2 v_3, t_3 v_3\}) // \sigma_{1,2} //$$

$$\sigma_{1,3} // \sigma_{2,3} // \text{Expand}$$

$$\{v_1, v_1 - t_1 v_1 + t_1 v_2 + w_1 \beta_{1,2}, v_1 - t_1 v_1 + t_1 v_2 - t_1 t_2 v_2 + t_1 t_2 v_3 + w_1 \beta_{1,3} + t_1 w_2 \beta_{2,3},$$

$$w_1, w_1 - t_1 w_1 + t_1 w_2, w_1 - t_1 w_1 + t_1 w_2 - t_1 t_2 w_2 + t_1 t_2 w_3,$$

$$t_1 v_1, t_2 v_1, t_3 v_1, t_1 v_1 - t_1^2 v_1 + t_1^2 v_2 + t_1 w_1 \alpha_{1,2} + t_1 w_1 \beta_{1,2},$$

$$t_2 v_1 - t_1 t_2 v_1 + t_1 t_2 v_2 - t_2 w_1 \alpha_{1,2} + t_2 w_1 \beta_{1,2}, t_3 v_1 - t_1 t_3 v_1 + t_1 t_3 v_2 + t_3 w_1 \beta_{1,2},$$

$$t_1 v_1 - t_1^2 v_1 + t_1^2 v_2 - t_1^2 t_2 v_2 + t_1^2 t_2 v_3 + t_1 w_1 \alpha_{1,3} + t_1 w_1 \beta_{1,3} + t_1^2 w_2 \beta_{2,3},$$

$$t_2 v_1 - t_1 t_2 v_1 + t_1 t_2 v_2 - t_1 t_2^2 v_2 + t_1 t_2^2 v_3 + t_1 t_2 w_2 \alpha_{2,3} + t_2 w_1 \beta_{1,3} + t_1 t_2 w_2 \beta_{2,3}, t_3 v_1 -$$

$$t_1 t_3 v_1 + t_1 t_3 v_2 - t_1 t_2 t_3 v_2 + t_1 t_2 t_3 v_3 - t_3 w_1 \alpha_{1,3} - t_1 t_3 w_2 \alpha_{2,3} + t_3 w_1 \beta_{1,3} + t_1 t_3 w_2 \beta_{2,3}\}$$

rhs =

$$(\{v_1, v_2, v_3, w_1, w_2, w_3, t_1 v_1, t_2 v_1, t_3 v_1, t_1 v_2, t_2 v_2, t_3 v_2, t_1 v_3, t_2 v_3, t_3 v_3\}) // \sigma_{2,3} //$$

$$\sigma_{1,3} // \sigma_{1,2} // \text{Expand}$$

$$\{v_1, v_1 - t_1 v_1 + t_1 v_2 + w_1 \beta_{1,2}, v_1 - t_1 v_1 + t_1 v_2 - t_1 t_2 v_2 + t_1 t_2 v_3 +$$

$$t_2 w_1 \alpha_{1,2} + w_1 \beta_{1,2} - t_2 w_1 \beta_{1,2} + t_2 w_1 \beta_{1,3} + w_1 \beta_{2,3} - t_1 w_1 \beta_{2,3} + t_1 w_2 \beta_{2,3},$$

$$w_1, w_1 - t_1 w_1 + t_1 w_2, w_1 - t_1 w_1 + t_1 w_2 - t_1 t_2 w_2 + t_1 t_2 w_3, t_1 v_1, t_2 v_1, t_3 v_1,$$

$$t_1 v_1 - t_1^2 v_1 + t_1^2 v_2 + t_1 w_1 \alpha_{1,2} + t_1 w_1 \beta_{1,2}, t_2 v_1 - t_1 t_2 v_1 + t_1 t_2 v_2 - t_2 w_1 \alpha_{1,2} + t_2 w_1 \beta_{1,2},$$

$$t_3 v_1 - t_1 t_3 v_1 + t_1 t_3 v_2 + t_3 w_1 \beta_{1,2}, t_1 v_1 - t_1^2 v_1 + t_1^2 v_2 - t_1^2 t_2 v_2 + t_1^2 t_2 v_3 + t_1 w_1 \alpha_{1,2} +$$

$$t_1 t_2 w_1 \alpha_{1,3} + t_1 w_1 \beta_{1,2} - t_1 t_2 w_1 \beta_{1,2} + t_1 t_2 w_1 \beta_{1,3} + t_1 w_1 \beta_{2,3} - t_1^2 w_1 \beta_{2,3} + t_1^2 w_2 \beta_{2,3},$$

$$t_2 v_1 - t_1 t_2 v_1 + t_1 t_2 v_2 - t_1 t_2^2 v_2 + t_1 t_2^2 v_3 - t_2 w_1 \alpha_{1,2} + 2 t_2^2 w_1 \alpha_{1,2} + t_2 w_1 \alpha_{2,3} - t_1 t_2 w_1 \alpha_{2,3} +$$

$$t_1 t_2 w_2 \alpha_{2,3} + t_2 w_1 \beta_{1,2} - t_2^2 w_1 \beta_{1,2} + t_2^2 w_1 \beta_{1,3} + t_2 w_1 \beta_{2,3} - t_1 t_2 w_1 \beta_{2,3} + t_1 t_2 w_2 \beta_{2,3},$$

$$t_3 v_1 - t_1 t_3 v_1 + t_1 t_3 v_2 - t_1 t_2 t_3 v_2 + t_1 t_2 t_3 v_3 + t_2 t_3 w_1 \alpha_{1,2} -$$

$$t_2 t_3 w_1 \alpha_{1,3} - t_3 w_1 \alpha_{2,3} + t_1 t_3 w_1 \alpha_{2,3} - t_1 t_3 w_2 \alpha_{2,3} + t_3 w_1 \beta_{1,2} -$$

$$t_2 t_3 w_1 \beta_{1,2} + t_2 t_3 w_1 \beta_{1,3} + t_3 w_1 \beta_{2,3} - t_1 t_3 w_1 \beta_{2,3} + t_1 t_3 w_2 \beta_{2,3}\}$$

lhs - rhs // Simplify

$$\{0, 0, -w_1 (\beta_{1,2} - \beta_{1,3} + t_2 (\alpha_{1,2} - \beta_{1,2} + \beta_{1,3}) + \beta_{2,3} - t_1 \beta_{2,3}), 0, 0, 0, 0, 0, 0, 0,$$

$$0, 0, 0, t_1 w_1 (-\alpha_{1,2} - (-1 + t_2) \alpha_{1,3} - \beta_{1,2} + t_2 \beta_{1,2} + \beta_{1,3} - t_2 \beta_{1,3} - \beta_{2,3} + t_1 \beta_{2,3}),$$

$$t_2 w_1 ((1 - 2 t_2) \alpha_{1,2} + (-1 + t_1) \alpha_{2,3} - \beta_{1,2} + t_2 \beta_{1,2} + \beta_{1,3} - t_2 \beta_{1,3} - \beta_{2,3} + t_1 \beta_{2,3}),$$

$$-t_3 w_1 (\alpha_{1,3} - \alpha_{2,3} + t_1 \alpha_{2,3} + \beta_{1,2} - \beta_{1,3} + t_2 (\alpha_{1,2} - \alpha_{1,3} - \beta_{1,2} + \beta_{1,3}) + \beta_{2,3} - t_1 \beta_{2,3})\}$$

$$\beta_{1,2} - \beta_{1,3} + t_2 (\alpha_{1,2} - \beta_{1,2} + \beta_{1,3}) + \beta_{2,3} - t_1 \beta_{2,3} /. \{\alpha_{i,j} \Rightarrow \alpha[t_i, t_j], \beta_{i,j} \Rightarrow \beta[t_i, t_j]\}$$

$$\beta[t_1, t_2] - \beta[t_1, t_3] + t_2 (\alpha[t_1, t_2] - \beta[t_1, t_2] + \beta[t_1, t_3]) + \beta[t_2, t_3] - t_1 \beta[t_2, t_3]$$

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D[β[t1, t2] - β[t1, t3] + t2 (α[t1, t2] - β[t1, t2] + β[t1, t3]) +
  β[t2, t3] - t1 β[t2, t3], t3] // Simplify
(-1 + t2) β(0,1)[t1, t3] - (-1 + t1) β(0,1)[t2, t3]

(((-1 + t2) β(0,1)[t1, t3] - (-1 + t1) β(0,1)[t2, t3]) / ((-1 + t2) (-1 + t1))) // Apart
β(0,1)[t1, t3] / (-1 + t1) - β(0,1)[t2, t3] / (-1 + t2)

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β[x_, y_] := γ[y] (x - 1) + δ[x]

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D[β[t1, t2] - β[t1, t3] + t2 (α[t1, t2] - β[t1, t2] + β[t1, t3]) +
  β[t2, t3] - t1 β[t2, t3], t3] // Simplify

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0

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Solve[

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  β[t1, t2] - β[t1, t3] + t2 (α[t1, t2] - β[t1, t2] + β[t1, t3]) + β[t2, t3] - t1 β[t2, t3] == 0,
  α[t1, t2]]

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{{α[t1, t2] → 1/t2 (γ[t2] - t1 γ[t2] - t2 γ[t2] + t1 t2 γ[t2] - δ[t2] + t1 δ[t2])}}

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(lhs - rhs /. {αi,j := (ti - 1), βi,j := ti}}) // Simplify

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{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

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lhs =

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({f[t1, t2, t3] v1, f[t1, t2, t3] v2, f[t1, t2, t3] v3, w1, w2, w3}) // σ1,2 // σ1,3 // σ2,3 //
Simplify

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{f[t1, t2, t3] v1, f[t1, t2, t3] (-(-1 + t1) v1 + t1 v2 + w1 β1,2) +
  w1 α1,2 (-t2 f(0,1,0)[t1, t2, t3] + t1 f(1,0,0)[t1, t2, t3]),
f[t1, t2, t3] (-(-1 + t1) v1 + w1 β1,3 + t1 (v2 - t2 v2 + t2 v3 + w2 β2,3)) -
  t3 (w1 α1,3 + t1 w2 α2,3) f(0,0,1)[t1, t2, t3] +
  t1 (t2 w2 α2,3 f(0,1,0)[t1, t2, t3] + w1 α1,3 f(1,0,0)[t1, t2, t3]),
w1, -(-1 + t1) w1 + t1 w2, -(-1 + t1) w1 + t1 (-(-1 + t2) w2 + t2 w3)}

```

rhs =

$(\{f[t_1, t_2, t_3] v_1, f[t_1, t_2, t_3] v_2, f[t_1, t_2, t_3] v_3, w_1, w_2, w_3\}) // \sigma_{2,3} // \sigma_{1,3} // \sigma_{1,2} //$

Simplify

$\{f[t_1, t_2, t_3] v_1, f[t_1, t_2, t_3] (-(-1+t_1) v_1 + t_1 v_2 + w_1 \beta_{1,2}) +$
 $w_1 \alpha_{1,2} (-t_2 f^{(0,1,0)}[t_1, t_2, t_3] + t_1 f^{(1,0,0)}[t_1, t_2, t_3]),$
 $f[t_1, t_2, t_3] (-1+t_1) (-1+t_2) v_1 - f[t_1, t_2, t_3] (-1+t_1) t_2 v_1 -$
 $f[t_1, t_2, t_3] t_1 (-1+t_2) v_2 + f[t_1, t_2, t_3] t_1 t_2 v_3 - f[t_1, t_2, t_3] (-1+t_2) w_1 \beta_{1,2} +$
 $f[t_1, t_2, t_3] t_2 w_1 \beta_{1,3} + f[t_1, t_2, t_3] (-(-1+t_1) w_1 + t_1 w_2) \beta_{2,3} +$
 $(-(-1+t_1) w_1 + t_1 w_2) \alpha_{2,3} (-t_3 f^{(0,0,1)}[t_1, t_2, t_3] + t_2 f^{(0,1,0)}[t_1, t_2, t_3]) +$
 $t_2 w_1 \alpha_{1,3} (-t_3 f^{(0,0,1)}[t_1, t_2, t_3] + t_1 f^{(1,0,0)}[t_1, t_2, t_3]) +$
 $w_1 \alpha_{1,2} (f[t_1, t_2, t_3] t_2 + (-1+t_2) (t_2 f^{(0,1,0)}[t_1, t_2, t_3] - t_1 f^{(1,0,0)}[t_1, t_2, t_3])) ,$
 $w_1, -(-1+t_1) w_1 + t_1 w_2, -(-1+t_1) w_1 + t_1 (-(-1+t_2) w_2 + t_2 w_3) \}$

$(lhs - rhs /. \{\alpha_{i,j} \rightarrow (t_i - 1), \beta_{i,j} \rightarrow t_i\}) // \text{Simplify}$

$\{0, 0, 0, 0, 0, 0\}$