Cheat Sheet OneCo

Deriving Gassner. $\mathcal{L}^{2Dw}$ is $\mathbb{Q}[\![b_i]\!]\langle a_{ij}\rangle$ modulo locality, $[a_{ij},a_{ik}]=0$, $[a_{ik},a_{jk}]=-[a_{ij},a_{jk}]=b_ja_{ik}-b_ia_{jk}$, and (mod $\langle a_{ii}\rangle$) $[a_{ij},a_{ji}]=b_ia_{ji}-b_ja_{ij}$. Acts on $V=\mathbb{Q}[\![b_i]\!]\langle x_i=a_{i\infty}\rangle$ by $[a_{ij},x_i]=0$, $[a_{ij},x_j]=b_ix_j-b_jx_i$. Hence $e^{\operatorname{ad}a_{ij}}x_i=x_i$, $e^{\operatorname{ad}a_{ij}}x_j=e^{b_i}x_j+\frac{b_j}{b_i}(1-e^{b_i})x_i$. Renaming $\bar{x}_i=x_i/b_i$, $t_i=e^{b_i}$, get $[e^{\operatorname{ad}a_{ij}}]_{\bar{x}_i,\bar{x}_j}=\begin{pmatrix}1 & 1-t_i\\ 0 & t_i\end{pmatrix}$.

The $\mathcal{L}^{2Dw}$ Adjoint representation. $e^{\operatorname{ad}a_{ij}}$ acts by

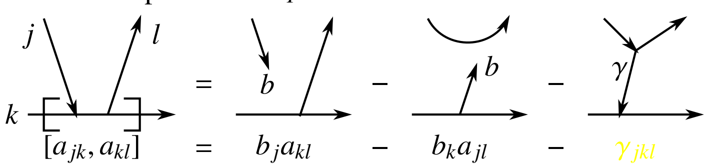
$$a_{kl}\mapsto a_{kl},\qquad a_{ik}\mapsto a_{ik},\qquad a_{kj}\mapsto e^{-b_i}a_{kj}+\frac{b_k}{b_i}(1-e^{-b_i})a_{ij},$$
$$a_{ki}\mapsto a_{ki}+(1-e^{-b_i})a_{kj}+b_k\frac{e^{-b_i}-1}{b_i}a_{ij},$$
$$a_{jk}\mapsto e^{b_i}a_{jk}+\frac{b_j}{b_i}(1-e^{b_i})a_{ik},\qquad a_{ji}\mapsto e^{b_i}a_{ji}+\frac{b_j}{b_i}(1-e^{b_i})a_{ij}.$$

Implementation/verification: pensieve://2015-04/nb/ZeroCo.pdf, pensieve://2016-04/nb/BurauAndAd.pdf.

Adjoint Gassner. Renaming $\bar{a}_{ij}=a_{ij}/b_i$ and $t_i=e^{b_i}$, get $[\bar{a}_{ij},\bar{a}_{ik}]=0$, $[\bar{a}_{ik},\bar{a}_{jk}]=-[\bar{a}_{ij},\bar{a}_{jk}]=\bar{a}_{ik}-\bar{a}_{jk}$, and (mod $\langle\bar{a}_{ii}\rangle$) $[\bar{a}_{ij},\bar{a}_{ji}]=\bar{a}_{ji}-\bar{a}_{ij}$, so

$$\bar{a}_{kj}\mapsto t_i^{-1}\bar{a}_{kj}+(1-t_i^{-1})\bar{a}_{ij},$$
$$\bar{a}_{ki}\mapsto\bar{a}_{ki}+(1-t_i^{-1})\bar{a}_{kj}+(t_i^{-1}-1)\bar{a}_{ij},$$
$$\bar{a}_{jk}\mapsto t_i\bar{a}_{jk}+(1-t_i)\bar{a}_{ik},\qquad \bar{a}_{ji}\mapsto t_i\bar{a}_{ji}+(1-t_i)\bar{a}_{ij}.$$

Question. Interpretation? π_T-Artin?



2Dv. b: bracket trace; c: cobracket trace; $\langle b,c\rangle=\delta\in\{0,1\}$; $\deg b_i=\deg c_j=\deg a_{ij}=\deg\delta=1$. Implementation/verification: pensieve://2015-08/nb/abc.pdf.

$\mathcal{A}^{2Dv}$ is $\mathbb{Q}[\![\delta]\!]FA(b_i,c_j,a_{ij})$ (so $\mathcal{L}^v=\{f+f^{ij}a_{ij}\}$) modulo locality,

tt. $[a_{jk},a_{jl}]=c_la_{jk}-c_ka_{jl}$,

hh. $[a_{jk},a_{ik}]=b_ia_{jk}-b_ja_{ik}$,

Swinging. $\delta a_{ij}a_{kl}-\delta a_{il}a_{kj}=b_kc_la_{ij}-b_ic_la_{kj}-b_kc_ja_{il}+b_ic_ja_{kl}$

ht. $[a_{jk},a_{kl}]=b_ja_{kl}-b_ka_{jl}-c_la_{jk}+c_ka_{jl}$,

ab,ac. $\operatorname{ad}a_{jk}\colon b_j,-b_k,-c_j,c_k\mapsto\gamma_{jk}:=\delta a_{jk}-b_jc_k$,

Backie. $[a_{jk},a_{kj}]=(b_j+c_k)a_{kj}-(b_k+c_j)a_{jk}+(b_j-c_j)a_{kk}-(b_k-c_k)a_{jj}+\gamma_{jk}-\gamma_{kj}$, with $\gamma_{jk}:=\delta a_{jk}-b_jc_k$,

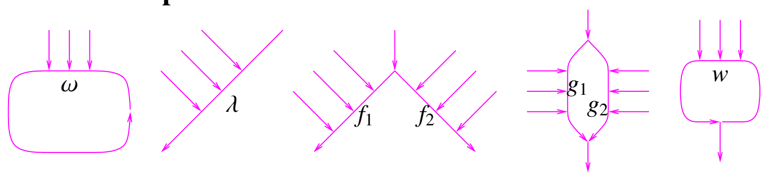
bc. $[b_i,c_j]=0$.

So $a_{ij}f=f^\delta a_{ij}-\frac{b_ic_j}{\delta}(f^\delta-f)$, $[a_{ij},f]=(f^\delta-f)\left(a_{ij}-\frac{b_ic_j}{\delta}\right)$, with $f^\delta:=f/\!/\begin{pmatrix}b_i\to b_i+\delta & b_j\to b_j-\delta\\ c_i\to c_i-\delta & c_j\to c_j+\delta\end{pmatrix}$.

The Ascending Algebra $\mathcal{A}_+^{2Dv}$. Same but with only a_{ij}, $i<j$.

The OneCo Sub-Quotient is $\langle a_{ij}\rangle$ modulo $\delta^2=\delta c_i=c_jc_k=0$, so $\mathcal{L}^{1\text{co}}$ is (coefficient functions non-central, in $\mathbb{Q}[\![b_i]\!]$)

The 1co Graphs.



The Abstract Context. (From LesDiablerets-1508)

Definition. A meta-monoid is a functor M: (finite sets, injections)→(sets) along with natural operations $*\colon M(S_1)\times M(S_2)\to M(S_1\sqcup S_2)$ whenever $S_1\cap S_2=\emptyset$ and $m^{ab}_c\colon M(S)\to M((S\setminus\{a,b\})\sqcup\{c\})$ whenever $a\neq b\in S$ and $c\notin S\setminus\{a,b\}$, such that

$$\text{meta-associativity:}\quad m^{ab}_y/\!/m^{yc}_x=m^{bc}_y/\!/m^{ay}_x$$
$$\text{meta-locality:}\quad m^{ab}_x/\!/m^{de}_y=m^{de}_y/\!/m^{ab}_x$$

and, with $\epsilon_b=M(S\hookrightarrow S\sqcup\{b\})$,

$$\text{meta-unit:}\quad \epsilon_b/\!/m^{ab}_a=Id=\epsilon_b/\!/m^{ba}_a.$$

Theorem. $S\mapsto\Gamma_0(S)$ is a meta-monoid and $z_0\colon P\!\!\!/T\to\Gamma_0$ is a morphism of meta-monoids.

Theorem. There exists an extension of Γ_0 to a bigger meta-monoid $\Gamma_{01}(S)=\Gamma_0(S)\times\Gamma_1(S)$, along with an extension of z_0 to $z_{01}\colon P\!\!\!/T\to\Gamma_{01}$, with

$$\Gamma_1(S)=R_S\oplus V\oplus V^{\otimes 2}\oplus V^{\otimes 3}\oplus\mathcal{S}^2(V)^{\otimes 2}\qquad(\text{with }V:=R_S\langle S\rangle).$$

Furthermore, upon reducing to a single variable everything is polynomial size and polynomial time.

Furthermore, Γ_{01} is given using a "meta-2-cocycle ρ^{ab}_c over Γ_0": In addition to $m^{ab}_c\to m^{ab}_{0c}$, there are R_S-linear $m^{ab}_{1c}\colon\Gamma_1(S\sqcup\{a,b\})\to\Gamma_1(S\sqcup\{c\})$, a meta-right-action $\alpha^{ab}\colon\Gamma_1(S)\times\Gamma_0(S)\to\Gamma_1(S)$ R_S-linear in the first variable, and a first order differential operator (over R_S) $\rho^{ab}_c\colon\Gamma_0(S\sqcup\{a,b\})\to\Gamma_1(S\sqcup\{c\})$ such that

$$(\zeta_0,\zeta_1)/\!/m^{ab}_c=\left(\zeta_0/\!/m^{ab}_{0c},(\zeta_1,\zeta_0)/\!/\alpha^{ab}/\!/m^{ab}_{1c}+\zeta_0/\!/\rho^{ab}_c\right)$$

OneCo Monoblog.

(1504) If $S_n:=\sum_{k=0}^{n-1}A^kCB^{n-1-k}$ then $AS_n-S_nB=A^nC-CB^n$ so $S_n=(L_a-R_B)^{-1}(A^nC-CB^n)$.

(160317) To do: For 0-co a and b, compute the 1-co part of $e^{-a}be^a$.

(151019d) Perhaps I should switch to a circuit algebra perspective, plus meta-monoid ops.

(151019c) Make the braid representation presentable?

(151019b) Switch to an EK basis?

(151019a) To do: Find and implement the group-like condition.