Shmuel Weinberger at Colloquium

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Milnor 1950's: $\exists M^7$ s.t. $M^7 \overset{①}{\underset{\text{homeo}}{\approx}} S^7$ but

$$M^7 \overset{②}{\underset{\text{diffeo}}{\not\approx}} S^7$$

①: By finding an explicit "Morse function".

②: Using "secondary invariants".

Let W^{4k} be a compact orientable mfld,

$$H^{2k}(W;\mathbb{R}) \times H^{2k}(W;\mathbb{R}) \xrightarrow{\int \cdot \cap} \mathbb{R}$$

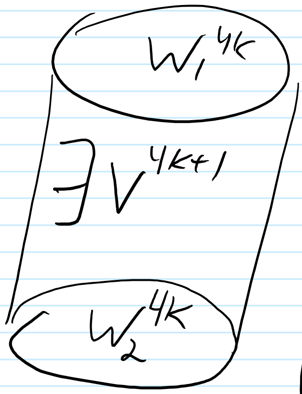
Symmetric non-degenerate.

Diagonalize to $\begin{pmatrix} 1 & & & & \\ & \ddots & & & \\ & & -1 & & \\ & & & \ddots & \\ & & & & -1 \end{pmatrix}$, get a "signature", $\operatorname{sign}(W)$.

This is a "primary invariant". Properties:

① oriented homotopy invt.

② "Cobordism invariance"



$\Longrightarrow \operatorname{sign}(W_1) = \operatorname{sign}(W_2)$

(Exercise:

$\operatorname{Im}(H^{2k}(V) \to H^{2k}(W))$ satisfies

1. $\cup$ is trivial on image

2. $\dim \operatorname{Im} = \frac{1}{2} \dim H^{2k}(W)$)

Back to exotics: $M^7 = \partial W^8$ w/ $QF = E_8$,

which is positive definite. Also, $\operatorname{sign} W^8 = 8$.
And W^8 has trivial normal bundle in $\mathbb{R}^{10}$.

Hirzebruch signature formula: $\operatorname{sign}(V^8) = \dfrac{7p_2 - p_1^2}{45}$