A question about faithfulness

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For the purpose of my slow-moving paper at http://drorbn.net/AcademicPensieve/Projects/ExQu/, I'm interested in the following question:

It is easy to see that every pure tangle group, even pure virtual tangle group, has a multiplicative expansion. When are these expansions faithful? Note that I'm not asking if the group detects the tangle, only if the expansion detects the identity of the group.

Let T be a pure (virtual) tangle, let G be its group, and let Z:G->A be an expansion for G, where A=gr(QG).

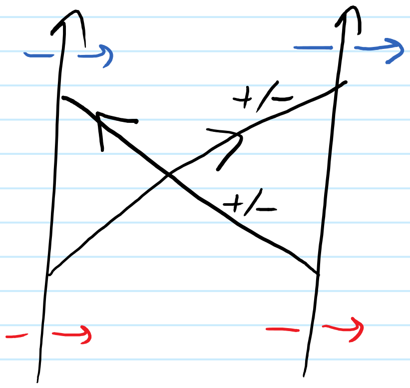
If T is a -strand u-tangle, Z is not faithful (unless T is trivial) because A is Abelian. For similar reasons Z is not faithful if T is obtained by injecting some non-trivial 1-strand u-tangle inside some other pure tangle.

On the other hand, if T is a braid then G is a free group and Z is faithful. More generally if T is "acyclic", meaning that you cannot form a cycle by walking along T in the direction of the strands while sometimes dropping down at a crossing, but never climbing up (example below). In that case G is again free so Z is faithful.

Do we know anything more? What if we restrict to u-tangles? (Restriction to w-tangles is irrelevant because they have the same fundamental groups as v-tangles).

An acyclic (virtual) pure tangle

I'm only showing a Gauss diagram, and the signs of the crossings do not matter:



The red elements generate the fundamental group, which is therefore free on two generators.

The blue elements do not generate the fundamental group.