Pensieve header: The Gauss-Gassner-Alexander scattering.

```
Simplify[x1 /. Solve[{
     x1 == a11 y1 + a21 y2 + a31 y3,
     x2 == a12 y1 + a22 y2 + a32 y3,
     x3 == a13 y1 + a23 y2 + a33 y3,
     x1 == y2, x2 == 0, y3 == 1},
    {x1, x2, x3, y1, y2, y3}]]
```

$$\left\{\frac{\text{a12 a31} - \text{a11 a32}}{\text{a12} - \text{a12 a21} + \text{a11 a22}}\right\}$$

```
Simplify[x2 /. Solve[{
     x1 == a11 y1 + a21 y2 + a31 y3,
     x2 == a12 y1 + a22 y2 + a32 y3,
     x3 == a13 y1 + a23 y2 + a33 y3,
     x2 == y3, x1 == 0, y2 == 1},
    {x1, x2, x3, y1, y2, y3}]]
```

$$\left\{\frac{-\text{a12 a21} + \text{a11 a22}}{\text{a11} + \text{a12 a31} - \text{a11 a32}}\right\}$$

```
Simplify[x1 /. Solve[{
     x1 == a11 y1 + a21 y2 + a31 y3,
     x2 == a12 y1 + a22 y2 + a32 y3,
     x3 == a13 y1 + a23 y2 + a33 y3,
     x1 == y2, x2 == 1, y3 == 0},
    {x1, x2, x3, y1, y2, y3}]]
```

$$\left\{\frac{\text{a11}}{\text{a12} - \text{a12 a21} + \text{a11 a22}}\right\}$$

```
Simplify[x2 /. Solve[{
     x1 == a11 y1 + a21 y2 + a31 y3,
     x2 == a12 y1 + a22 y2 + a32 y3,
     x3 == a13 y1 + a23 y2 + a33 y3,
     x2 == y3, x1 == 1, y2 == 0},
    {x1, x2, x3, y1, y2, y3}]]
```

$$\left\{\frac{\text{a12}}{\text{a11} + \text{a12 a31} - \text{a11 a32}}\right\}$$

$$\begin{pmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \end{pmatrix} = \text{Inverse}\left[\begin{pmatrix} b11 & b12 & b13 \\ b21 & b22 & b23 \\ b31 & b32 & b33 \end{pmatrix}\right]$$

```
{{(-b23 b32 + b22 b33) /
    (-b13 b22 b31 + b12 b23 b31 + b13 b21 b32 - b11 b23 b32 - b12 b21 b33 + b11 b22 b33),
   (b13 b32 - b12 b33) / (-b13 b22 b31 + b12 b23 b31 + b13 b21 b32 -
      b11 b23 b32 - b12 b21 b33 + b11 b22 b33), (-b13 b22 + b12 b23) /
    (-b13 b22 b31 + b12 b23 b31 + b13 b21 b32 - b11 b23 b32 - b12 b21 b33 + b11 b22 b33)},
  {(b23 b31 - b21 b33) / (-b13 b22 b31 + b12 b23 b31 + b13 b21 b32 -
      b11 b23 b32 - b12 b21 b33 + b11 b22 b33), (-b13 b31 + b11 b33) /
    (-b13 b22 b31 + b12 b23 b31 + b13 b21 b32 - b11 b23 b32 - b12 b21 b33 + b11 b22 b33),
   (b13 b21 - b11 b23) / (-b13 b22 b31 + b12 b23 b31 + b13 b21 b32 -
      b11 b23 b32 - b12 b21 b33 + b11 b22 b33)}, {(-b22 b31 + b21 b32) /
    (-b13 b22 b31 + b12 b23 b31 + b13 b21 b32 - b11 b23 b32 - b12 b21 b33 + b11 b22 b33),
   (b12 b31 - b11 b32) / (-b13 b22 b31 + b12 b23 b31 + b13 b21 b32 -
      b11 b23 b32 - b12 b21 b33 + b11 b22 b33), (-b12 b21 + b11 b22) /
    (-b13 b22 b31 + b12 b23 b31 + b13 b21 b32 - b11 b23 b32 - b12 b21 b33 + b11 b22 b33)}}
```

$$\left\{\frac{a11}{a12 - a12\,a21 + a11\,a22}, \frac{a12}{a11 + a12\,a31 - a11\,a32}\right\} \text{ // Simplify}$$

$$\left\{\frac{-b23\,b32 + b22\,b33}{b13\,b32 + b33 - b12\,b33}, \frac{b13\,b32 - b12\,b33}{b32 - b23\,b32 + b22\,b33}\right\}$$

```
Simplify[x1 /. Solve[{
     x1 == a11 y1 + a21 y2 + a31 y3,
     x2 == a12 y1 + a22 y2 + a32 y3,
     x3 == a13 y1 + a23 y2 + a33 y3,
     x1 == y2, x2 == -1, y3 == 1},
    {x1, x2, x3, y1, y2, y3}]]
```

$$\left\{\frac{a12\,a31 - a11\,(1 + a32)}{a12 - a12\,a21 + a11\,a22}\right\}$$

```
Simplify[x2 /. Solve[{
     x1 == a11 y1 + a21 y2 + a31 y3,
     x2 == a12 y1 + a22 y2 + a32 y3,
     x3 == a13 y1 + a23 y2 + a33 y3,
     x2 == y3, x1 == -1 y2 == 1},
    {x1, x2, x3, y1, y2, y3}]]
```

$$\left\{\frac{a12 + a12\,a21 - a11\,a22}{a11 + a12\,a31 - a11\,a32}\right\}$$